Temperature Variation With Input Power in an Electrical Furnace

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Temperature control of an electrical furnace is generally accomplished by means of a servo mechanism. For simple laboratory experiments, however, it may be desirable to produce a selected temperature rise by manual control of a variac or rheostat. A procedure is offered here for producing a linear temperature rise in a furnace at desired points called temperature controlled points. A power function dependent only on time and/or temperature is used. This function is a correspondence of current needed in the coil (to produce the desired temperature rise) versus time and/or temperature.

Two procedures are offered. In the first method we derive an equation for the power function from considerations of Newton's law of cooling and energy conservation. This is applicable only in the near-reversible processes where some thermodynamic system (including the temperature-controlled points and the heating coil) is in near-equilibrium throughout the process. A well insulated furnace with a highly conductive solid block and a well distributed coil would satisfy this criterion at a sufficiently low heating rate.

The second procedure (more generally applicable) is completely empirical. The heating rate of the furnace is observed as a function of furnace temperature and current. This three-variable relationship is transformed to the power function.

Apparatus

The furnace used in this experiment is designed for Differential Thermal Analysis (DTA). DTA is a method of observing the transitions and reactions that a substance undergoes on heating. For this experiment it was desired that the temperature about the samples in the furnace increase linearly. The sample holder is the flat end of a solid right circular cylinder as shown in Figure (1). The sample to be analyzed and the reference sample are placed about the temperature controlled points A and A' respectively. The points A, A' and C (a temperature checking point) are equally spaced positions on a circle centered on the center of the end of the cylinder. Heat comes radially from the outside since the coil evenly surrounds the cylinder. Thus the temperature at these three points should be equal. In general then, the symmetry of the furnace permits us to have more than one temperature con-

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trolled point. Without some symmetry it might be very difficult or impossible to control the temperature of more than one point.

For development of a power function which will reproduce temperature curves consistently, the conditions of the surroundings must be constant. To avoid the irregularities of radiation and air convection with changing conditions of the atmosphere, the following procedure was used to virtually eliminate these losses: the furnace was surrounded by a protective sand bath and metal container and was immersed in a boiling water bath. Thus the surface of the container was put at a fixed temperature, $T_0 = 100^\circ$C. Radiation was minimized by shielding with aluminum foil.

**Development of Power Function**

**Method One. Theoretical Approach**

The temperature rise of the furnace in a reversible process is related to the heat input by

$$dQ = C_p dT,$$

where $dQ$ is the heat input, $T$ is the temperature of the furnace (measured at the temperature checking point) and $C_p$ is the heat capacity for the constant pressure process. Dividing by $dt$, we have

$$\frac{dQ}{dt} = C_p \frac{dT}{dt}. \quad (1)$$

However, the furnace is not in equilibrium, so the process is not reversible. The theory will be presented here, however, to provide a limiting case of a very well-insulated furnace heated uniformly.
at a slow rate. Thus the furnace will be nearly uniform in temperature and the theory should apply.

The power term, \( \frac{dQ}{dt} \), is dependent on the input power of the coil and the loss of heat to the surroundings. The loss could be due to conduction, convection, or radiation. Since we have virtually eliminated the latter two we can apply Newton's Law of Cooling to find the rate of heat loss due to conduction.

\[
\frac{dQ_1}{dt} = -C(T - T_0),
\]

where \( C \) is a constant, \( Q_1 \) is the heat exchange due to conduction loss, \( T \) is the furnace temperature (at the temperature checking point) and \( T_0 \) is the temperature of the outer surface of the container.

The coil provides a power proportional to the current squared since a manganin coil was used here. The resistance of manganin is nearly independent of temperature.

\[
\frac{dQ_0}{dt} = RI^2,
\]

where \( R \) is the resistance of the coil, \( Q_0 \) is the heat input of the coil and \( I \) is the current.

The net input power then by combining Eqs. (2) and (3) is

\[
\frac{dQ}{dt} = \frac{dQ_0}{dt} + \frac{dQ_1}{dt} = RI^2 - C(T - T_0).
\]

Equating Eq. (1) and (4)

\[
\frac{dQ}{dt} = RI^2 - C(T - T_0) = C_p \frac{dT}{dt}.
\]

Dividing by \( C_p \), redefining constants, and making a change of variables we have

\[
\frac{dT'}{dt} = A_1I^2 - A_2T',
\]

where \( T' = T - T_0 \), \( A_1 = R/C_p \), \( A_2 = C/C_p \).

