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The angles were drawn with black india ink on white cards 10 cm. square and exhibited to the observer in the Michotte tachistoscope. The observer reproduced the angles with a pencil on a piece of white paper, 10 cm. square. The position of the reproduced legs of the angle was then measured. The size of the angle and the position of the bisector were computed subsequently. Position was measured in terms of a circular protractor placed in a vertical plane and rotated until 0° was uppermost. The protractor was also turned so as to read in the clockwise direction. Twelve sizes of angles were used. viz. 10°, 30°, 45°, 60°, 75°, 90°, 105°, 120°, 135°, 150°, 170°, and 180°. Each angle was shown in 16 different positions, viz. with the bisector at 0°, 22°, 45°, 67°, 90°, 112°, 145°, 167°, 180°, 202°, 225°, 247°, 270°, 292°, 315°, 337°. There were therefore 192 angles in all. There were six observers, and each was shown all the angles 10 times. Consequently 11520 reproductions were made and since 4 measures were made of each reproduction, the raw data consists of 46080 numbers. Differences between the standard angles shown to the observers and their reproductions will be called "errors." Overestimations and underestimations will be styled "positive" and "negative" errors respectively. As regards the position of a line, a negative error means a counterclockwise displacement and a positive error is the clockwise displacement.

Table I — The Position of Straight Angles

0°	11°	22°	34°	45°	56°	67°	79°
0	+11.7	+9.2	+4.3	+0.6	—6.4	—10.4	—11.5
90°	101°	112°	124°	135°	146°	157°	169°
0	+9.3	+7.6	+3.6	—1.7	—6.7	—11.0	—13.0

The first line of numbers indicates the positions of the 16 standard straight angles used in the experiment, and the second line gives in degrees the arithmetical mean of the position errors. In interpreting the results we assume that imaginary vertical and hori-

Table II — Overestimations and Underestimations

	10°	30°	45°	60°	75°	90°	105°	120°	135°	150°	170°
0°	+ 5.4 (+11.5)	+ 2.7 (+10.8)	+ 0.6 (+ 9.4)	— 0.8 (+ 6.8)	+ 0.8 (+ 3.6)	+ 4.3 (0)	+ 3.0 (— 3.6)	— 0.6 (— 6.8)	— 4.7 (— 9.4)	—10.7 (—10.8)	—14.8 (—11.5)
45°	+ 5.1 (— 2.4)	— 0.5 (— 6.8)	— 7.0 (— 9.4)	—10.7 (—10.8)	—10.9 (—11.5)	+ 0.2 (0)	+10.3 (+11.5)	+ 7.9 (+10.8)	+ 1.6 (+ 9.4)	— 5.0 (+ 6.8)	—10.3 (+ 2.4)
22°	+ 4.6 (— 1.5)	+ 0.2 (— 3.9	— 5.6 (— 0.3)	— 0.4 (+ 4.0)	+ 2.0 (+ 1.9)	+ 2.9 (0)	— 2.5 (— 1.9	— 3.9 (— 4.0)	— 0.5 (+ 0.3)	— 5.7 (+ 3.9)	—11.5 (+ 1.5)
67°	+ 4.8 (— 1.3)	— 0.7 (— 4.0)	— 4.9 (+ 0.3)	— 4.1 (+ 3.9)	+ 0.8 (+ 2.2)	+ 2.4 (0)	— 1.8 (— 2.2)	+ 0.1 (— 3.9)	— 1.1 (—0.3)	— 5.7 (+ 4.0)	—11.9 (+ 1.3)

Table III — Errors in Positions of Bisectors

	10°	30°	45°	60°	75°	90°	105°	120°	135°	150°	170°
0°	—0.9	—1.4	—1.2	—0.1	—0.4	—0.9	—0.2	—0.9	—1.0	—1.1	—1.3
45°	—1.4	—2.8	—2.4	—1.9	—2.5	—1.0	—1.1	—0.7	—1.1	—3.0	—1.7
22°	+5.4 (+4.0)	—0.5 (+3.4)	—4.2 (—0.2)	—7.9 (—4.2)	—7.1 (—5.0)	—8.5 (—5.4)	—6.5 (—5.0)	—4.6 (—4.2)	—1.1 (—0.2)	+0.6 (+3.4)	+5.1 (+4.0)
67°	—8.0 (—5.1)	—4.5 (—4.2)	+1.2 (—0.2)	+3.6 (+3.4)	+3.5 (+3.9)	+4.7 (+4.2)	+5.1 (+3.9)	+1.0 (+3.4)	—1.5 (—0.2)	—3.9 (—4.2)	—7.9 (—5.1)

zontal meridians passing through the center of the field of vision dominate the perception of straight lines in the following ways:

1. The nearest meridian "repels" the reproduced line, the repulsion varying indirectly with the standard's distance from the meridian.
2. Lines coinciding with the meridians show no error at all.

The last fact raises two new problems:

1. Would lines less than 11° from a meridian be "attracted" or "repelled?"
2. Would such repulsion be greater or less than for lines 11° distant?

