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THE ELECTRICAL CAPACITY OF SIMILAR, NON-PARALLEL PLANE PLATES, AND ITS APPLICATION WHERE THE PLATES ARE NON-RECTANGULAR.

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An investigation in which it was necessary to find the distance between plane, non-parallel plates in terms of their electrical capacity raised the question of the amount of error in applying the equation for parallel plates. An equation for non-parallel plates was derived.

By a special transformation with conjugate functions it is shown that in an extensive divided conducting plane, where the line of division is straight, if a positive charge resides on the conductor on one side of the line, and a corresponding negative charge on the other, the lines of force are arcs of coaxial circles having the line of division for axis. The equipotentials are thus planes passed through the line of division. If two of these equipotentials be taken with a small included angle, and if equal and opposite limited areas of the equipotentials be considered, figure 33, the case is that of the non-parallel plates in the experiment, neglecting edge corrections. If the limited areas are supposed rectangular the capacity equation is

\[ C = \frac{W}{4\pi} \log\left(\frac{r_2}{r_1}\right) \frac{1}{\sqrt{\epsilon}} \]  
(see figure 35) (1)

This equation is obtained by assigning definite potentials to the two plates and integrating the expression for density of sur-

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face charge $\sigma$ between the limits $r_1$, $r_2$, which results in an expression for $Q$, the total charge on one of the plates. Omission of the potential difference from this expression leaves the equation for capacity in terms of distance. The capacity for any shape of the opposed limited areas of the equipotentials can be similarly obtained by suitable integration of $\sigma$ over one of the areas, neglecting edge corrections for actual plates when their distance of separation is small compared with the other dimensions.

In the experiment referred to the plates were non-rectangular, figure 36. Choose $x$-and $y$-axes as indicated in the figure. Considering the element of area, $dy \, dx$, its charge is $dq = \sigma \, dy \, dx$, where $\sigma = 1/4 \pi (V_1 - V_2)/\Theta \cdot 1/y$. Integrate first over the strip parallel to the $x$-axis of width $dy$, and let the charge on the strip be represented by $dQ$. Thus,

$$dQ = 1/4 \cdot \pi (V_1 - V_2)/\Theta \cdot dy / y \cdot x$$

where $x = 1/m \cdot (y + mx_1 - y_1)$, if $m$ is the slope of the slant edge. The equation for this edge is, $y = mx + b$, where $m = (r_2 - r_1)/(w_1 - w_2)$, and $b = r_2 - mw_1$. (In figure 36, $y_1 = r_1$, $y_2 = r_2$, $w_1 = x_2$, $w_2 = x_1$). The next integration is that of the strips between the limits $r_1$, $r_2$, and we get, after canceling the potential difference,

$$C = 1/4 \pi \cdot [(w_2 r_2 - w_1 r_1)/(r_2 - r_1) \cdot \log (r_2/r_1) \cdot (w_2 - w_1)] / \Theta$$
It is seen that in the case of rectangular plates equation (3) reduces to equation (1).

The neglect of edge corrections in equations (1) and (3) is justified on the same grounds as the similar neglect in the well-known formula for parallel plates, \( C = \frac{S}{4 \pi d} \).

In equations (1) and (3) \( \Theta \) may be expressed in terms of \( r \) and \( d \), figure 34, where \( d \) the chord is taken equal to the arc. Arbitrary selection of \( r \) will then fix the points on the plates between which \( d \) is measured. Take \( r = \frac{(r_1 + r_2)}{2} \) in equations (1) and (3) to express the mean distance between the non-parallel plates in the two cases. Suppose the equation for parallel plates to be applied to non-parallel plates. Equation (3) is applicable in the experiment, while the formula for the former case was used. Equating the capacities for the two cases, we obtain

\[
d_e = k \cdot d,
\]

where \( d \) is the computed distance for parallel plates of a given capacity, \( d_e \) is the correct mean value for the actual non-parallel plates having the same capacity, and \( k \) the correction factor.

Thus,

\[
k = \frac{(r_1 + r_2)}{(r_2 - r_1)} \cdot \frac{(w_1 + w_2)}{(w_2 r_2 - w_1 r_1)} \cdot \frac{(r_2 - r_1)}{\log(r_2 - r_1) - (w_2 - w_1)}
\]

In the experiment the value of \( k \) was not far from unity.