Proceedings of the Iowa Academy of Science

Volume 31 | Annual Issue Article 114

1924

A New Interpolation Formula

John F. Reilly
University of Iowa

Recommended Citation
Reilly, John F. (1924) "A New Interpolation Formula," *Proceedings of the Iowa Academy of Science, 31(1),* 369-371.
Available at: https://scholarworks.uni.edu/pias/vol31/iss1/114

A NEW INTERPOLATION FORMULA

JOHN. F. REILLY

If we wish to interpolate values of a function between two given values y_1, and y_2, we may employ a polynomial in $x-1$. When the only conditions to be satisfied are that the function take the value y_1 when $x=1$, and the value y_2 when $x=2$, the polynominal of minimum degree is of the form $a_0+a_1(x-1)$. Interpolation by means of this polynomial is that ordinarily employed when using trigonometric and logarithmic tables, and is spoken of as interpolation by first differences. If, however, additional conditions are imposed the degree of the polynomial will increase one unit for each condition. For example, if the interpolated values between y_1 and y_2 are dependent upon the conditions that the function takes the value y_0 when $x=0$, and the value y_3 when $x=3$, then the interpolation polynomial is of the form $a_0+a_1(x-1)+a_2(x-1)^2+a_3(x-1)^3$.

It should be stated that the polynomial in $x-1$ need not be of minimum degree, nor is it the only function that can be employed, any function containing as many undetermined constants as conditions imposed will suffice; neither is it necessary that the conditions be known function values, anything that will enable us to determine the constants is all that is essential, for example, rate of change of a function value.

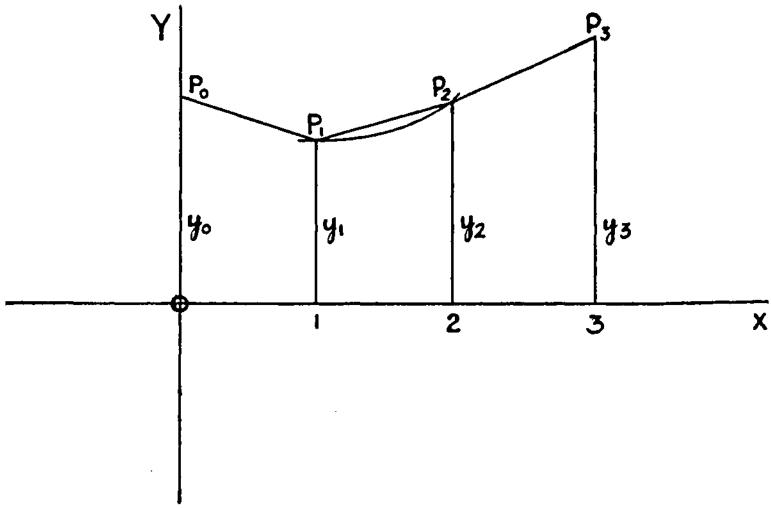
It is evident that the number of interpolation functions is infinite even if we limit ourselves to polynomials of minimum degree in $x-1$, since the number and nature of the conditions that may be imposed is arbitrary.

The conditions imposed by Newton were known equidistant function values, those by Lagrange were known function values not necessarily equidistant, while those imposed by Sprague were in part known function values and in part rates of change.

In any particular case what interpolation function should be used? In general, this question cannot be answered, although it seems reasonable that some discretion can be exercised in selecting it.

To add one more to the already long list of interpolation functions is the object of this note.

Given four values y_0, y_1, y_2, y_3, let it be required to interpolate between y_1 and y_2. Let P_0, P_1, P_2, P_3 be the points $(0, y_0)$, $(1, y_1)$, $(2, y_2)$, $(3, y_3)$ respectively. Assume an interpolation function of the form $a_0+a_1(x-1)+a_2(x-1)^2+a_3(x-1)^3$, and for the determination of the a's impose the conditions that the function have the value y_1 when $x=1$, and the value y_2 when



$x=2$, that the graph of this function have at the P_1 the slope $[y_1-y_0+2(y_2-y_1)]/3$ and at the point P_2 the slope $[2(y_2-y_1)+(y_3-y_2)]/3$. These slopes are weighted arithmetic means of the slopes of the straight lines $P_0 P_1$, $P_1 P_2$, $P_2 P_3$. It seems reasonable that the slope of the straight line $P_1 P_2$ should have more weight when interpolating between P_1 and P_2 than the slopes of lines external to this interval. The slope of $P_1 P_2$ is therefore given twice as much weight as the slopes of $P_0 P_1$ and $P_2 P_3$.

These conditions lead to the system of equations:

$$a_0=y_1$$
$$a_0+a_1+a_2+a_3=y_2$$
$$a_1=(2y_2-y_1-y_0)/3$$
$$a_1+2a_2+3a_3=(y_2+y_3-2y_1)/3.$$

Hence

$$a_0=y_1$$
$$a_1=(2y_2-y_1-y_0)/3$$
$$a_2=(-y_3+4y_2-5y_1+2y_0)/3$$
$$a_3=(y_3-3y_2-3y_1-y_0)/3,$$

and the function is

$$y_1+(1/3)(2y_2-y_1-y_0)(x-1)-(1/3)(y_3-4y_2+5y_1-2y_0)$$
$$(x-1)^2+(1/3)(y_3-3y_2+3y_1-y_0)(x-1)^3.$$

When y_1, y_2, y_3 are expressed in terms of y_0 and its leading differences this function becomes

$$y_0+x\Delta y_0+(1/3)(x^2-1)\Delta^2 y_0+(1/3)(x-1)^2(x-2)\Delta^3 y_0.$$

UNIVERSITY OF IOWA.