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An Interesting Case of Resonance in an Alternating Current Circuit

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AN INTERESTING CASE OF RESONANCE IN AN ALTERNATING CURRENT CIRCUIT.

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The phenomena of voltage and current resonance are familiar to all students of alternating currents. The former occurs in series circuits and complete resonance is secured when the condensive reactance is equal to the inductive reactance. The latter occurs in connection with parallel circuits, the necessary condition being that the sum of all the susceptances, both condensive and inductive, equals zero. The expression for the impedance of a series circuit is $Z = \sqrt{r^2 + (2\pi f L - \frac{1}{2\pi f C})^2}$ This becomes a minimum when the condensive reactance, $\frac{1}{2\pi f C}$, just balances the inductive reactance, $2\pi f L$. This occurs at a frequency $f = \frac{1}{2\pi\sqrt{LC}}$. As the current is equal to E/Z, this is also the condition for maximum current and as the current is in phase with the voltage, the power-factor is unity.

If the frequency is less than that determined by the above expression then the reactance of the condenser becomes greater and that of the inductance less. The result is that the current becomes smaller and smaller with decrease in frequency and leads by an increasing angle. If, on the other hand, the frequency is increased, the inductive reactance is made more prominent and the condensive reactance is reduced. The current becomes smaller and smaller and lags by an increasing angle.

Thus we see that in a series circuit, as the frequency is increased the current begins at a small value, increases to a maximum and then returns to a small value again. At the same time the power-factor increases to unity and then decreases. The current leads for the lower frequencies and lags for the higher. Therefore, with a given voltage, as the frequency is increased every value of current or power-factor occurs twice, since each value of current or power-factor that is obtained at a frequency less than that required for resonance occurs again at some higher frequency.

These facts are clearly brought out in figure 23 (c) in which is plotted the curve representing the variation with change of frequency of the admittance[1] of the series portion of the circuit represented diagrammatically in the figure. Figure 23 (b), which is a polar diagram, shows the admittance plotted vectorially. It shows that at a frequency of thirty cycles the current leads the applied voltage by nearly ninety degrees. As the frequency is increased the phase angle becomes less and the current greater, the change in both being very pronounced in the neighborhood of the resonance frequency, which is a trifle less than sixty cycles.

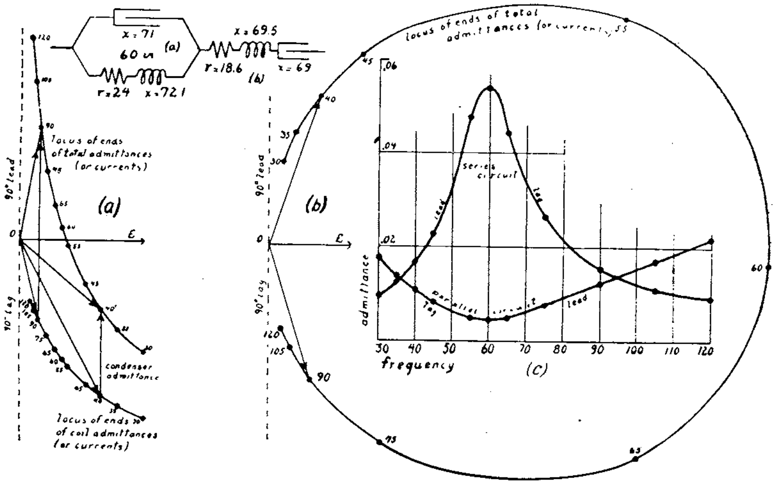

Figure 23

Just the opposite occurs in a circuit in which the inductance and capacitance are in parallel. The current becomes a minimum at resonance, lags for low frequencies and leads for high frequencies. This is brought out in figure 23 (c), and in the polar diagram, figure 23 (a). As it is not practicable to draw a large number of vectors in this and other figures only a few are shown. The end points of a few more are indicated by large dots marked with the corresponding frequency. These are sufficient in number not only to determine the hodograph which is shown as a heavy continuous line but also to enable one to estimate the vector corresponding to any frequency.

