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Cognitive ability and mathematics instruction as factors in mathematical problem-solving achievement of fifth graders

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Abstract

Problem solving has been an area of educational concern for many years. Research completed as far back as 1926 and 1932 centered on comparing methods of solving problems or identifying types of errors students made when solving verbal problems. Research has focused on characteristics of the problems, characteristics of those who are successful or unsuccessful at solving problems, and teaching strategies that may improve students' abilities in problem solving.

COGNITIVE ABILITY AND MATHEMATICS INSTRUCTION AS FACTORS IN MATHEMATICAL PROBLEM-SOLVING ACHIEVEMENT OF FIFTH GRADERS

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MATHEMATICAL PROBLEM-SOLVING ACHIEVEMENT

Introduction

Review of the Literature

Problem solving has been an area of educational concern for many years. Research completed as far back as 1926 and 1932 centered on comparing methods of solving problems or identifying types of errors students made when solving verbal problems. Research has focused on characteristics of the problems, characteristics of those who are successful or unsuccessful at solving problems, and teaching strategies that may improve students' abilities in problem solving.

More recent research centers on determining the nature of the skills and abilities which contribute toward student success in solving verbal problems. The possibility of investigating skills and abilities in combination as well as in isolation has become desirable when viewed from the perspective that research implications could provide direction for classroom teachers for implementing more effective instructional approaches.

Since the 1970s, problem solving has been a topic of increased popularity and emphasis in mathematics education. Since the National Council of Teachers of Mathematics (1980) recommended that problem solving should be the focus of school mathematics, problem solving is now considered to be an ultimate goal of mathematics instruction. Research continues to focus on the nature of problem solving and implications for classroom application that will enable students to become better problem solvers.

Frank (1988) poses the question, "How can we get students to become better problem solvers?" Answers to this question have focused on instructional techniques. She proposes that students may not be able to become better problem solvers unless they change their beliefs about mathematics. What goes on in the classroom strongly influences the mathematical beliefs of students. Consequently, she advocates a focus on solutions, not answers; the use of small groups; and a belief that problem solving, not computation, should be the focus of mathematics instruction if students are to become better problem solvers. Frank's belief is in agreement with the National Council of Teachers of Mathematics 1980 recommendation.

There seems to be some conflict of opinion regarding curriculum. The back-to-the-basics movement was interpreted by the textbook companies to be a directive to stress computational skills, virtually omitting real problem-solving skills (Ward, 1979).

In his study Ward included the following recommendations which were based upon the National Assessment of Education Progress's (NAEP) second assessment of mathematics:

1. Students must be introduced to exercises involving higher-level, as well as lower-level cognitive processes.
2. Textbooks should emphasize the ability to analyze a problem situation.
3. Problem solving needs to be taught.

4. Teachers should be prepared to teach problem solving.
5. Further research on methods of teaching problem solving is necessary.
6. Further research into causes and remedies for the disparities in group performance that exist is needed (Ward, 1979).

Hence, the new directive in mathematics is to place increased emphasis on enhancing problem-solving skills.

Problem solving, as defined in this paper, is similar to Wheatley's definition: "Problem solving is what you do when you don't know what to do" (Wheatley, 1984). It enlists use of Polya's (1957) four main stages in problem solving. Problem solving is, therefore, a process. In the process of problem solving, individuals use previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation. Students synthesize information and apply it to the new situation. The focus is on the process, not on the answer. It entails more than thinking skills and problem-solving strategies. It is a tool, a means of thinking, a philosophy, and a predisposition to learn. Whereas arithmetic word problems usually measure a student's ability to apply a fact, rule, or procedure to obtain the right answer. A problem is a situation in which there is a goal blocked by an obstacle with no apparent solution. Problem solving is <u>not</u> story problems that drill a concept in a textbook lesson or disguised exercises.

Problem-solving skills should be taught as distinct skills. Math problem-solving skills are defined here as higher-level thinking skills. They are needed to solve the multi-step and process problems which have no pre-determined plan for solution. Whereas basic (lower-level) math skills hinge upon knowing the basic facts and using them to solve problems which inherently define specific use of operations (addition, subtraction, multiplication, division) required for obtaining correct answers. In true problem solving there may be two or more acceptable strategies for solving the problem. Strategies (heuristics, plans) used in problem solving include act it out, draw a diagram or make a model, make an organized list or table, look for a pattern, work backward, logical thinking, reasoning, or intuitive thinking.

