Calibration of a Gold Leaf Electrometer for Ionization Work

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CALIBRATION OF A GOLD LEAF ELECTROMETER FOR IONIZATION WORK

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The ordinary type of ionization electroscope having a straight, horizontal scale of 100 divisions in the focal plane of its microscope, over which the image of the gold leaf is seen to move obliquely (Fig. 1), is subject to well marked variations which, for certain kinds of work, necessitate its calibration.

It will be assumed that the voltage at which the electroscope operates is sufficient to maintain the ionization current at saturation, and that the actual discharge rate, in units of electricity per unit time, is always proportional to the rate at which the air is being ionized. This means that for a constant rate of ionization, such as that produced by a radio-active material of long period, equal quantities of discharge should take place in equal times. But if the electroscope be actually exposed to a constant source and the scale readings plotted against time, the result will be, not a straight line as it would be if the scale intervals represented equal amounts of discharge, but a curve similar to that shown in figure 2.

While a common procedure is to compare ionization currents by observing the respective times required for the leaf to drop from one stated reading to another, there are applications in which this would not be feasible; as when one is working with a short-lived product, such as thorium emanation, whose half value period is only about fifty-four seconds. There would then not be time to recharge the electroscope during the progress of the experiment.

The problem is, therefore, to translate the arbitrary graduation of the scale into one on which different intervals shall be in exact proportion to the quantities of discharge to which they correspond.
Let the readings on the actual scale, running from 0 to 100, be denoted by \( S \), and those of the corresponding standardized or uniform discharge scale by \( S' \); the two scales coinciding at their extremities, 0 and 100, but, in general, nowhere else.

Let the electroscope be tested at a constant discharge rate of moderate value, as with the gamma-rays from a little radium; or, data from observations on the natural leak may be utilized. Under such circumstances the relation between the time \( t \) and the true scale reading \( S' \) must be a linear one:

\[
t = S' + t_o,
\]
where \( t_o \) is the time when the zero of the scale is reached; or letting \( t - t_o = T \),

\[
T = hS',
\]
(1)

For the actual scale, the writer has found that the relation is quite satisfactorily represented by a quadratic, the most convenient form of which is

\[
t = k\ (S^2 - aS) + t_o
\]
(2)
or

\[
T = k\ (S^2 - aS).
\]
(3)

\( h, k \) and \( a \) in these equations are constants.

We may now equate (1) and (3), giving

\[
h\ S' = k\ S^2 - k\ a\ S.
\]
(4)

When \( S=100, S'=100 \); therefore, substituting in (4)

\[
100\ h = 10,000\ k - 100\ k\ a,
\]
or

\[
h = 100\ k - k\ a.
\]
(5)

On putting this for \( h \) in (4), \( k \) cancels out, leaving

\[
(a - 100)\ S' = a\ S - S'
\]
CALIBRATION OF GOLD LEAF ELECTROMETER

or

\[ S' = \frac{aS - S^2}{a - 100}. \]  

(6)

Thus the corrected reading \( S' \) is expressed in terms of the actual reading \( S \) and a quantity \( a \), which may be called the constant of the scale, and whose value is to be found. It is independent of the rate of discharge used in the experiment.

The constant may be determined by taking time and scale readings over a wide range of the scale, using (2) as an observation equation and adjusting the observations by least squares for the most probable values of \( k, a \) and \( t_0 \). A more elementary procedure is to select from the plotted curve three typical readings of time and scale, and substitute them in (2):

\[
\begin{align*}
  t_1 &= k (S_1^2 - a S_1) + t_0, \\
  t_2 &= k (S_2^2 - a S_2) + t_0, \\
  t_3 &= k (S_3^2 - a S_3) + t_0.
\end{align*}
\]  

(7)

Eliminating \( t_0 \) and \( k \), we have finally

\[ a = \frac{(t_3 - t_1) (S_1^2 - S_3^2) - (t_2 - t_1) (S_1^2 - S_2^2)}{(t_3 - t_1) (S_1 - S_3) - (t_2 - t_1) (S_1 - S_2)}. \]  

(8)

Example—Time observations made on the discharge of a certain straight-scale electroscope in the Coe laboratory under the influence of 1 mgr. radium enclosed in lead at a short distance gave for \( S \) and \( t \):

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>( t ) (sec.)</td>
<td>( S' )</td>
</tr>
<tr>
<td>100</td>
<td>3 *</td>
<td>100.000</td>
</tr>
<tr>
<td>90</td>
<td>60</td>
<td>91.346</td>
</tr>
<tr>
<td>80</td>
<td>120</td>
<td>82.362</td>
</tr>
<tr>
<td>70</td>
<td>182</td>
<td>73.086</td>
</tr>
<tr>
<td>60</td>
<td>238 *</td>
<td>63.518</td>
</tr>
<tr>
<td>50</td>
<td>301</td>
<td>53.660</td>
</tr>
<tr>
<td>40</td>
<td>367</td>
<td>43.510</td>
</tr>
<tr>
<td>30</td>
<td>434</td>
<td>33.070</td>
</tr>
<tr>
<td>20</td>
<td>503 *</td>
<td>22.338</td>
</tr>
<tr>
<td>10</td>
<td>571</td>
<td>11.314</td>
</tr>
<tr>
<td>0</td>
<td>640</td>
<td>0.000</td>
</tr>
</tbody>
</table>

The values selected as \( S_1, S_2, S_3, t_1, t_2, t_3 \), are the ones starred, and these give, on substitution in (8), \( a = 786.7 \). The required calibration formula for this instrument, is therefore, from (6),

\[ S' = 1.146 S - 0.001456 S'. \]

The values in the third column above are thus calculated.

It is often desirable to compare different rates of ionization by means of rates of fall observed on different parts of the scale, as in the case of short-lived products above referred to. Differentiating (6) with respect to \( t \)

\[ \frac{dS'}{dt} = \frac{a - 2 S}{a - 100} \frac{dS}{dt}. \]
That is, if the rate of fall observed on the actual scale is $R$, the corresponding rate on the true scale is

$$R' = \frac{a - 2S}{a - 100} R.$$  \hspace{1cm} (10)

It is clear that for a constant ionization current, $R'$ must remain fixed, though $R$ will be variable.

Example—Some thorium emanation was being examined by means of an electroscope whose constant had been found to be $a = 392.7$. Inspection of the results showed that when $S$ was 67.8 the rate of fall was $R = 0.725$ scale divisions per second. Substituting in (10), $R' = 0.878R = 0.637$ div. per sec. A few moments later, when $S = 34.5$, $R = 0.270$; so that on this part of the scale $R' = 1.11 R = 0.299$ div. per sec. The necessity for the correction is thus quite apparent.

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