[1]The admittance of an alternating current circuit is that factor which multiplied by the applied voltage gives the current. It is therefore proportional to the current and equal to the current flowing with unit voltage impressed on the circuit.

One example will show how the positions of the points on such a hodograph are determined. If we assume a voltage of forty volts to be impressed upon the parallel portion (a) of the circuit the coil branch receives a current which can be represented by the vector 0 40, while the condenser branch receives a current 40 40. The total current will be represented by the vector sum of the two, which is the vector 0 40. It is possible to compute the magnitude and phase of the current for each frequency, as indicated by the lower hodograph and to add to these the corresponding currents in the condenser branch giving the upper hodograph as the locus of the ends of the total currents or total current hodograph. As shown by the vector diagram and the curve of figure 23 (c) the total current in such a parallel circuit lags at low frequencies and with increase of frequency decreases in magnitude, passes through a minimum near the frequency corresponding to unity power-factor, and then increases as a leading current. Thus in the case of inductance and capitance in parallel there are two frequencies at which the current has the same value and two frequencies at which the power-factor has the same value.

A consideration of the characteristic current curves for series and parallel circuits containing both inductive and condensive reactance led the writer to believe that it might be possible to combine a series and a parallel circuit in such a way as to obtain much more complicated phenomena than those just described. It was conceived as possible that identical values of current and power-factor might be obtained at more than two frequencies. A little thought at once revealed the inadequacy of any conclusion based on current (admittance) curves like those of figure 23 (c) for they take no account of phase relations. Also it is from the standpoint of impedance that circuits are added in series. Some rough calculations led to the belief that such a circuit as is represented diagramatically in figure 23 would yield interesting results.

The circuit was made up of inductance coils, non-inductive resistances, and telephone condensers. A variable frequency generator was employed and the voltage kept constant at fifty volts. The current curve represented by the heavy line of figure 24 (e) was secured. This curve was something of a disappointment but the power-factor curve, figure 24 (d) was more encouraging. Two attempts were then made to adjust the con-

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stants of the circuit so as to improve the results but they proved unsuccessful, showing that a careful analysis had to be made before any accurate prediction of the effect of changes in the circuit could be made. Some of this work has already been described.

Although the current characteristics of the circuits gave the clue to the results to be expected it is, of course, as impedances that the two circuits must be added. The impedance of the parallel portion (a) and the series portion (b) of the total