Research indicates that several factors may affect problem-solving ability. These factors include characteristics of good problem solvers (ability, intelligence, sex, and aptitude), type of instruction (especially small-group instruction), and interaction of ability and instruction.

Characteristics of Problem Solvers. Several studies discussed characteristics of good problem solvers. Sowder (1984) examined grade level, sex differences, reading ability, and general ability of good problem solvers. In his title Sowder used the words "Mathematical Problem Solving" and in his report he equated that phrase with "typical mathematics story problems or multistep story problems." The literature (Polya, 1957; Wheatley, 1984) does not support the use of that term to describe the true problem-solving dimension. Multistep problems in texts have a definite operational procedure which can be determined, and that procedure brings a correct answer; however, problem solving as defined in the literature indicates that problems do not have a pre-determined operation. Rather, students need to find a strategy that helps them solve the problem. One problem may have several applicable strategies.

Suydam (1970) focused on several characteristics of problem solvers: skill in computation, reading comprehension, and understanding of mathematical concepts. Suydam's work gives evidence that intelligence is related to problem-solving ability, whereas sex differences have not been found to be related to that ability.

Several comprehensive research studies (Beckerman & Good, 1981; Hungerman, 1981; Meyer, 1978a, 1978b; Silver, 1977; Talton, 1973) have addressed the factors of IQ, sex, and aptitude in relation to mathematical problem-solving performance. Meyer (1978a) investigated relations between intellectual abilities and mathematical problem-solving performance. She attempted to identify a structure of mental abilities related to problem-solving performance. Her study used 179 fourth-grade children involved in a specific math program, hence the sample group shared similar experiential background. Location was selected for the convenience of the investigator. Twenty tests were administered. The means, standard deviation (SD), and Hoyt analysis of variance reliability estimates were determined. Ten of these estimates were greater than or equal to .80; however, the reliability of the problem-solving part was relatively low. Factor analysis was also used. Generalizability of the results was limited by the nonrandom sample and the difficulty of the problem-solving questions. According to Meyer (1978a), the problem solving mean was only 3.47, SD was 2.40, and range of correct responses was 0-13.

In her conclusions Meyer (1978a) suggested the following ideas:

1. Intellectual structures contain a specific mathematics ability. Verbal, reasoning, numerical, and perceptual-speed abilities are present in intellectual structures.
2. Prerequisite mathematics concepts and skills are related to, and may account for, some of the variance of problem solving; however, knowing skills does not guarantee successful problem solving.

Silver (1977) examined the relation between a student's use of mathematical structure and his problem-solving ability. Analyses of the data were used to identify relations between students' performance on ability measures and their perception of problem relatedness. Silver (1977) states that Polya suggests one should try to think of a related problem when devising a plan for solving a problem. The significant positive relation, $\underline{r} = .54$, $\underline{p} < .001$ between structure DPS (a measure of the degree to which students perceive the mathematical structure of word problems) and problem-solving ability remained significant in the partial correlation analysis.

Students utilizing mathematical structure for classifying problems tended to be high in mathematical ability (nonverbal IQ), computational ability, conceptual and reasoning ability, or problem-solving ability. When the effects of verbal and nonverbal IQ, conceptual knowledge, and computational ability were all controlled, the relation remained significant, $p < .01$. Silver (1977) recommended the development of instructional strategies designed to teach low-ability students to disregard pseudostructure (quantity measured, such as age or weight) dimensions of problems, because highly capable students perceive the structure of problems and relate problems to that perception.

Researchers (Hungerman, 1981; Meyer, 1978b; Talton, 1973) also strongly suggest that intelligence is highly related to mathematical problem-solving achievement. Talton's comprehensive dissertation (1973) states that the level of intellectual development (abstract thinking) is related to mathematical verbal problem-solving ability. The ability to complete mental operations for solving problems is related to the ability to solve verbal problems. The high correlation coefficients indicated that intelligence is highly related to the ability to solve verbal problems in mathematics. The area of arithmetic concepts (the ability to apply basic principles) was also found to be highly related to solving verbal problems in her study.

Both sex-related differences and intelligence as factors in mathematics achievement were studied by Meyer (1978b) and Hungerman (1981). Essentially they were in agreement with the conclusion that elementary males and females performed equally well on all tests of mathematics achievement. Meyer (1978b) adds that each sex may, however, prefer different intellectual processes for problem solving.

