

1936

## The Decomposition of $4(x^p-1)/(x-1)$

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### Recommended Citation

Gouwens, Cornelius (1936) "The Decomposition of  $4(x^p-1)/(x-1)$ ," *Proceedings of the Iowa Academy of Science*, 43(1), 255-262.

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THE DECOMPOSITION OF  $4(x^p-1)/(x-1)$ 

CORNELIUS GOUWENS

If  $4X = 4(x^p-1)/(x-1)$  where  $p$  is an odd prime then  $4X = Y^2 - (-1)^{(p-1)/2} pZ^2$ , where  $Y$  and  $Z$  are polynomials in  $x$  with integral coefficients. For  $p \leq 37$  we find the decomposition cited in "Recherches sur la theorie des nombres" by M. Kraitchik (1924) p. 126. For  $37 \leq p \leq 61$  the decomposition is given by Pocklington in "Nature," Vol. 107 (1921) pp. 456 and 587. For  $67 \leq p \leq 97$  the results are given by Gouwens in "The Mathematical Monthly, Vol. 43, (1936) page 283.

Herewith are presented the results for  $101 \leq p \leq 199$ . For all decompositions  $Y$  is a polynomial of degree  $(p-1)/2$  and  $Z$  is a polynomial of degree  $(p-3)/2$ .  $Y$  is listed first in each case, then  $Z$ .

$$p = 101$$

2, 1, 26, -12, 40, -65, 33, 9, 40, 49, -110, -4, 13, 63, 86, -166, -3, 6, 90, 77, -185, 19, 53, 76, 66, -206, 66, 76, 53, 19, -185, 77, 90, 6, -3, -166, 86, 63, 13, -4, -110, 49, 40, 9, 33, -65, 40, -12, 26, 1, 2, -1, 0, -4, 4, -3, 5, -5, -6, 3, 0, 10, -5, -11, 4, 0, 15, -8, -12, 5, 1, 13, -11, -14, 10, 0, 10, -14, -11, 13, 1, 5, -12, -8, 15, 0, 4, -11, -5, 10, 0, 3, -6, -5, 5, -3, 4, -4, 0, -1.

$$p = 103$$

2, 1, -25, -38, 44, 122, 26, -123, -96, -22, 10, 150, 221, -45, -334, -213, 93, 269, 293, 82, -296, -441, -129, 297, 426, 195, -195, -426, -297, 129, 441, 296, -82, -293, -269, -93, 213, 334, 45, -221, -150, -10, 22, 96, 123, -26, -122, -44, 38, 25, -1, -2, -1, -1, 4, 8, -2, -14, -7, 6, 8, 10, 10, -13, -31, -8, 25, 29, 11, -13, -36, -30, 11, 43, 33, -6, -37, -37, -6, 33, 43, 11, -30, -36, -13, 11, 29, 25, -8, -31, -13, 10, 10, 8, 6, -7, -14, -2, 8, 4, -1, -1.

$$p = 107$$

2, 1, -26, 14, 39, -104, 83, 100, -254, 200, 83, -412, 372, 12, -447, 563, -120, -406, 598, -285, -318, 515, -253, -194, 403, -78, -237, 237, 78, -403, 194, 253, -515, 318, 285, -598, 406, 120, -563, 447, -12, -372, 412, -83, -200, 254, -100, -83, 104, -39, -14, 26, -1, -2.

-1, 0, 4, -5, 0, 13, -18, 2, 22, -35, 14, 28, -48, 31, 21, -58, 42, 6,  
-53, 48, 1, -37, 38, 3, -32, 15, 15, -32, 3, 38, -37, 1, 48, -53, 6,  
42, -58, 21, 31, -48, 28, 14, -35, 22, 2, -18, 13, 0, -5, 4, 0, -1.

$p = 109$

2, 1, 28, -13, 84, -77, 211, -170, 372, -339, 599, -543, 883, -874,  
1236, -1225, 1603, -1634, 1966, -2004, 2308, -2408, 2631, -2689,  
2842, -2909, 2947, -2931, 2947, -2909, 2842, -2689, 2631, -2408,  
2308, -2004, 1966, -1634, 1603, -1225, 1236, -874, 883, -543, 599,  
-339, 372, -170, 211, -77, 84, -13, 28, 1, 2.

-1, 0, -5, 4, -13, 11, -28, 24, -45, 41, -71, 67, -100, 100, -137,  
137, -170, 174, -206, 212, -236, 245, -265, 270, -277, 281, -285,  
281, -277, 270, -265, 245, -236, 212, -206, 174, -170, 137, -137,  
100, -100, 67, -71, 41, -45, 24, -28, 11, -13, 4, -5, 0, -1.