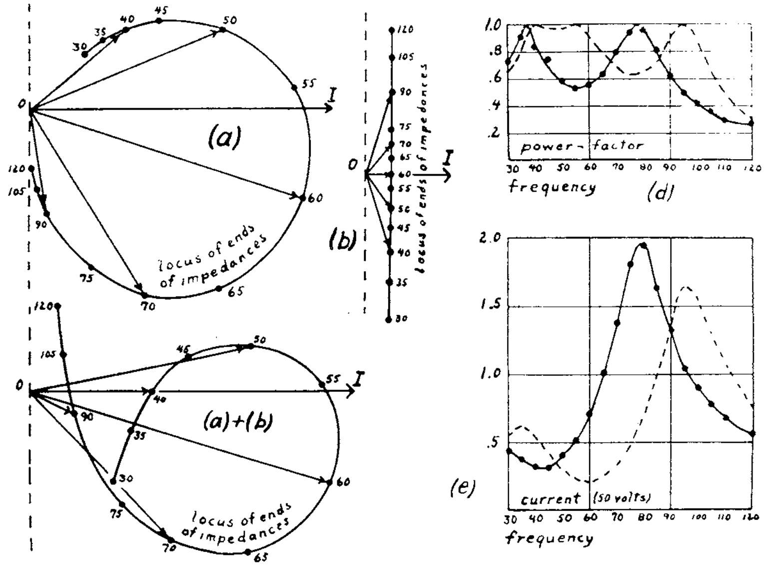
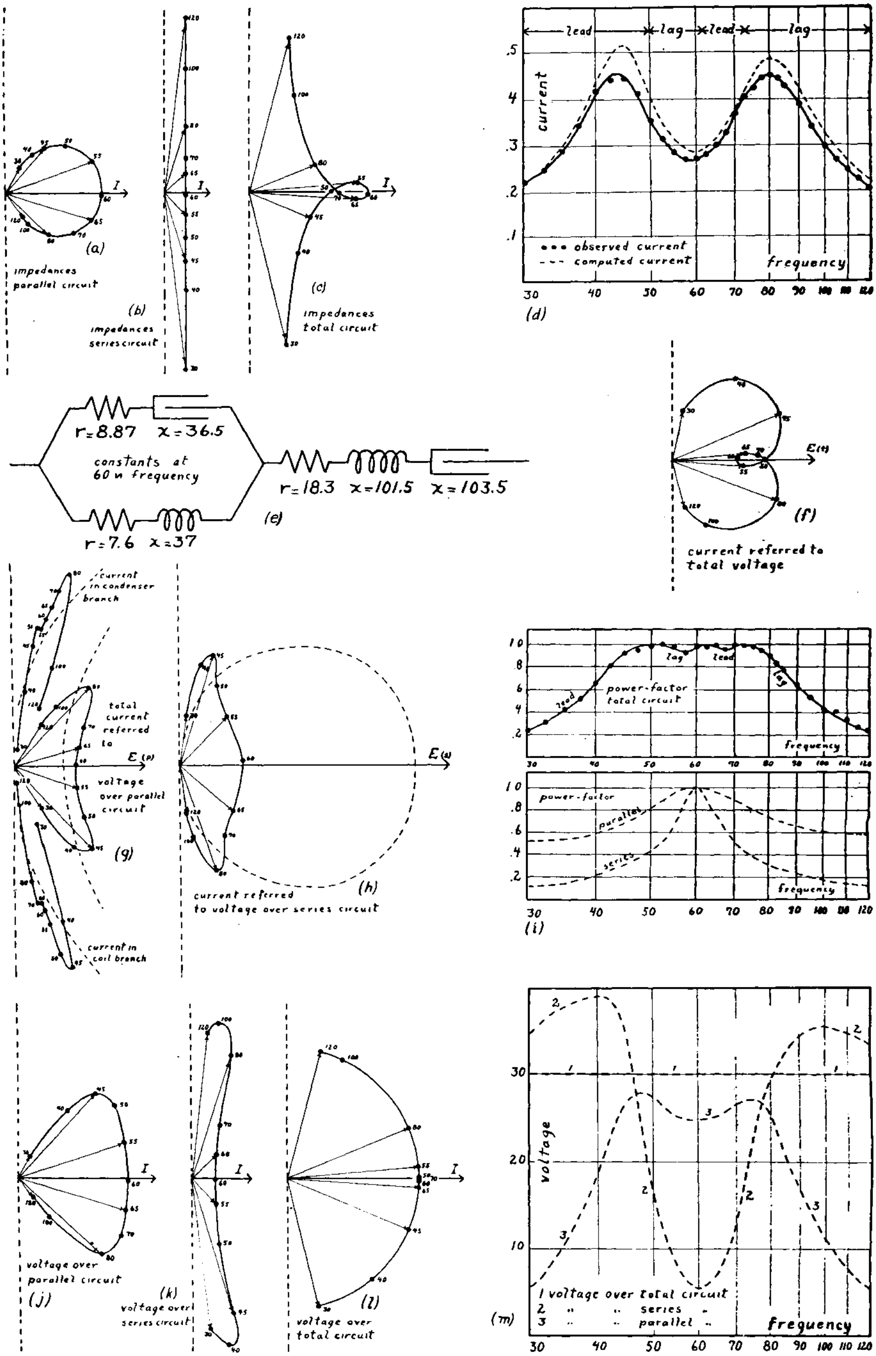


FIGURE 24

circuit are plotted in figure 24 (a) and (b). They are plotted vectorially and when added give figure 24 (a)+(b) which represents the change in the total impedance with variation of frequency. From this figure the broken curves of figure 24 (d) and (e) are computed.

If the total impedance hodograph is followed one will see that with increase of frequency (voltage constant) the corresponding current must increase slightly from thirty to thirty-five cycles and then decreases until sixty cycles is reached. It will then increase to a maximum at about ninety-five cycles and finally decrease. The power-factor increases rapidly to unity at forty cycles. Between forty and fifty-five cycles it is very nearly unity and the current is lagging. From fifty-five

PLATE X.