Hungerman (1981) concluded that girls and boys at the highest intelligence level are most similar in their mathematics performance. She also states that when the program of instruction is controlled, there were no significant sex-related differences at the highest intelligence level.

Instructional Methods. What are some components of effective instructional techniques for teaching problem solving? Teaching specific strategies, creating a warm classroom climate, using effective questioning, and implementing small-group instruction are suggested techniques that can be utilized by the classroom teacher.

Do instructional techniques for teaching problem-solving skills affect students' performance on problem-solving tests? Research results reflect the importance of these techniques in problem-solving achievement.

Suydam (1970) states that systematic instruction not only in how-to-solve-a-problem, but in why-that-process-is-appropriate has been found to be effective in increasing problem-solving achievement and understanding. She compared a group of fourth graders who used this type of instruction with another group which merely solved the problems. The first group showed statistically significant gains on tests of problem solving. Sixth grade classes which used specific procedures achieved higher mean gains on problem-solving tests than did control groups who followed the regular textbook program.

Interaction of Ability and Instruction. Research by Peterson, Swing, Braverman, and Buss (1982) addressed the relationship between specific cognitive processes (such as use of the overview as a teaching strategy) that students used to understand the problem and student aptitude on achievement. Fifth- and sixth-grade students (N = 72) were randomly assigned to classes that followed the direct instruction model (Peterson et al., 1982). Reliable, standardized tests were used to measure outcomes. The total number of specific cognitive strategies mentioned by the students was positively related to their achievement scores and to their ability scores. The specific strategy most frequently mentioned by students was "trying to understand the teacher or a problem." These responses were significantly positively related to achievement and aptitude. Since a common limitation of verbal reports is that they are data that may be influenced by experimenter cues, the study group endeavored to overcome this limitation by using questions that avoided cues.

A fairly frequent variable in related studies was the use of small-group instruction. Some research studies suggest that small-group instruction is not equally effective for all students. A correlational study by Swing and Peterson (1981) suggests that task-related interaction in a small group enhanced the achievement and retention of high- and low-ability students, but did not facilitate the achievement of medium-ability students. It also supported findings that low-ability students not only benefited from a small-group approach, but that they learned best in mixed-ability small groups. Their study of fifth graders used a treatment group that was trained in small-group interaction. The students worked in mixed-ability groups of four. The control group received regular math instruction. Weaknesses of the study may be that two intact classes from one school were used (chunk bias) and that the two teachers volunteered (judgment bias) to take part in the study. Reliable tests were used to measure outcomes. The ability score was a sum of the student's two scores on Raven's Progressive Matrices and the Mathematical Computations subtest. The trained and control groups within each ability level were compared using a one-tailed Mann-Whitney U test. An overall type I error of .12 was used in testing the differences between the comparison groups. A correlational analysis was used to determine the relation of small-group interaction to performance. The correlations were tested for significance with a one-tailed test. Results from the study indicated that task-related small-group interaction was highly related to the academic achievement of low-ability students. Achievement of high-ability students was also positively related, but to a lesser extent. The same interaction was unrelated to the achievement of medium-ability students. Descriptive statistics showed that trained low-ability students outperformed control low-ability students by median differences of .8, .5, and .4 standard deviation units based on mean differences. In conclusion, the most important finding of this study was that the effects of small-group interaction depend on the ability level of the students.

A very similar study by Peterson and Janicki (1979) investigated aptitude-treatment interactions in children's learning in a large-group approach and in a small-group approach. Many of the same tests were used to measure outcomes. Generalizability was assessed by the same procedure used in the Swing and Peterson (1981) study. Results were that neither the large-group approach nor the small-group approach was more effective for all students. In this study the low-ability students did better on the delayed test in the large-group approach. Speculation was made that they probably needed more direction and help provided by the teacher in the large-group approach.

Two studies (Beckerman & Good, 1981; Peterson et al., 1982) discussed student aptitude in relation to achievement. Both studies showed that aptitude (a characteristic that predicts a student's probability of success in a given approach which can include ability and attitude) is positively related to achievement. Beckerman and Good's (1981) study investigated how the types of students present in classrooms influence instructional process and outcomes, while the Peterson et al. (1982) study suggested cognitive processes that define ability and produce achievement.

Taken together, these results suggest that a variety of factors (such as ability, intelligence, and instructional techniques) influence student's mathematical problem-solving achievement. Moreover, based on these results, there is still much to be examined regarding how these factors singly and interactively influence students' math problem-solving achievement.