$p = 113$

2, 1, 29, 43, 80, 185, 179, 255, 345, 194, 286, 273, 78, 301, 262,  
193, 473, 376, 353, 537, 436, 431, 583, 504, 437, 539, 370, 292, 424,  
292, 370, 539, 437, 504, 583, 431, 436, 537, 353, 376, 473, 193, 262,  
301, 78, 273, 286, 194, 345, 255, 179, 185, 80, 43, 29, 1, 2.

-1, -1, -5, -10, -11, -23, -25, -21, -34, -20, -15, -30, -11, -22,  
-39, -23, -40, -47, -33, -46, -51, -41, -50, -53, -35, -40, -38,  
-22, -38, -40, -35, -53, -50, -41, -51, -46, -33, -47, -40, -23,  
-39, -22, -11, -30, -15, -20, -34, -21, -25, -23, -11, -10, -5, -1, -1.

$p = 127$

2, 1, -31, -47, 70, 190, 26, -248, -169, 45, 11, 74, 399, 242, -447,  
-521, 39, 191, 34, 374, 498, -294, -778, -231, 258, 136, 319, 513,  
-79, -636, -346, 16, -16, 346, 636, 79, -513, -319, -136, -258, 231,  
778, 294, -498, -374, -34, -191, -39, 521, 447, -242, -399, -74, -11,  
-45, 169, 248, -26, -190, -70, 47, 31, -1, -2.

-1, -1, 5, 10, -4, -22, -10, 15, 13, 5, 18, 3, -44, -39, 23, 39, 7,  
12, 16, -38, -66, 2, 53, 26, 10, 15, -25, -63, -16, 32, 24, 24, 32,  
-16, -63, -25, 15, 10, 26, 53, 2, -66, -38, 16, 12, 7, 39, 23, -39,  
-44, 3, 18, 5, 13, 15, -10, -22, -4, 10, 5, -1, -1.

$p = 131$

2, 1, -32, 17, 64, -152, 79, 195, -333, 144, 294, -422, 65, 390, -438,  
22, 357, -374, 124, 169, -297, 206, 112, -460, 412, 16, -468, 414,  
37, -386, 306, 94, -411, 411, -94, -306, 386, -37, -414, 468, -16,  
-412, 460, -112, -206, 297, -169, -124, 374, -357, -22, 438, -390,  
-65, 422, -294, -144, 333, -195, -79, 152, -64, -17, 32, -1, -2.

-1, 0, 5, -6, -2, 19, -19, -5, 32, -28, -12, 39, -24, -20, 42, -23, -10,  
28, -23, 1, 22, -34, 12, 28, -48, 24, 22, -37, 12, 26, -38, 17, 17,

-38, 26, 12, -37, 22, 24, -48, 28, 12, -34, 22, 1, -23, 28, -10, -23, 42, -20, -24, 39, -12, -28, 32, -5, -19, 19, -2, -6, 5, 0, -1.

$p = 137$

2, 1, 35, 52, 114, 267, 279, 445, 597, 404, 603, 532, 213, 600, 422, 344, 847, 536, 582, 783, 486, 631, 763, 940, 1046, 1184, 1372, 895, 994, 906, 384, 970, 805, 716, 1326, 716, 805, 970, 384, 906, 994, 895, 1372, 1184, 1046, 940, 763, 631, 486, 783, 582, 536, 847, 344, 422, 600, 213, 532, 603, 404, 597, 445, 279, 267, 114, 52, 35, 1, 2.

-1, -1, -6, -12, -15, -33, -37, -37, -58, -37, -34, -53, -18, -40, -60, -31, -66, -62, -39, -60, -53, -61, -76, -98, -106, -90, -111, -72, -52, -84, -40, -73, -100, -56, -100, -73, -40, -84, -52, -72, -111, -90, -106, -98, -76, -61, -53, -60, -39, -62, -66, -31, -60, -40, -18, -53, -34, -37, -58, -37, -37, -33, -15, -12, -6, -1, -1.

$p = 139$

2, 1, -34, 18, 120, -170, -179, 499, -10, -828, 520, 787, -1047, -285, 1140, -288, -696, 461, 120, -88, 81, -467, 286, 727, -988, -390, 1494, -387, -1435, 1051, 869, -1165, -328, 730, 290, -290, -730, 328, 1165, -869, -1051, 1435, 387, -1494, 390, 988, -727, -286, 467, -81, 88, -120, -461, 696, 288, -1140, 285, 1047, -787, -520, 828, 10, -499, 179, 170, -120, -18, 34, -1, -2.