(a)
impedances parallel circuit
(b)
impedances series circuit
(c)
impedances total circuit
lead
lag
lead
lag
current
observed current
computed current
frequency
(d)
r = 8.87 x = 36.5
constants at 60 ~ frequency
r = 18.3 x = 101.5 x = 103.5
r = 7.6 x = 37
(e)
E(t)
(f)
current referred to total voltage
current in condenser branch
total current referred to voltage over parallel circuit
E(p)
(g)
current in coil branch
E(s)
(h)
current referred to voltage over series circuit
power-factor total circuit
lead
lag
lead
lag
frequency
power-factor
parallel
series
frequency
(i)
(j)
voltage over parallel circuit
(k)
voltage over series circuit
(l)
voltage over total circuit
voltage
1 voltage over total circuit
2 " " series "
3 " " parallel "
frequency
(m)

to ninety-five cycles the power-factor decreases, reaching a minimum again and thereafter increasing rapidly. The broken curves of current and power-factor, figure 24 (d) and (e), are computed from the impedance diagram and their wide divergence from the curves showing the actual conditions is undoubtedly evidence, not of the inaccuracy of the experimental and theoretical work, but of the disturbing effect of higher harmonics in the voltage wave.

A consideration of both the observed and computed results led to the conclusion that it would be well to make the resistances of the two branches of the parallel circuit practically equal and to equalize the reactances of both the parallel and series circuits for sixty cycles, this frequency being mid-way (in effect upon reactance) between thirty and 120 cycles, the limits of the generator. It was also thought best to reduce the reactances of the parallel circuit and to increase those of the series circuit. After a few trials the circuit of Plate X (e) was built up with very satisfactory results which will be discussed in some detail. In order that higher harmonics might be eliminated as far as possible a coil of seventy ohms reactance (sixty cycles) was inserted in series with the circuit to be studied. This served its purpose very well as is indicated in Plate X (d) in which the computed current and observed current do not differ materially.

Plate X (a) is a polar diagram of the impedances of the parallel circuit. The symmetry of the figure should be noted, the result of balancing the susceptances at sixty cycles and working over a range of thirty to 120 cycles. Plate X (b) is a similar diagram for the series portion of the circuit. These two diagrams bring out in a very striking way the difference in the characteristics of the two circuits. The phase relations are just opposite, as is also the manner in which the magnitude of the impedance changes with variation in the frequency. By adding the impedances of the two circuits, frequency by frequency, Plate X (c) is obtained. At extreme frequencies the effect of the series circuit predominates. Near sixty cycles the effect of the parallel branch becomes more important and as a result the total impedance passes through two minima and one maximum. The total impedance hodograph winds up on itself and takes on a form very much like that of a strophoid. This diagram also shows how the voltage is first lagging, then leading, then lagging, and finally leading at the higher frequencies. It is also clear that the power-factor passes through unity at three different values of frequency.

The current in the circuit, as the frequency is varied (constant voltage of thirty volts), can be computed from the total impedances and is represented as a curve in Plate X (d) and vectorially in Plate X (f). The current passes through two maxima and one minimum and as the frequency is increased changes from leading to lagging to leading to lagging. The vector diagram shows clearly how the power-factor becomes unity at three different values of frequency. The current hodograph winds up on itself and is similar in form to the limacon.

Several very interesting results have been accomplished with this circuit. The curves and diagrams have all been made symmetrical. The current has been made to pass through two maxima and the minimum has been so controlled as to be practically equal to the end values. The power-factor has been made to remain at practically unity for a range of frequency from forty-five to seventy-eight cycles and to drop off rapidly outside of these frequencies.

We will now examine the conditions existing in the circuit in detail so as to see the causes which contribute to and control these results. We have already seen that the choice of constants, so as to secure balanced reactances at sixty cycles, is responsible for symmetrical results with a range of frequency of thirty to 120 cycles.

