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On the Summation Σx^n

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ON THE SUMMATION Σx^n

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The symbol Σx^n will be used to denote the sum of the *n* th powers of the first x positive integers, that is, $1^n + 2^n + 3^n + \dots + x^n$, n integral. It is well known that this sum is equivalent to a polynomial of degree $(n+1)$, having no constant term.

The formula by means of which Σx^n is usually computed is

$$* \Sigma x^n = \frac{x^{n+1}}{n+1} + \frac{x^n}{2} + \frac{B_1}{2!} n x^{n-1} - \frac{B_3}{4!} n (n-1)(n-2) x^{n-3} + \frac{B_5}{6!} n(n-1)(n-2)(n-3)(n-4) x^{n-5} - \dots$$

In this formula the B's are Bernoulli's numbers, and have the values

$$B_1 = \frac{1}{6}, \quad B_3 = \frac{1}{30}, \quad B_5 = \frac{1}{42}, \dots$$

We note that this formula for Σx^n contains only a finite number of non-zero terms, and from this we conclude that it is desirable to write the final non-zero term. It is

$$(-1)^{\frac{n-1}{2}} \frac{B_n}{n+1} \quad \text{when n is an odd integer}$$

$$\text{and } (-1)^{\frac{n}{2}-1} B_{n-1} x \quad \text{when n is an even integer.}$$

The student who is not acquainted with the derivation of this formula, and who is not aware of its limitations, would naturally in his applications of it make use of all the non-zero terms. That this cannot be done is seen immediately by noting that when *n* is an odd integer the formula contains a constant term, while Σx^n does not. If the user of this formula can remember that he must drop the final term when it is a constant he will always obtain the correct result, but there is nothing in the formula to indicate this. A formula to be complete should not leave this to the memory.

A much better method of procedure would be to introduce a constant making the formula read

* Higher Algebra, Hall and Knight, Fourth edition, 1920, page 337.

$$\Sigma x^n = C_n + \frac{x^{n+1}}{n+1} + \frac{x^n}{2} + \frac{B_1}{2!} n\, x^{n-1} - \frac{B_3}{4!} n(n-1)(n-2)\, x^{n-3} + \ldots$$

and then to determine C_n so that in the application of it all the non-zero terms would be made use of.

If *n* is an odd integer the formula is

$$\Sigma x^n = C_n + \frac{x^{n+1}}{n+1} + \frac{x^n}{2} + \frac{B_1}{2!} n\, x^{n-1} - \frac{B_3}{4!} n(n-1)(n-2)\, x^{n-3} + \ldots$$
$$+ (-1)^{\frac{n-1}{2}} \frac{B_n}{n-1}.$$

Putting $x = 0$, there results $0 = C_n + (-1)^{\frac{n-1}{2}} \frac{B_n}{n+1}$ whence $C_n = (-1)^{\frac{n+1}{2}} \frac{B_n}{n+1}$.

If *n* is an even integer the formula is

$$\Sigma x^n = C_n + \frac{x^{n+1}}{n+1} + \frac{x^n}{2} + \frac{B_1}{2!} n\, x^{n-1} - \frac{B_3}{4!} n(n-1)(n-2)\, x^{n-3} + \ldots$$
$$+ (-1)^{\frac{n}{2}-1} B_{n-1}\, x.$$

Putting $x = o$, there results $o = C_n + o$. Whence $C_n = o$.

If B_{-1} is defined as zero, it will be necessary to take $C_0 = -\frac{1}{2}$ if the formula is to hold true for $n=o$.

The constant thus has the value $(-1)^{\frac{n+1}{2}} \frac{B_n}{n+1}$, if n is odd, o, if n is even, and $-\frac{1}{2}$, if n is zero. The complete formula then would contain the constant C_n with a statement of its value.

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STATISTICAL CONTROL OF A GRADING SYSTEM

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"Statistical Control" is a phrase which is very popular in business circles. What does it mean? Let us examine two types, the statistical control of the quality of a manufactured product, and the statistical control of a purchasing department. In the former case, over a sufficient period of time, records are made of the number of defects per 1000 (say) found in the product manufactured. From these records statistical standards are set up consisting of the averages and distributions of the occurrence of defects. By comparing any subsequent sample with such standards, the statistician is able to assert that the occurrence of defects in this sample is within or without the ordinary limits of random sampling. If without, the manufacturing department is notified of the excess of defects and is expected to remedy the situation. In the case of the purchase of commodities, the standard (price to be paid) is ill defined or lacking. In this type of control, the distinguishing feature in the estimation of the proper price to be paid is the collection and use of a group of statistical facts concerning correlated, independent variables. Some such are (1) the trend of prices in the commodity itself with due attention to seasonal and cyclical fluctuations; (2) the level of general commodity prices, (3) the facility of credit, etc. From such data, the statistician fixes an average price to be paid for the commodity and a proper time for its purchase.