Control experiments on exclusively straight lines were performed by the same six observers; twice as many positions were employed. It was found that a line 6° from a meridian is generally repelled as strongly as those at 11°, but all repulsions were much weaker and much less regular.

If the perception of the other angles is influenced by the same vertical and horizontal meridians, those angles fall into four classes, i.e. according as their bisectors (1) coincide with a meridian, (2) lie halfway between meridians, (3) lie 22° from a meridian in the clockwise direction, or (4) lie 67° from a meridian in the same clockwise direction. We shall henceforth designate the position of these angles as 0°, 45°, 22° and 67° respectively.

This table gives the average errors in the sizes of the angles in the four different positions. If the legs of these angles were repelled by the imaginary meridians in the same direction, *and half as much*, as straight lines are, the angles would have the errors expressed by the numbers given in parentheses.

Conclusions

1. Position affects the apparent size of all except the 2 smallest angles.
2. Since in the 45° position, the errors of 45° — 120° angles correspond surprisingly well to the expected errors, these angles must be seen as two almost independent lines.
3. Independently of position, 10° angles are strongly overestimated and 150° and 170° angles are underestimated.

The numbers in parentheses give the errors to be expected if the legs are repelled by the meridians in the same direction, and half as much as the straight lines are.

Previous experiments on qualitative differences in the form of angles had indicated that if the observer rids himself of the attitude of formal mathematics:

1. small angles tend to be seen as highly integrated wholes viz. as "sharp points" or "triangular wedges,"
2. medium-sized angles are seen as "two (almost independent) lines meeting at a point,"
3. and large angles are again seen as unitary wholes viz. as "broken, curved or straight lines."

Consequently the position of a:

1. small angle should be the position of its bisector,
2. medium angle should be the position of its 2 legs, or at least determined by the position of one of them,
3. large angle should be the position of a straight line joining its 2 extremities.

Errors in the positions of the bisectors are, therefore, significant data for interpreting the qualitative forms in which angles are seen. If an angle is seen as a highly integrated whole, its bisector should be repelled by the nearest meridian in the same direction, though not necessarily the same distance, as a straight line is. If, on the contrary, an angle is seen as two quasi-independent legs, each leg should be repelled from the nearest meridian in the same direction, and presumably half as far, as straight lines are, and the positions of the bisectors would, of course, follow the repulsions of the legs. At the 0° and 45° positions, the bisectors should, in either alternative, suffer small negative errors corresponding closely to those found in Table 3. At the 22° position the bisectors of 10°, 30°, 150° and 170° angles should undergo positive repulsions in both alternatives, and the same angles at the 67° position should show negative repulsions.

But at the 22° position the bisectors of 45°-135° angles, if seen as:

1. unitary wholes, should all suffer positive repulsions,
2. two independent legs, should suffer the repulsions indicated in Table 3 by the numbers in parentheses.

Likewise at the 67° position, the bisectors of 45°-135° angles seen as:

1. unitary wholes, should all suffer negative repulsions
2. two independent legs, should suffer the repulsions indicated in Table 3 by the numbers in parentheses.

The study of Tables 2 and 3 reveals an apparent contradiction.

1. The bisectors of 45°-135° angles show the errors to be expected if their legs were seen as independent straight lines.
2. The magnitudes of 45°-120° angles do not conform, definitely at least, with what should be expected from such a repulsion of the legs.

We can avoid the contradiction by concluding that one leg of 45°-120° angles is seen as a quasi-independent straight line and undergoes the corresponding repulsion, while either the other leg is confusedly perceived in the same fashion or else the size of the angle is confusedly but directly apprehended.

Summary of conclusions as to the qualitative forms of angles.

1. The apparent size of 10° and 30° angles is independent of position; this indicates that they are seen as unitary wholes.
2. The apparent size of 150°, 170° and, possibly, of 135° angles is independent of position except at the 0° position; we could plausibly conclude that these angles are seen as "broken lines" and that in the 0° position, the meridian to which they are parallel causes the "brokenness" to be overestimated.
3. Save for the just-mentioned exception, the apparent size of angles at the 0° position is not affected by position; this indicates that all angles at the 0° position are seen as unitary wholes.
4. 45°-120° angles at the 45°, 22° and 67° positions are seen as 2 quasi-independent straight lines.

Finally we must give some attention to one of the most singular phenomena encountered in these experiments. Examination of Table 1 shows that negative repulsions of straight lines are on the average 2.2° stronger than positive repulsions. Likewise, at the 0° and 45° positions, bisectors should show no errors at all; in reality there is always a small negative error averaging 0.9°and 1.7° respectively. At the 22° and 67° positions, the bisectors in angles of the same size ought to suffer opposite repulsions of equal amounts; as a matter of fact, the negative repulsions average 3.1° more.

When this constant counterclockwise displacement was noted, the apparatus used in exhibiting and measuring the angles was carefully examined, but no fault was discovered. Nor can we suppose that the apparent position of the vertical and horizontal meridians suffers a negative displacement, for straight lines coinciding with the meridians are reproduced without any errors at all. It seems that we are dealing here with a primitive fact. It enhanced greatly the difficulty of interpreting the data furnished by these experiments.