The control of the relative magnitude of the current at extreme frequencies and the minimum value is through the choice of the resistance and reactance of the series circuit (a given parallel circuit being assumed). If the resistance be made greater there is very little difference in the total impedance at thirty and 120 cycles. Change of resistance is, however, of great importance in the neighborhood of sixty cycles for there the series impedance is made up entirely of resistance. Thus a variation of the series resistance can be made to control the minimum value of the total current.

On the other hand the magnitude of the impedance of the series circuit at the extreme frequencies is changed enormously by any change in the inductance or capacity. Thus these values of current may be controlled by a variation in the reactances of the series circuit.

It is quite difficult to see the effect of a change in the constants of the parallel circuit. In general we may say that with increase of resistance in the branches the end points of the impedance vectors are crowded around on the hodograph toward the axis, so that the circuit becomes less sensitive to changes in frequency. If, however, the resistance is small the crowding is at the ends and the impedance sweeps around through a large angle with a very slight change of frequency in the neighborhood of resonance and the change for extreme frequencies is correspondingly smaller. A change in the reactance tends to make the whole figure smaller or larger, but unless the resistance is changed in the same proportion the character of the figure will be altered as already explained.

The control of the power-factor is of course dependent upon the same factors as the control of the current. If the power-factor is to be made to remain at values close to unity over a wide range of frequency it is necessary that the "wound up" portion of the impedance hodograph represent a large part of the total frequency variation. This may be done by decreasing the reactances of the series branch. If, on the contrary, the series reactances are made greater the range over which the power-factor is practically unity is made smaller but over this range the power-factor remains very much closer to unity.

Thus far we have interpreted the circuit from the standpoint of impedances. It is very helpful to study the relations of the current to the voltages over the two portions of the circuit and to see how the current divides in the parallel circuit. Plate X (g) shows how the branch currents vary with frequency, and they, together with the total current, are plotted vectorially with the voltage over the parallel circuit as the axis of reference. One notices that the current in the condenser branch always leads by a large angle and that the current in the coil branch lags by a large angle. It is evident that the changes in phase and magnitude of the two currents, as the frequency changes, are just opposite. When the branch currents are added vectorially, frequency by frequency, the total current hodograph is obtained. The lengths of these total current vectors are equal, at each frequency, to those of diagram (f). The difference in the shape of the hodographs lies in the fact that diagram (f) is plotted with the total voltage as reference and diagram (g) with the voltage over the parallel circuit as reference. Plate X (h) shows this same total current plotted with the voltage over the series circuit as the axis of reference.

Just why the current hodographs have these particular shapes is best understood by a consideration of the voltages over the two parts of the circuit. Plate X (j) is a polar diagram of the voltage over the parallel portion and Plate X (k) is a similar diagram for the series circuit. If these two voltages are added, frequency by frequency, the total voltage diagram, Plate X (l) is obtained.

Returning to diagram (g) which represents the currents in the parallel circuit, we find three broken curves. These hodographs represent the currents that would exist in the parallel circuit with a constant voltage of twenty volts. With increase of frequency the condenser current would become greater, while at the same time the coil current would become smaller. At sixty cycles the two would be equal. The resultant total current vectors would all end along the broken line symmetrical with the voltage axis. The current would be a minimum at sixty cycles and in phase with the voltage. It would increase in magnitude and lag by a larger angle with decreasing frequencies and with increase of frequency would increase in value and lead by an increasing amount. In order for the hodograph to bend back violently, as it actually does, the voltage must be greatly reduced at the more extreme values of frequency. If we examine diagram (j) we find that such is the case and if we examine diagram (k) we see the reason, for the series circuit with its high impedance at extreme frequencies requires by far the greater voltage.

If we now look to the broken curve of diagram (h) which is the current hodograph for the series circuit on the basis of a constant voltage of twenty volts, we see that the current has been reduced far below normal for the frequencies close to sixty cycles. It must be that at these frequencies the voltage is very much reduced for the series circuit and correspondingly large for the parallel circuit. A glance at diagrams (k) and (j) shows this to be the case.

