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Le Roy D. Weld
Coe College

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LE ROY D. WELD

One year ago the writer reported to this Academy the results of some preliminary calculations of the intensities of the three components assumed by Millikan to compose the cosmic radiation, based upon his last published data.[1] At that time the Academy was so kind as to set apart a grant for a continuance of this research. Soon afterward, Dr. Millikan informed me that he was engaged upon a more elaborate series of depth-ionization measurements which promised greater precision, and extended the hope that the results would be available by fall. It was, however, not until February that the new data were received, and it was then found that the results would entail considerably more calculation than had been anticipated from the earlier set.

In their previous paper (*loc. cit.*) Milliken and Cameron assumed *three* components having absorption coefficients 0.04, 0.08, 0.35 per meter, respectively. From the more recent results,[2] they now conclude that there are at least *four* components, the absorption coefficients of which are estimated to be 0.03, 0.10, 0.20, and 0.80, and whose intensities are approximately 33, 80, 130 and 141,000. The exceedingly powerful fourth component, if such exists, is so rapidly absorbed that its effect disappears almost completely within a few meters of depth.

The theory that the cosmic rays enter the absorbing water uniformly from all directions gives rise to an expression for the intensity i at any depth H:

$$i = I_1 G(\mu_1 H) + I_2 G(\mu_2 H) + I_3 G(\mu_3 H) + I_4 G(\mu_4 H) + R, \qquad (1)$$

in which I is the initial intensity and the function G is the integral:[3]

$$G(\mu H) = \int_1^\infty x^{-2} e^{-\mu H x} dx. \qquad (2)$$

R is a term which is added to represent the leakage of the electroscope and to cover any feeble residual radiation not included in

1 Millikan and Cameron, *Phys. Rev.* 31, p. 921 (1928).
2 Millikan and Cameron. Phys. Rev. 37, p. 235 (1931).
3 In the paper cited in footnote 1, this integral appears with the coefficient 2π, which is an error borrowed, apparently, from Gold's paper (Proc. Roy. Soc. A82, p. 62, 1909).

the four principal components. The preliminary estimate of R is 1.2 ions per cc. per sec.

The equation (1) may be regarded as an observation equation containing nine unknowns, I_1, I_2, I_3, I_4, μ_1, μ_2, μ_3, μ_4, and R, the observed quantities being H and i. In a paper read before the American Physical Society in December, 1929, the writer showed that this equation may be transformed into a linear one, in which the unknowns are not the quantities immediately sought, but are the *small corrections* which should be applied to the approximate values already estimated. If for each component, $I = I' + c$, where I' is the known approximate value of I and c the correction, and likewise $\mu = \mu' + k$; and if $R = R' + r$; then the new, linear observation equation may be written

$$A_1c_1 + A_2c_2 + A_3c_3 + A_4c_4 + B_1k_1 + B_2k_2 + B_3k_3 + B_4k_4 + r = s, \quad (3)$$

in which

$$\left.\begin{aligned} A &= G(\mu'H), \\ B &= \frac{I'}{\mu'}\left[G(\mu'H) - e^{-\mu'H}\right], \\ s &= i - \left[\sum_1^4 I'G(\mu'H) + R'\right]. \end{aligned}\right\} \quad (4)$$

In their latest paper (footnote 2), Millikan and Cameron conclude that the absorption coefficients μ, as measured, are not quite constant, but that they are smaller at first and increase to equilibrium values as the depth of peneration becomes greater. This is attributed to the effect of the Compton secondaries. For this reason the writer thought best to divide the range of the observations into two parts, and to analyze each separately, in order, if possible, to detect this change in absorption rate. Further, the large absorption coefficient estimated for the assumed fourth component makes it questionable whether the value of the necessary correction k_4 is small enough to neglect its higher powers. For small depths H these appear with appreciable coefficients in the complete expansion, from which the linear equation (3) is derived by dropping the higher power terms. But at greater depths, these questionable higher terms rapidly die out for all four components. The coefficients of all higher powers of the c's are, in fact, zero; while the terms containing the higher powers of the k's, if the k's are small, become negligible compared with the other terms for the greater depths. It was therefore thought best to analyze the lower half of the observations first, especially as the assumed very absorbable fourth component now disappears altogether, leaving the observations with only seven unknowns instead of nine.