-1, 0, 6, -6, -14, 27, 11, -58, 20, 74, -69, -49, 99, -2, -82, 38, 33, -28, 4, -17, 3, 56, -53, -54, 110, 2, -131, 65, 101, -101, -47, 84, 20, -40, -40, 20, 84, -47, -101, 101, 65, -131, 2, 110, -54, -53, 56, 3, -17, 4, -28, 33, 38, -82, -2, 99, -49, -69, 74, 20, -58, 11, 27, -14, -6, 6, 0, -1.

$p = 149$

2, 1, 38, -18, 96, -152, 109, -101, 194, 117, -90, -113, -40, -27, 554, -279, 51, -323, -45, 462, -4, -160, -48, -481, 631, -70, -40, 38, -564, 499, 71, -120, 273, -510, 317, 255, -388, 475, -388, 255, 317, -510, 273, -120, 71, 499, -564, 38, -40, -70, 631, -481, -48, -160, -4, 462, -45, -323, 51, -279, 554, -27, -40, -113, -90, 117, 194, -101, 109, -152, 96, -18, 38, 1, 2.

-1, 0, -6, 6, -8, 15, -13, -4, -9, -2, 17, 10, -27, 4, -33, 39, 11, -11, -6, -26, 12, 42, -27, 19, -40, 4, 42, -24, 21, -29, -18, 43, -28, 11, -3, -36, 49, -36, -3, 11, -28, 43, -18, -29, 21, -24, 42, 4, -40, 19, -27, 42, 12, -26, -6, -11, 11, 39, -33, 4, -27, 10, 17, -2, -9, -4, -13, 15, -8, 6, -6, 0, -1.

$p = 151$

2, 1, -37, -56, 102, 273, -14, -466, -116, 528, -42, -850, 255, 1459,

- 184, -1859, 63, 1771, -539, -1856, 1301, 2496, -1322, -2626, 1059, 1820, -1688, -1461, 2483, 1995, -2153, -2072, 1339, 1170, -1543, -794, 2258, 1750, -1750, -2258, 794, 1543, -1170, -1339, 2072, 2153, -1995, -2483, 1461, 1688, -1820, -1059, 2626, 1322, -2496, -1301, 1856, 539, -1771, -63, 1859, 184, -1459, -255, 850, 42, -528, 116, 466, 14, -273, -102, 56, 37, -1, -2.

-1, -1, 6, 12, -7, -32, -6, 40, 6, -52, 14, 93, -21, -140, 7, 151, -17, -141, 78, 175, -116, -220, 92, 185, -102, -122, 177, 137, -201, -177, 138, 137, -104, -65, 160, 96, -178, -178, 96, 160, -65, -104, 137, 138, -177, -201, 137, 177, -122, -102, 185, 92, -220, -116, 175, 78, -141, -17, 151, 7, -140, -21, 93, 14, -52, 6, 40, -6, -32, -7, 12, 6, -1, -1.

$p = 157$

2, 1, 40, -19, 160, -170, 446, -483, 813, -933, 1048, -1149, 1038, -1055, 805, -661, 650, -385, 668, -242, 604, -158, 181, 3, -262, 7, -286, -240, 325, -582, 982, -695, 1271, -706, 1193, -957, 1130, -1410, 1166, -1683, 1166, -1410, 1130, -957, 1193, -706, 1271, -695, 982, -582, 325, -240, -286, 7, -262, 3, 181, -158, 604, -242, 668, -385, 650, -661, 805, -1055, 1038, -1149, 1084, -933, 813, -483, 446, -170, 160, -19, 40, 1, 2.

-1, 0, -7, 6, -22, 24, -51, 57, -77, 86, -89, 92, -73, 69, -57, 40, -51, 24, -54, 16, -34, 7, 5, -4, 29, 8, 2, 33, -54, 54, -95, 55, -100, 63, -91, 94, -92, 128, -93, 128, -92, 94, -91, 63, -100, 55, -95, 54, -54, 33, 2, 8, 29, -4, 5, 7, -34, 16, -54, 24, -51, 40, -57, 69, -73, 92, -89, 86, -77, 57, -51, 24, -22, 6, -7, 0, -1.