The hovering of the power-factor around unity must of necessity appear in diagram (l) in which the total voltage is plotted vectorially with the total current as the axis of reference. As the frequency increases the voltage vector swings from a lagging position up beyond the axis. It then swings below the axis and finally swinging across the axis for the third time continues to lead the current at an increasing angle.

The lower part of Plate X (i) shows the power-factors of the parallel and series circuits for comparison with the power-factor of the whole circuit. In Plate X (m) are plotted the values of the three voltages.

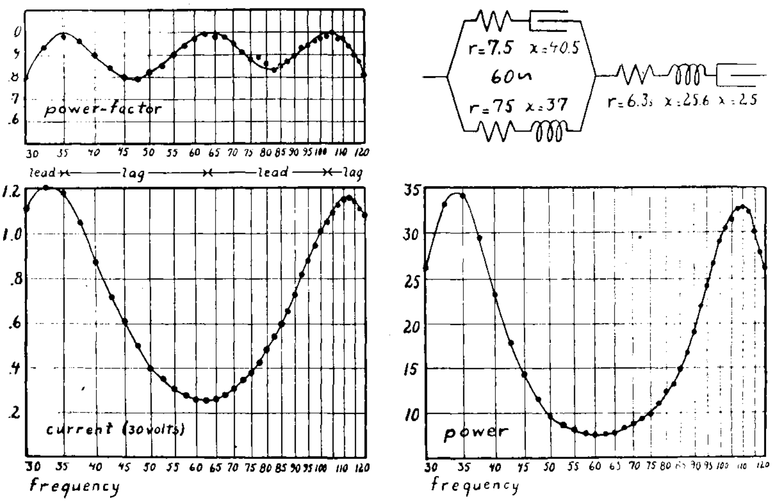


FIGURE 25

The control which can be exercised over the results is very well illustrated in figure 25. It was decided, if possible, to cause the power-factor to have two minimum values equal to each other and equal to the values at the extreme frequencies. A glance at Plate X (c), (f), or (l) will show that this necessitated a nice adjustment of the constants. Figure 25 gives the circuit and the results. The current curve and the power curve are also interesting.

In the two circuits that have just been described an attempt was made to secure certain definite results, one of them being that the curves should be of symmetrical form. It is perfectly evident that current and power-factor curves of various forms might be secured by the proper choice of constants. For instance, if the frequencies at which the two portions of the circuit passed through resonance were not the same, the "wind up" parts of the total impedance and current hodographs could be made to occur either above or below the axis or be brought tangent to the axis. In this way the current curves could be made to assume a large variety of forms and the power-factor to hover around any chosen value. Many other results could be obtained by the variation of the fundamental constants together with the frequency or with the frequency constant. The present discussion is intended to be suggestive of the possibilities of an alternating current circuit. By variation of the frequency it was possible to make every reactance change and secure particularly complicated and unusual results. Equally interesting results could be obtained by keeping the frequency constant and varying one or more of the constants of the circuit, there being an infinite number of ways in which this could be done.

In working out this problem the writer was not concerned with any particular application. But since every alternating current circuit and machine can be reduced to an equivalent circuit of this kind a study of an artificial circuit cannot but be suggestive. In fact variation of frequency is nothing but change of speed and the starting of a synchronous motor may be compared to what goes on in this circuit. In actual practice the constants of a circuit often depends upon the other constants or upon the current, or still other factors. If these relationships are known the equivalent artificial circuit may be altered accordingly.

Again, the different frequencies may be regarded as representing the various harmonics usually present in the voltage of a generator. The response of a circuit to the various harmonics will, as is well known, always cause conditions in the circuit quite different from what would be expected from sine wave theory. Usually the assumption is made that inductance tends to choke out the higher harmonics and capacitance to emphasize them. However, the circuits which have been considered make it perfectly evident that a complex circuit may exercise a surprising selective action.

Such a circuit as this is also suggestive in connection with telephone engineering, where a large range of frequency must be considered, and would have direct bearing upon the construction of any instrument or circuit in which any factor, as for instance, the current or the impedance, was required to vary with the frequency according to some arbitrary law. But even though practical applications may be remote it is hoped that attention has been called to phenomena which are at least curious, interesting and suggestive.

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