I shall draw your attention to three features of the application of statistical control to a grading system, discussing them somewhat in detail.

First, the Standards Set Up. Since the grading systems of educational institutions usually do not spring full-grown from Jovian brows, the standards are pretty well fixed. Some modifications are possible from time to time, but sudden changes should be avoided. The effort should be to evaluate and unify the existing standards rather than to impose new ones. In our department, for example, the average grade found in the fall of 1928 was set up with only slight modification as a standard. The modification was made in order to bring our departmental standard into conformity with that of the institution. Some change in the distribution of the grades was effected with the same purpose. Statistical control has for its purpose unification and standardization rather than change.

Second, the Correlated, Independent Variables. Such are (1) high school average; (2) aptitude test score; (3) a training test in a particular subject such as English, chemistry or mathematics; and (4) a grade on the preliminary work done by the student in the college. We should like to have information on other variables, for example, health and emotional balance, but such is not at present available. In the case of mathematics, these variables are correlated with the first quarter's grade in magnitudes increasing in the order in which the variables are mentioned above. The values of these four variables for the individual students are rather easily obtained. Three of them may be made available, if desired, even before the student enters college. Using these data to estimate in advance the grade in mathematics, we obtained a multiple correlation of 0.79 after the control was put into effect. In the preceding year, this correlation was 0.76. The small difference indicates that although an improvement was made in the uniformity of the departmental grading, there was no pronounced effect on the grades of individual students. This is as it should be.

After we collected the values of the independent variables, their averages were computed for each group of students. These averages were then substituted in a regression equation to obtain an estimated group average. This brings us to a consideration of

Third, the Group Averages of Estimated Grades, which constitute the mode of statistical control. These averages are estimates, based on past experience, of the grade to be expected in such a group. They possess the usual degree of stability, the probable deviations of the mean grades of our classes of 25 students being less than 1% for averages whose normal value is 82%. Based on this average, a distribution of grades may be suggested for each group, and the instructor may be requested to make his grades conform as closely as may seem practicable, due allowance being made for the usual variation of random sampling. Attention is called particularly to the fact that it is the average of the groups which is controlled, not the grades of the individuals. Furthermore, the average of each group is determined from the estimated abilities of the individuals, so that group differences in ability are evaluated and duly considered.

In conclusion, I shall mention a few advantages of such a statistical control. It should be emphasized in advance that grades play an important role in the student's life. Not only is his graduation determined by grades, but his access to prizes and awards, his election to honorary and social groups, and even his participation in student activities. Any measure taken to standardize grading practice is highly desirable from the student's standpoint.

1. While grading standards are set by the concurrent practice of the majority of the instructors, there are usually in every institution a few whose standards seem to be different. This creates a difficult situation for both students and instructors. The matter is easily adjusted by an adequate statistical control.

2. The average abilities of groups of students vary greatly, especially in cases where sections are formed on the basis of ability. It is not only difficult for an instructor to accurately gauge the level of his class but it is sometimes awkward for him to convince his colleagues that his judgment is correct.

3. Young and inexperienced instructors as well as those newly come from other colleges find such a control valuable in enabling them to align themselves with the institutional standards.

4. Departments and colleges in the same institution frequently have different standards of grading. This leads to or perpetuates different academic standards, and makes the grades of a student in one group non-comparable with those of a student in another. An adequate control of grades would reduce such inequalities, perhaps to a minimum.

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AN ALGORITHM FOR WRITING THE COEFFICIENTS OF A POLYNOMIAL WITH GIVEN ZEROS

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Given a set of numbers $\{A_1\}=a_1 \ldots, a_n$, we may represent the sum of the products, taken r at a time, of the first k numbers of the set by $S(a_i,r,k)$. Obviously $S(a_i,r,k)=S(a_i,r,k-1)+a_kS(a_i,r-1,k-1)$ for r, $k>1$. If in addition we define $S(a_i,o,k)=1$ for $k>0$ the relation holds for $r,k=0$. Also $S(a_i,r,k)=0$ for $k<r$.

This suggests an arrangement of these sums in an array in which r is the number of the row and k is the number of the column with a_k at the head of the column.

For example let the set $\{a_1\}$ be the numbers 2, 3, 4, 5. Then the arrangement will be as follows:

	$k=1$ 2	$k=2$ 3	$k=3$ 4	$k=4$ 5
$r=0$	1	1	1	1
$r=1$	2	5	9	14
$r=2$		6	26	71
$r=3$			24	124
$r=4$				120

and each kth column contains the coefficients, in order, of an equation whose roots are $-a_1$, $-a_2$, ---------- a_k. When the roots are all equal to -1, the rth row will be the figurate numbers of order $r+1$.

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