$p = 163$

2, 1, -40, 21, 161, -230, -238, 757, -197, -1155, 1307, 440, -1969, 1373, 573, -2103, 2216, -275, -2898, 3887, -493, -3978, 4418, -646, -3356, 4659, -2449, -2461, 6155, -3970, -2790, 6872, -3973, -2389, 6067, -4454, -865, 5757, -5688, -48, 6114, -6114, 48, 5688, -5757, 865, 4454, -6067, 2389, 3973, -6872, 2790, 3970, -6155, 2461, 2449, -4659, 3356, 646, -4418, 3978, 493, -3887, 2898, 275, -2216, 2103, -573, -1373, 1969, -440, -1307, 1155, 197, -757, 238, 230, -161, -21, 40, -1, -2.

-1, 0, 7, -7, -18, 36, 9, -81, 55, 77, -142, 33, 123, -161, 65, 102, -239, 174, 127, -361, 218, 160, -368, 250, 55, -347, 395, -53, -418, 494, -54, -425, 469, -101, -326, 479, -219, -274, 542, -276, -276, 542, -274, -219, 479, -326, -101, 469, -425, -54, 494, -418, -53, 395, -347, 55, 250, -368, 160, 218, -361, 127, 174, -239, 102, 65, -161, 123, 33, -142, 77, 55, -81, 9, 36, -18, -7, 7, 0, -1.

$$p = 167$$

2, 1, -41, -62, 71, 281, 355, 103, -561, -1170, -1028, -40, 1242, 1798, 1030, -396, -1323, -955, 440, 1448, 1178, -27, -1184, -1318, -546, 173, 205, -340, -656, -351, 231, 634, 431, -74, -235, -5, 452, 649, 357, 12, -247, -189, 189, 247, -12, -357, -649, -452, 5, 235, 74, -431, -634, -231, 351, 656, 340, -205, -173, 546, 1318, 1184, 27, -1178, -1448, -440, 955, 1323, 396, -1030, -1798, -1242, 40, 1028, 1170, 561, -103, -355, -281, -71, 62, 41, -1, -2.

-1, -1, 6, 13, 5, -19, -49, -55, -6, 74, 124, 96, -8, -106, -116, -35, 71, 100, 22, -82, -127, -80, 12, 60, 39, -9, -22, 16, 55, 59, 24, -19, -20, 9, 33, 30, -7, -35, -36, -21, 7, 7, -21, -36, -35, -7, 30, 33, 9, -20, -19, 24, 59, 55, 16, -22, -9, 39, 60, 12, -80, -127, -82, 22, 100, 71, -35, -116, -106, -8, 96, 124, 74, -6, -55, -49, -19, 5, 13, 6, -1, -1.

$$p = 173$$

2, 1, 44, -21, 133, -209, 140, -216, 57, 371, -625, 811, -960, 594, 338, -1138, 1698, -2046, 1615, -71, -1397, 2191, -3037, 2644, -626, -1001, 2262, -3412, 3260, -1503, -414, 1691, -3074, 3599, -2023, 203, 909, -2589, 3516, -2105, 592, 589, -2324, 3235, -2324, 589, 592, -2105, 3516, -2589, 909, 203, -2023, 3599, -3074, 1691, -414, -1503, 3260, -3412, 2262, -1001, -626, 2644, -3037, 2191, -1397, -71, 1615, -2046, 1698, -1138, 338, 594, -960, 811, -625, 371, 57, -216, 140, -209, 133, -21, 44, 1, 2.

-1, 0, -7, 7, -11, 22, -10, -3, 15, -49, 75, -64, 34, 8, -84, 150, -150, 97, -29, -87, 213, -217, 160, -92, -67, 216, -248, 210, -141, 0, 177, -242, 213, -183, 63, 131, -207, 196, -199, 84, 99, -184, 197, -184, 99, 84, -199, 196, -207, 131, 63, -183, 213, -242, 177, 0, -141, 210, -248, 216, -67, -92, 160, -217, 213, -87, -29, 97, -150, 150, -84, 8, 34, -64, 75, -49, 15, -3, -10, 22, -11, 7, -7, 0, -1.

$$p = 179$$

2, 1, -44, 23, 132, -275, 91, 532, -799, 210, 826, -1148, 302, 552, -823, 535, -125, -453, 1275, -883, -579, 1749, -1110, -580, 879, -388, 184, -668, 354, 1175, -1455, 396, 938, -953, 443, -645, -17, 1258, -1957, 301, 2085, -2112, 547, 1138, -1385, 1385, -1138, -547, 2112, -2085, -301, 1957, -1258, 17, 645, -443, 953, -938, -396, 1455, -1175, -354, 668, -184, 388, -879, 580, 1110, -1749, 579, 883, -1275, 453, 125, -535, 823, -552, -302, 1148, -826, -210, 799, -532, -91, 275, -132, -23, 44, -1, -2.

