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A PHOTOGRAPHIC DETERMINATION OF THE FORM
OF AN AIRPLANE LOOP, AND THE DY-
NAMICS OF THE LOOP

L. P. SIEG

Introduction. During a greater part of the years 1918 and 1919, the writer was engaged at Langley Field, Virginia, on experimental work connected with the determination of bomb trajectories. The work was under the direction of Dr. A. Wilmer Duff, as Engineer-in-charge, and the writer as assistant engineer. The credit for the development of the photographic method, described below, is due to Doctor Duff. The details connected with this paper, have been worked out by the writer as an interesting and important side issue of the main problem.

Method and Apparatus. In order to gain information as to the actual path of an airplane during any manoeuvre, for example the loop, it is necessary to have some method of accurately surveying its positions in space as a function of the time. Whatever method could be used for accurately surveying the path of a falling bomb, could obviously be used for the present problem. If the flying is done at night, if there is a light on the airplane sufficiently bright to affect a photographic plate at a distance of, say two miles, and if, to record the path of this light, there are two cameras which are situated at the ends of a measured base line, are raised to known elevations, and are pointing in known directions; then it is possible to reduce the two plates so as to give the X, Y, and Z co-ordinates of the path of the plane, referred to any arbitrary origin. If in addition one has a means of obscuring the camera lenses simultaneously at regular intervals, say every second, he has in addition the necessary time element to completely define the path of the plane as a function of the time. Such apparatus was installed at Langley Field for the regular program of trajectory determinations, and it served of course equally well for the present problem. There is neither space nor occasion for an elaborate description of this apparatus, nor of the labor involved in meeting the practical difficulties one by one until accurate results were possible. But the briefest description, therefore, will be given of the apparatus and method.

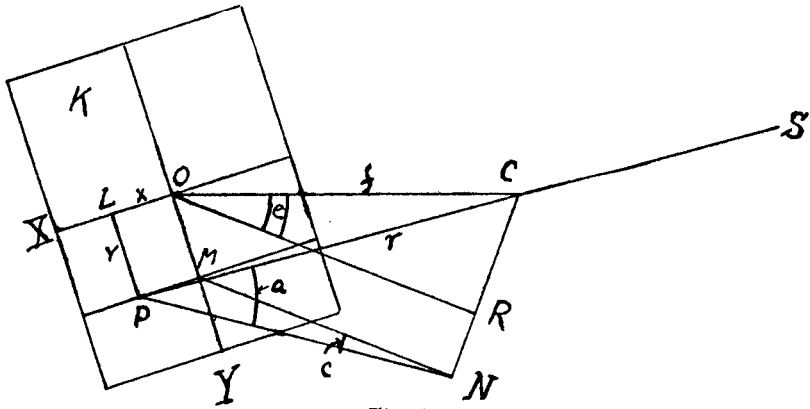


Fig 41

In figure 41 let K be the plate in the tilted camera. Let P be the image of a point on the path of the loop, the ray coming from the direction S , through the center of the lens C . The focal length f is given by the distance CO . Take the horizontal line OX and the line OY perpendicular to it as axes of reference. CRN is a vertical line through C . OY is in the vertical plane through OC . Horizontal lines OR and MN in this vertical plane intersect CRN in R and N respectively. The angle MNP , called c , is the angle which a vertical plane through SCP makes with the vertical plane through the axis OC . This angle is the azimuth of the ray SP referred to the horizontal direction MN . The angle CPN , called a , is the elevation of the ray SCP above the horizontal plane. Letting $CP=r$, then

$$r^2 = x^2 + y^2 + f^2 \dots \dots \dots (1)$$

$$CN = CR + RN = f \sin e + y \cos e,$$

where e is the elevation of the axis of the camera above the horizontal plane. Then

$$\sin a = \frac{y \cos e + f \sin e}{r} \dots \dots \dots (2)$$

and since

$$PN = r \cos a$$

$$\sin c = \frac{x}{r \cos a} \dots \dots \dots (3)$$

We thus have formulas for determining, as viewed from one camera the elevation of a given point on the path of the airplane above the horizontal plane, and its azimuth with reference to a fixed direction from the base line. A great deal is here omitted in regard to the problem of choice of camera, plates, camera adjustment, lens distortion, determination of focal length, determination of the azimuth of the camera axis, etc. One also

has results for the azimuth and elevation of the *same* point on the airplane's path as measured from the plate of the other camera. Call these angles c' and a' , respectively.