We wish to develop the power function \( I(t) \) such that the heating rate is constant.

\[
\frac{dT}{dt} = \frac{dT'}{dt} = K.
\]

Thus from Eqs. (5) and (6)

\[
A_1I^2 + A_2T' = K.
\]

Solving for \( I \) we have

\[
I = \sqrt{\frac{K + A_2T'}{A_1}}.
\]

By integrating Eq. 6 we have

\[
T = Kt + T_s,
\]
where $T_s$ is some starting temperature for the experiment. Thus we have a linear temperature rise at rate $K$. The experimenter selects $T_s = T_0$ for the particular experiment. Subsacting $T_0$ we have

$$T' = Kt + T_s - T_0 = Kt + T_s', \quad (9)$$

Inserting Eq. (9) into Eq. (7) we have

$$I = \frac{\sqrt{\frac{K + A_2 (Kt + T_s')}{A_1}}} {\sqrt{\frac{K(1 + A_2 t) + A_2 T_s'}{A_1}}} \quad (10)$$

**Determination of constants.** To determine the constants $A_1$ and $A_2$, we consider two conditions of the furnace.

1. The furnace is heated to a high temperature and is allowed to cool with $I = 0$. Furnace temperature is measured against time by means of a thermocouple and chart recorder potentiometer. From Eq. (5)

$$\frac{dT'}{dt} = -A_2 T'.$$

Integrating,

$$\ln T' = C - A_2 t.$$

Differentiating with respect to $t$,

$$\frac{d(\ln T')}{dt} = A_2.$$

Thus, $A_2$ can be found by plotting $\ln T'$ versus time and finding the slope.

2. A fixed current is maintained and the furnace is allowed to reach a constant equilibrium temperature such that the power input equals the heat loss through conduction.

Thus $dT/dt = 0$.

Inserting this condition into Eq. (5)

$$A_1 I^2 = A_2 T'$$

$$A_1 = A_2 T'/I^2.$$

**RESULTS**

The experimentally determined rate of heating did not correspond well with the theoretical prediction, perhaps because the furnace was not sufficiently insulated or uniformly heated to permit the process to be reversible. A graph of temperature versus time using the power function of time evaluated for $K = 12$ deg/min is given in Fig. (2). The heating rate begins at 38
deg/min and ends at 10 deg/min. The temperature rises most steeply at the beginning of the curve, and the furnace does not lose heat as fast as the theory predicts.

**Method Two, Empirical Approach**

In this procedure, the heating rate, $R$, is simply observed as a function of current and furnace temperature. The furnace is heated at a constant current and the function of temperature, $T$, versus time, $t$, is converted to $T$ vs $R$ for that current. By using several ($N$) currents we have a group of $N$ functions as in Figure (3). These plots were taken only from the parts of the curves where $R$ was near 18°C/min.

By picking a line of constant $R$, intersecting it with the constant current curves, and recording $I$ and $T$ for the intersections, we have a set of $N$ points of $I$ vs $T$ for the selected $R$. Then since we have a linear temperature relationship, the $N$ temperature points can be converted to time points by using Eq. (8).

$$t_n = \frac{T_n - T_s}{K} \quad n = 1, 2, 3 \ldots N$$

The resulting array of $N$ ($I$ versus $t$) points for the given $R$ is plotted and the power function, $I(t)$, is graphically determined. The power function for $R = 18.25$°C/min is shown in Figure (4). The result is shown in Figure (5). The slope tended to increase slightly at higher temperatures but the result agrees fairly well with the theoretical prediction.

All results shown in this paper were computed manually by graphical means. A computer program is being prepared by the author (1) for a more precise calculation of the power function (by the empirical method) for an arbitrary value of $R$. Also an iteration of this procedure has been successfully tried to greatly improve the linearity of the result. Details are available from the author, and they may be published shortly.

**Literature Cited**

Figure 2. (upper left) Result of the power function from the first method.
Figure 3. (upper right) Temperature versus rate, \( R \), from seven (\( N = 7 \)) constant current, \( T \) versus \( t \) curves. Note that each plot has taken over a very small temperature interval and the \( R \) axis is considerably expanded. Thus well defined temperatures are indicated by the intersections with the selected \( R \) line.
Figure 4. (lower left) Power function for \( R = 18.25 \). This was plotted from the plots of Figure (3).
Figure 5. (lower right) Result of applying the power function of Figure (4). The slope is 18.66° C/min which is 2.2% high.