Maxwell's Treatment of Networks Applied to the Delta Connection of a Three-Phase Power Source

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MAXWELL’S TREATMENT OF NETWORKS APPLIED TO THE DELTA CONNECTION OF A THREE-PHASE POWER SERVICE

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There are two purposes in this paper. The more subordinate one is to give the results of the particular problem. The more important reason for the paper is that it may suggest a means of solution of similar problems.

The question arose as to the relation between the electromotive forces of line currents and the electromotive forces of the currents in the delta connected load lines of a three-phase service. There are four possible arrangements. The alternator coils may be in Y or delta connection and with either of these there may be a Y or delta connection between the main and load lines. In this particular problem the alternator coils were in Y connection and the load lines in delta connection.

Figure 1 is the wiring diagram. The alternator coils are 01, 02, and 03; the loads are 12, 23, and 31; the lines extending to other loads are 1T, 2T, and 3T. For a single phase service there would be a return wire OT, but due to the fact that the sum of the impressed electromotive forces at any instant is zero, there would be no current in OT, so it is omitted.

Figure 2 represents the same wiring in the conventional network form. The problem consists of a five-point network with one conductor omitted, and is treated according to Kirchhoff’s laws. In such a wiring arrangement the alternator coils are balanced and the loads are balanced both in the delta connection and in the complete line, and we may assume the following conductances:

\[
\begin{align*}
K_{01} &= K_{02} = K_{03} = K_0 \\
K_{12} &= K_{23} = K_{31} = K \\
K_{1t} &= K_{2t} = K_{3t} = K_t \\
K_{0t} &= 0
\end{align*}
\]

In order to simplify the problem it is assumed that one point has a potential of zero and the potentials of the other points are compared to the potential of this point.

Therefore let \( V_0 = 0 \)

The phase interval in a three-phase system is 120 degrees and
the electro-motive force is proportional to the \( \cos 2\pi nt \), at any time \( t \) where \( n \) is the frequency of oscillation. \( E \) is the maximum electromotive force. Then the three electromotive forces are given in the following equations:

\[
\begin{align*}
E_{01} &= E \cos 2\pi nt \\
E_{02} &= E \cos (2\pi nt + 120^0) \\
E_{03} &= E \cos (2\pi nt + 240^0)
\end{align*}
\]

Kirchhoff’s first law states that the sum of the currents flowing to any point in a network at any instant is equal to the sum of the currents flowing from the point at that instant. The following equations are derived from this law:

\[
\begin{align*}
C_{01} + C_{12} + C_{21} + C_{31} &= 0 \\
C_{02} + C_{12} + C_{22} + C_{32} &= 0 \\
C_{03} + C_{13} + C_{23} + C_{33} &= 0 \\
C_{12} + C_{21} + C_{13} + C_{31} &= 0 \\
C_{22} + C_{32} + C_{23} + C_{32} &= 0 \\
C_{33} + C_{33} + C_{33} + C_{33} &= 0
\end{align*}
\]

The current in any conductor is equal to the potential difference between its terminals plus any electromotive force impressed within the conductor, multiplied by the conductance of the conductor. The following equations express the ten currents in this form:

\[
\begin{align*}
C_{01} &= 0 \\
C_{01} &= (E_{01} - V_1) K_0 \\
C_{02} &= (E_{02} - V_2) K_0 \\
C_{03} &= (E_{03} - V_3) K_0 \\
C_{12} &= (V_1 - V_2) K \\
C_{22} &= (V_2 - V_3) K \\
C_{33} &= (V_3 - V_1) K \\
C_{11} &= (V_1 - V_t) K_t \\
C_{21} &= (V_2 - V_t) K_t \\
C_{31} &= (V_3 - V_t) K_t
\end{align*}
\]

These ten current values in terms of the conductance and electromotive force are put into equations (1), (2), (3), (4), and (5). The following equations result:

\[
\begin{align*}
V_1 + V_2 + V_3 &= E_{01} + E_{02} + E_{03} = 0 \\
(V_1 + V_2) K - (K_0 + 3K + K_t) V_3 + V_1 K_t &= -K_0 E_{03} \\
(V_1 + V_3) K - (K_0 + 3K + K_t) V_2 + V_1 K_t &= -K_0 E_{02} \\
(V_2 + V_3) K - (K_0 + 3K + K_t) V_1 + V_2 K_t &= -K_0 E_{01} \\
V_1 + V_2 + V_3 &= 3V_t = 0
\end{align*}
\]

These five equations are solved simultaneously for \( V_1 \), \( V_2 \), \( V_3 \), and \( V_t \), and the following values result:

\[
\begin{align*}
V_t &= 0 \\
V_1 &= [K/(K_0 + 3K + K_t)] E_{01}
\end{align*}
\]
\[ V_2 = \frac{K}{(K_0 + 3K + K_t)} E_{02} \]
\[ V_3 = \frac{K}{(K_0 + 3K + K_t)} E_{03} \]

These values in equations (10), (11), (12) obtain the following results:

\[ C_{12} = \frac{K K_0}{(K_0 + 3K + K_t)} (E_{01} - E_{02}) \]
\[ C_{23} = \frac{K K_0}{(K_0 + 3K + K_t)} (E_{02} - E_{03}) \]
\[ C_{31} = \frac{K K_0}{(K_0 + 3K + K_t)} (E_{03} - E_{01}) \]

If the values \( E_{01}, E_{02}, \) and \( E_{03} \) are now replaced by their values in terms of \( E \cos 2\pi nt \), there is obtained:

\[ C_{12} = \frac{K K_0}{(K_0 + 3K + K_t)} [E \sqrt{3} \cos (2\pi nt - 30°)] \]
\[ C_{23} = \frac{K K_0}{(K_0 + 3K + K_t)} [E \sqrt{3} \cos (2\pi nt - 90°)] \]
\[ C_{31} = \frac{K K_0}{(K_0 + 3K + K_t)} [E \sqrt{3} \cos (2\pi nt - 210°)] \]

Figure 3 is the graphic interpretation of these results compared with \( E_{01}, E_{02}, \) and \( E_{03} \). As can be seen, \( E_{12} \) lags 30 degrees behind \( E_{01}, E_{23} \) is 30 degrees behind \( E_{02}, \) and \( E_{31} \) is 30 degrees behind \( E_{03} \). Then the whole delta connection lags 30 degrees behind the alternator coils in electromotive force. The curve also shows that the maximum voltages in the delta connected load lines are higher than the maximum impressed voltages in the ratio \( \sqrt{3} : 1 \). In the first solution of the problem \( E_{12} \) is the difference between \( E_{01} \) and \( E_{02} \). The ordinate of \( E_{12} \) at any time is the actual distance between \( E_{01} \) and \( E_{02} \). At the maximum value of \( E_{12}, E_{02} \) is negative and the difference between \( E_{01} \) and \( E_{02} \) is their numerical sum which is greater than the maximum of \( E_{01} \). This explains the \( \sqrt{3} : 1 \) ratio.

In conclusion it should be said that there are probably many problems in wiring across different voltages and especially in polyphase service, which may be solved by the Maxwell method. The solution will give the current, electromotive force, and lag due to the wiring arrangement. Especially in instruction this method is desirable because the mathematical and physical theory are closely and clearly associated.

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