We have now to find the co-ordinates of the point of the path of the airplane from these four angles, and the length of the base line. In figure 42, let L be the length of the base line (in

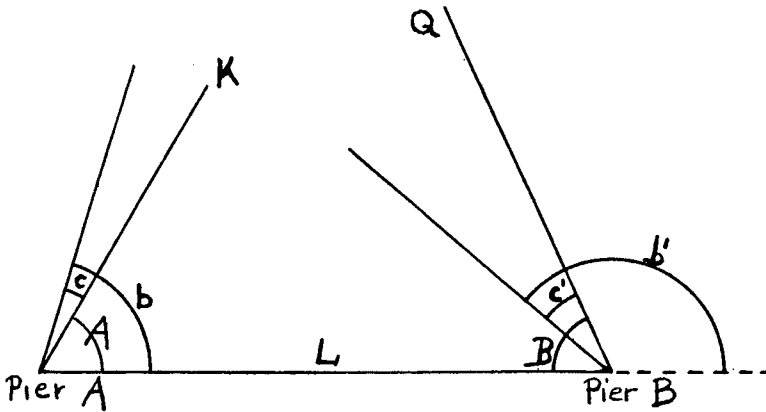


Fig. 42

our work, about one-half mile) between the two piers A and B . Let the line K to pier A be the projection of the axis of camera A , and Q to pier B , be the projection of the axis of camera B . Let the angle b be the azimuth of the ray in question from camera A , referred to the base line, and b' the azimuth determined from camera B . Then

$$b = A + c \dots \dots \dots (4)$$

$$b' = 180^\circ - B + c' \dots \dots \dots (5)$$

where the angles A and B have previously been accurately surveyed. We have given then (see Fig. 43) the angles a , b , a' and b' , and the length L of the base line. S is the point to be determined, and X , Y , and Z referred for convenience to pier A as origin, its co-ordinates. Then

$$X = R \cos b$$

$$Y = R \sin b$$

$$Z = R \tan a$$

$$X = R' \cos b' + L$$

$$Y = R' \sin b'$$

$$Z = R' \tan a'$$

and if we eliminate R and R' from these equations, we obtain,

$$X = \frac{\cos b \sin b'}{\sin (b' - b)} \dots \dots \dots (6)$$

$$Y = X \tan b \dots \dots \dots (7)$$

$$Z = X \frac{\tan a}{\cos b} \dots \dots \dots (8)$$

$$Z = Y \frac{\tan a'}{\sin b'} \dots \dots \dots (9)$$

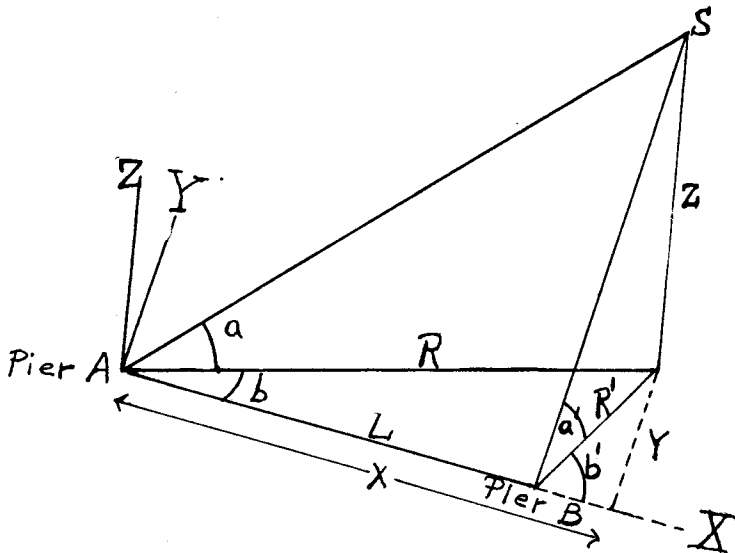


Fig. 43

Equations (1) to (8) are sufficient to determine X, Y, and Z. Equation (9) is a valuable check equation for the latter quantity.

The original plate measurements, corrections, etc., that were employed to obtain the X, Y, and Z of the loop will not be given here. In the actual determination there was a wind blowing transversely to the loop, which carried the airplane out of a plane. By appropriate methods the wind was determined, and the path of the plane reduced relative to the air. Lastly the Y-co-ordinate was eliminated by rotating the plane of the loop into the X-Z plane.

In figure 44 is drawn the resultant loop referred to the X-Z plane. The distances are in feet. The numbers beside the various points of the curve represent the time in seconds from the beginning, arbitrarily chosen at a given point as zero. The loop is executed from right to left.

Forces Involved in Looping. At any given point in the loop there are the following forces to consider: the weight of the airplane (taken as 2000 lbs.), the centrifugal force, the engine thrust, the air lift on the wings, and the resultant of all these, yielding tangential acceleration in the path of the plane. Any other forces, such as parasite resistance, are merged into the above forces. The weight vector is simply obtained by drawing a fixed vector downward at each point to a scale representing 2000 pounds. The engine thrust is taken as 450 pounds when

