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orientation would be more nearly the same as that for 0° than is shown by the results.

The depression of the wet bulb temperature of the wet and dry bulb hygrometer is evidently an average value for the cooling on the different parts of the surface and further study of the effect of the shape of the cooling surface may lead to a better agreement between the theory and experiments for the wet and dry bulb hygrometer.

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HEAT FLOW IN A FINITE CYLINDER WITH VARIABLE SURFACE TEMPERATURE

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If heat be supplied at a constant rate to a liquid which is kept at uniform temperature throughout by stirring, and if this liquid lose heat according to Newton's law of cooling, we get

$$\frac{d\Theta}{dt} = \frac{\beta}{C} - \alpha\Theta. \qquad (1)$$

for the differential equation from which to obtain the temperature, Θ, as a function of time. $\beta/_c$ is the rate of heat supply divided by thermal capacity and $\alpha\Theta$ the rate of cooling. The surrounding medium is assumed to be at zero temperature. This equation is equally valid if the liquid be replaced by a solid of very high diffusivity. Equation (1) assumes the thermal capacity of the liquid to be independent of temperature.

Equation (1) when integrated subject to the condition that $\Theta = \Theta_0$, when $t = 0$, gives

$$\Theta = \Theta_\infty - (\Theta_\infty - \Theta_0)\, e^{-\alpha t} \qquad (2)$$

where

$$\Theta_\infty = \frac{\beta}{C\alpha}$$

It is seen from (2) that the temperature approaches asymtotically the value Θ_∞ as t approaches infinity.

Let us now immerse a small solid cylinder, of small thermal capacity in comparison to the liquid, in the liquid. It is required to find the temperature of any point of the cylinder at any time t. We assume the initial interior temperature of the cylinder uniform throughout and equal to Θ_1.

We have then to solve the partial differential equation

$$\frac{\delta \Theta}{\delta t} = k \left\{ \frac{\delta^2 \Theta}{\delta t^2} + \frac{1}{r} \frac{\delta \Theta}{\delta r} + \frac{\delta^2 \Theta}{\delta z^2} \right\}$$

subject to the conditions:

$$\Theta = (\Theta_\infty - \Theta_1) - (\Theta_\infty - \Theta_0)\, e^{-\alpha t} \text{ at } r = a \text{ and } z = \pm 1$$

and $\Theta = 0$ at all points of the cylinder when $t = 0$. We can reduce this to a problem of constant surface temperature by using Duhamel's theorem. By this theorem we replace t by λ in the surface temperature function given above and by $t - \lambda$ in the interior temperature function,

$$\Theta_1 = \sum_\mu \sum_1^\infty A_{\mu, m} \quad e^{-k \left\{ \mu^2 + \frac{m^2 \pi^2}{l^2} \right\} t} \; J_0 \; (\mu r) \; \sin \frac{m \pi}{2 l} (z + 1)$$

We then form the function

$$\Theta^1 = \left[(\Theta_\infty - \Theta_1) - (\Theta_\infty - \Theta_0)\, e^{-\alpha t} \right] \left[1 - \Theta_1 \right]$$

which reduces the interior temperature of the cylinder to zero. This function is then differentiated with respect to t and this result integrated with respect to λ, giving a final expression for the interior temperature as follows:

$$\Theta^1 = \Theta_\infty - \sum_\mu \sum_1^\infty A_{\mu, m} \left[(\Theta_\infty - \Theta_1)\, e^{-Rt} - (\Theta_\infty - \Theta_0) \left\{ \frac{R}{R - \alpha} \right\} \left\{ e^{-Rt} - e^{-\alpha t} \right\} \right] J_0 \; (\mu r) \; \sin \; \frac{m \pi}{2 l} (z + 1)$$

where

$$R = K \left\{ \mu^2 + \frac{m^2 \pi^2}{l^2} \right\}$$

The value of the constant $A_{\mu, m}$ is

$$\frac{4 \; (\cos m \pi - 1)}{m \pi \; (\mu a) \; J_1 \; (\mu a)}$$

In case the liquid reaches the boiling point at time t_1 the interior temperature after time t is, for $t > t_1$,

$$\Theta^{11} = \Theta_\infty - (\Theta_\infty - \Theta_0)\, e^{-\alpha t_1} - \sum_\mu \sum_1^\infty A_{\mu, m} \Big[\Theta_1 + (\Theta_\infty - \Theta_0)\, e^{-\alpha t_1} + (\Theta_\infty - \Theta_1)\, e^{-R t_1} - (\Theta_\infty - \Theta_0) \left\{ \frac{R}{R - \alpha} \right\} \left\{ e^{-R t_1} - e^{-\alpha t_1} \right\} \Big] e^{-R (t - t_1)} \quad J_0 \; (\mu r) \; \sin \frac{m \pi}{2 l} (z + 1)$$

For the case in which no heat is supplied to the liquid, $\beta = 0$ and $\Theta_\infty = 0$, and the interior temperature will approach that of the surrounding medium.

In the accompanying figure, curve (1) represents the temperature of the liquid in which the cylinder is immersed. The liquid is radiating into a medium at zero temperature. The cylinder which had an initial uniform interior temperature of 30° heats according to curve (2). In case the liquid boils at 80° and maintains this temperature indefinitely the cylinder heats according to curve (3).

The constants used in the equations were

$$\Theta_\infty = 100^\circ; \quad \Theta_0 = 0; \quad \alpha = 0.04; \quad R = 0.109.$$

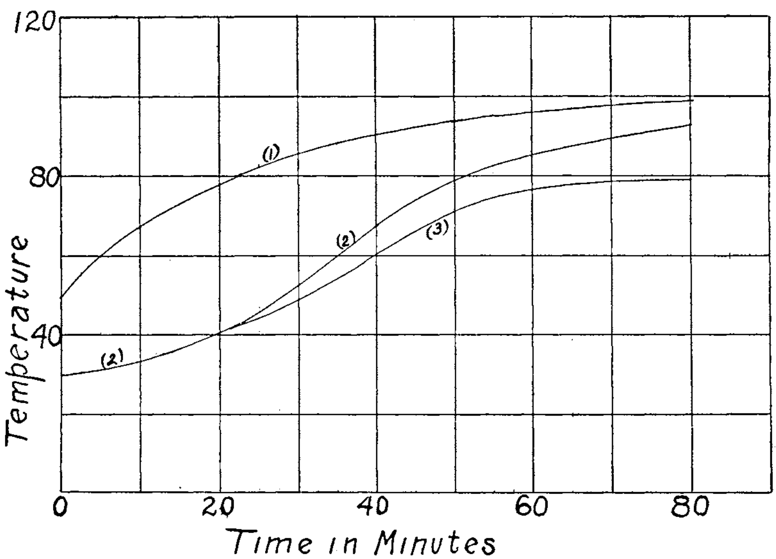

Figure I

Iowa State College.

VELOCITY OF CADMIUM ATOMS REGULARLY REFLECTED FROM A ROCK SALT CRYSTAL

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We have previously shown that a beam of Cadmium atoms incident upon a cleavage face of a rock salt crystal is reflected so that the incident and reflected beams make equal angles with the normal to the crystal surface. At that time we suggested that this phenomenon could be interpreted in terms of the phase waves of de Broglie. The existence of a reflected beam making the same angle with the normal as does the incident beam suggests at once the possibility that we have here a situation in which the phase waves behave as X-rays do in the Bragg type of reflection.