-1, 0, 7, -8, -7, 35, -32, -23, 74, -60, -24, 78, -56, 5, 37, -51, 61, -9, -91, 115, -27, -102, 96, -11, -18, 2, -10, 90, -75, -39, 108, -76,

-13, 19, -41, 93, -72, -51, 183, -103, -68, 155, -112, 21, 21, -112, 155, -68, -103, 183, -51, -72, 93, -41, 19, -13, -76, 108, -39, -75, 90, -10, 2, -18, -11, 96, -102, -27, 115, -91, -9, 61, -51, 37, 5, -56, 78, -24, -60, 74, -23, -32, 35, -7, -8, 7, 0, -1.

p = 181

2, 1, 46, -22, 207, -230, 664, -728, 1580, -1806, 2936, -3335, 4866, -5485, 7090, -8109, 10103, -11270, 13468, -14970, 17577, -18770, 21600, -22766, 25530, -26276, 28766, -29499, 31304, -31843, 32994, -33428, 34053, -34050, 34363, -34021, 34324, -33374, 33535, -32707, 32665, -31761, 31534, -31425, 30939, -30987, 30939, -31425, 31534, -31761, 32665, -32707, 33535, -33374, 34324, -34021, 34363, -34050, 34053, -33428, 32994, -31843, 31304, -29499, 28766, -26276, 25530, -22766, 21600, -18770, 17577, -14970, 13468, -11270, 10103, -8109, 7090, -5485, 4866, -3335, 2936, -1806, 1580, -728, 664, -230, 207, -22, 46, 1, 2.

-1, 0, -8, 7, -28, 32, -80, 88, -162, 188, -287, 320, -439, 504, -633, 711, -872, 976, -1148, 1249, -1460, 1548, -1752, 1824, -2028, 2080, -2237, 2286, -2399, 2436, -2496, 2513, -2550, 2539, -2555, 2504, -2530, 2461, -2457, 2391, -2391, 2348, -2309, 2315, -2301, 2315, -2309, 2348, -2391, 2391, -2457, 2461, -2530, 2504, -2555, 2539, -2550, 2513, -2496, 2436, -2399, 2286, -2237, 2080, -2028, 1824, -1752, 1548, -1460, 1249, -1148, 976, -872, 711, -633, 504, -439, 320, -287, 188, -162, 88, -80, 32, -28, 7, -8, 0, -1.

p = 191

2, 1, -47, -71, 105, 381, 397, -27, -658, -1152, -1400, -1128, 65, 1877, 3222, 3382, 2572, 1047, -1099, -3156, -4013, -3540, -2765, -2053, -908, 493, 1523, 2326, 3410, 4023, 2915, 462, -1879, -3393, -3947, -2791, 195, 3189, 4381, 3720, 2050, -267, -2549, -3330, -2437, -1300, -771, -330, 330, 771, 1300, 2437, 3330, 2549, 267, -2050, -3720, -4381, -3189, -195, 2791, 3947, 3393, 1879, -462, -2915, -4023, -3410, -2326, -1523, -493, 908, 2053, 2765, 3540, 4013, 3156, 1099, -1047, -2572, -3382, -3222, -1877, -65, 1128, 1400, 1152, 658, 27, -397, -381, -105, 71, 47, -1, -2.

-1, -1, 7, 15, 3, -29, -55, -50, -16, 40, 118, 183, 171, 66, -74, -194, -279, -299, -218, -67, 60, 129, 187, 236, 235, 195, 156, 90, -57, -225, -296, -245, -123, 49, 241, 331, 239, 41, -140, -256, -295, -215, -50, 73, 102, 101, 116, 116, 101, 102, 73, -50, -215, -295, -256, -140, 41, 239, 331, 241, 49, -123, -245, -296, -225, -57, 90, 156, 195, 235, 236, 187, 129, 60, -67, -218, -299, -279, -194, -74, 66, 171, 183, 118, 40, -16, -50, -55, -29, 3, 15, 7, -1, -1.

$p = 193$

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DEPARTMENT OF MATHEMATICS,  
IOWA STATE COLLEGE,  
AMES, IOWA.