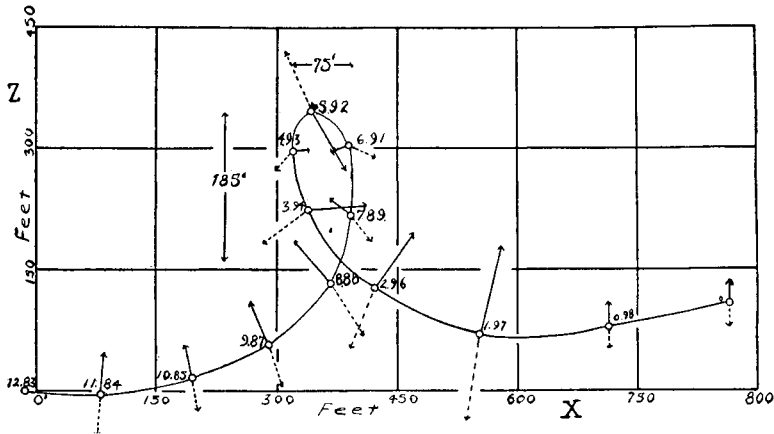


Fig. 44

the r.p.m. are 1400, and the speed of flight 70 miles per hour. For higher speeds, with the same r.p.m., the thrust is less. Near point 5.92 the engine was cut off and not turned on again until near point 10.83. The centrifugal force and the tangential acceleration at each point were obtained by a method of graphic differentiation, as follows. First, with dividers the length of the curve was determined, and platted as a function of the time.

Then for the various points the linear velocities $\frac{ds}{dt}$ were found

from the slopes of the tangent to the curve at these points. Next these velocities were platted as functions of the time, and the

slopes of this latter curve, $\frac{dv}{dt}$, or $\frac{d^2s}{dt^2}$ at these various points

yielded the tangential accelerations. These were sometimes positive, and sometimes negative. The centrifugal force was determined from Wv^2/gr , where r the radius of curvature was determined graphically from the original curve. The tangential force was determined from $W/g(dv/dt)$, and lastly the air pressure vector was determined as the difference between the vector for the tangential acceleration and the resultant vector of weight, centrifugal force, and engine thrust. On the curve (Fig. 44) are drawn to scale the air pressure vectors in solid lines, and the vector resultant of weight, centrifugal force, and engine thrust as dotted lines. The resultant of these two at each point represents the tangential accelerating force on the airplane. The details of these various forces are given in table I.

TABLE I

$W = 2000$ LBS.

POINT	TOTAL DIST. s (FT)	ds/dt $= v$ (FT/SEC)	dv/dt $= a$ (FT/SEC ²)	Wa/g $= f$ (LBS.)	RADIUS OF CURV. $= R$ (FT)	WV^2/gR (LBS.)	ENGINE THRUST $= T$ (LBS.)	(VECTOR SUM) $T + W + WV^2/gR$ (LBS.)	DIRECTION N THROUGH E (DEGREES)	AIR LIFT (LBS.)	DIRECTION N THROUGH E (DEGREES)
0.	0	150	+4	+250	Inf.	0	0	2000	180	1980	353
0.98	145	153	0	+10	Inf.	0	0	2000	180	2000	0
1.97	307	148	-8	-510	270	5040	10	7500	189	7500	13
2.96	455	137	-17	-1040	320	3640	100	5470	203	5330	35
3.94	580	103	-46	-2860	180	3660	410	4650	235	4930	88
4.93	657	59	-25	-1550	165	1310	450	2100	223	1450	88
5.02	715	56	+13	+810	25	7790	off	5550	338	5670	150
6.91	778	75	+17	+1060	145	2410	off	2450	115	1600	250
7.89	865	90	+13	+810	295	1700	off	2820	142	2300	307
8.88	957	100	+8	+520	180	3450	off	4920	147	4780	320
9.87	1069	107	+4	+240	370	1920	off	3720	162	3660	338
10.85	1172	107	+1	+90	745	950	off	2900	171	2850	350
11.84	1287	110	+1	+60	500	1500	350	3600	184	3550	3

Results and Conclusions.—It will be noted first that the loop is far from circular, contrary to the opinions of many pilots I have interviewed on this matter. This is to be expected, since the velocity of the airplane decreases toward the top of the loop. To create a sufficient centrifugal force here, then, a smaller radius of curvature is required. The dimensions of this loop are, as noted from figure 44, 75 feet broad by 185 feet high. The time consumed in making the loop proper is a trifle over five seconds. The greatest stresses involved are at point 1.97 where the air pressure is nearly four times the normal, and at point 5.92 where it is nearly three times the normal. It is thus evident that a large factor of safety is required in this manoeuvre. In fact in these airplanes, the J.N.4 H., the factor of safety is supposed to be in the neighborhood of ten to twelve. It will be noted further that the greatest stresses in this present case occur before the loop proper is executed. These stresses follow always when an airplane, following a steep dive, is too suddenly leveled out, or directed again upward. In the present case a velocity of about 100 miles per hour was attained. The high stress at point 1.97 came from the high velocity, while at point 5.92 it came from the small radius of curvature.

No two loops are ever alike as, for example in the present case, a close inspection of the figure reveals that the angle of attack varies rapidly from point to point. This is in the pilot's control, and he can never operate his controls with perfect uniformity.

Altitude is invariably lost on a loop; in this case about seventy feet as shown in figure 44, but more was lost actually as the first part of the approach is not recorded.

This method of survey, while somewhat laborious, is capable of yielding very accurate results. In the present case there was but one light on the airplane, and so only the path of the plane as a whole was determined. If three lights were mounted, one would have exact information as to the angle of attack, and other matters, all of which information would be of the utmost value. A great deal of stress has been, and is being laid on tests of airplane performances. This method would in the end yield more accurate results with far less labor than would many of the methods which have hitherto been pursued.

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