Before proceeding with the adjustment, it was necessary to extend the table of the integral, Eq. (2) above, as published in Gold's paper (footnote 3), to larger values of the argument μ H, and to insert values between those given, for greater accuracy in interpolation. This involved many days of labor. The numerical work of least-square adjustment was done on a calculating machine, and was carefully checked at every stage. The values obtained were found to satisfy the normal equations exactly, and to give smaller residuals, when substituted in the linear observation equations, than Millikan's own estimates.[4]

It is therefore somewhat disconcerting to have to report that these adjusted results yield an altogether impossible set of absorption coefficients and intensities, some of which even turn out negative.

The writer can arrive at only one interpretation of this, namely, that the assumptions upon which the analysis was made are not justified. If, as Millikan and Cameron conclude, the cosmic radiation were composed of four distinct principal bands having absorption coefficients and intensities approximately those assigned to them, and if a sufficiently accurate and continuous series of depth-ionization measurements were available, it should be perfectly possible to determine the constants for each component. We have here applied that analysis which, since the time of Gauss, has been accepted as yielding the most probable values of such constants which can be deduced from a set of observations, namely, the method of least squares; yet the resulting adjusted values prove to be physically meaningless and impossible.

Millikan and Cameron, using a meter-by-meter, trial-and-error method, have succeeded, it must be admitted, in constructing an empirical formula which represents their results with considerable accuracy. But that these results are incapable of uniquely determining rational constants for the assumed formula, the least square analysis just set forth has amply demonstrated. If this formula were a genuine expression of the absorption law under examination, the small correction to be applied to each assumed constant would have appeared as a matter of routine adjustment. But some of the corrections turn out several times larger than the quantities to be corrected; which indicates some altogether untenable premise.

4 Or more properly stated, the sum of the squares of the residuals is distinctly less for these adjusted values than for Millikan's estimates; as it should be in accordance with the principle of least squares.

If the influence of the Compton effect is responsible for the difficulty, the absorption coefficients being in fact variables, this influence must persist at depths of from 20 to 80 meters. Millikan and Cameron remark in their latest article that this phenomenon renders futile a more precise analysis of the curve than that which they have given; a fact which is only too evident if this is indeed the explanation. In that case the obvious course would be to take series of many readings at small, regular depth intervals, not "bunched" as is the case with the published series, and to apply least-square analysis to one section of the curve at a time, so short that the coefficients are sensibly constant throughout the section. In this way the four coefficients, if they exist, could be ascertained to approach true limiting values.

It is interesting to note that in a paper shortly to be read before the Physical Society, Mr. Chas. M. Olmsted contends that Millikan and Cameron's curve can be reproduced just as well on the assumption of a continuous ultra-gamma spectrum beginning abruptly at $\mu = 0.03$ and extending on to much less penetrating radiation. Doubtless there are many other ways in which the curve could be reproduced.

The present investigation does not undertake to decide between conflicting theories, or to interpret the nature of the cosmic rays. But whether the uncertainties here manifest arise from insufficiently accurate tentative values of the coefficients, from the assumption of too few or of too many components, from the distorting effect of Compton scattering, or from an erroneous concept of the physical nature of the whole phenomenon; not to mention the admitted shortcomings of the Klein-Nishina formula; it seems clear that there is every reason to look with reserve upon any such far-reaching conclusions as Millikan and Cameron have deduced from the results of their admirable experimental work.

The writer wishes to express his gratitude to the Academy for the grant which has made this analysis possible; also to express his appreciation to his student assistant, Miss Edna Kerchmar, whose painstaking accuracy throughout the long and laborious calculation has supplied the necessary figures.

Coe College,
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