Statement of Purpose

The main purpose of this study was to examine the effects of ability and instruction on the mathematical achievement of fifth graders.

The study attempted to determine differences in mathematical achievement for both basic skills and problem-solving skills of high- and low-ability students when a program of basic textbook instruction was used in the classroom as opposed to a classroom program of basic textbook instruction combined with problem-solving instruction. The goal was to determine which students were benefiting most from the additional problem-solving instruction.

As noted in the literature review, problem solving is a major area of concern in mathematics education. The mathematics assessment panel prescribed more emphasis on the teaching of problem solving and recommended further research on the methods of teaching problem solving (Ward, 1979).

Hypothesis

It was hypothesized that there would be an interaction between student ability and instructional program for problem-solving achievement of fifth graders.

Method

Subjects

The subjects were 91 fifth-grade students from two elementary schools in the Cedar Falls Public School System. The school population was representative of those in other cities of its size (35,000) in the state. These schools were comparable in socioeconomic status and distribution of sex and race of students. They were representative of the city's total school population. The investigation was restricted to fifth-grade students in classrooms which were participating in the Cedar Falls Problem Solving (CFPS) Project.

The 91 subjects (43 male and 48 female) in the study represented four intact classrooms. Intact group selection was used rather than random assignment, because random assignment was not possible in this field experiment. Experimental and control groups were established, using fifth-grade CFPS Project participants as experimental group instructors and fifth-grade non-participants as instructors of control groups. The experimental groups had 48 subjects and the control groups totaled 43 subjects.

Several control features were incorporated to minimize possible confounding variables and error. Nearly all subjects in the two experimental groups had two years of formal problem-solving instruction, whereas none of the subjects in the two control groups had any formal problem-solving instruction. Within-group instructors were selected on the basis of similar training in problem solving, socioeconomic status of schools, number of students in classrooms, and similarity of subjects' involvement in problem-solving education.

The two control-group teachers had no specific training in problem solving; whereas the two teachers of the experimental groups had identical training in mathematics problem solving. They attended the CFPS Project's 3-week 1987 summer session, then continued their training by attending monthly in-service sessions throughout the 1987-88 school year. Problem-solving instructors and consultants provided research-based effective mathematics problem-solving instruction at these sessions. In addition, teachers were given opportunities to share problems, insights, and problem-solving techniques.

Research Design

Because random assignment of subjects was not feasible, this study was a quasi-experimental factorial non-equivalent control group research design. A 2 (teaching method) x 2 (ability) x 2 (time of testing) design was employed. For each of the independent variables two levels were used: teaching method (problem-solving vs. basic skills control), ability (high vs. low), and time of testing (pretest vs. posttest). The between-subjects factors were teaching method and ability; whereas, the within-subjects factor was time of testing. The dependent variables were student achievement scores on mathematics tests (three scores from CFPS Project test and three scores from Iowa Test of Basic Skills).

Instruments

Three instruments were administered during this study: CFPS Project Test, Iowa Test of Basic Skills (ITBS), and Cognitive Ability Test (CAT).

Scoring, Reliability, and Validity. The CFPS Project test was designed by project personnel. The test is a measure of student achievement in problem solving, using the various strategies taught. Raw scores reflect three dimensions: understanding the problem, planning to solve the problem by using a strategy, and answering the problem by finding a correct solution. Standard printed instructions were provided on the test.

Both the ITBS and the CAT are norm-referenced standardized tests. The ITBS mathematics section is designed to measure basic skills. The three mathematics subtests provide a valid measure of curricular mathematics content achievement in the areas of concepts, (word) problems, and computation. The instrument also demonstrates high reliability. High validity and reliability coefficients have been demonstrated for the CAT, as well.

Procedure

This study was conducted as a result of the author's involvement with the Cedar Falls Problem Solving (CFPS) Project. The CFPS Project was implemented in the summer of 1986 due to a grant from the National Science Foundation. Cedar Falls teachers were provided an opportunity to improve mathematics problem-solving instruction.

Pretesting and Posttesting. Pretests were given to subjects in the fall of 1987; posttests were given in May, 1988. The CFPS Project pretests were administered to subjects in the four study groups early in the fall, before formal instruction in mathematics problem solving was begun for the 1987-88 school year. The posttests were given to the subjects in May. These pretests and posttests were based upon the same strategies, but used different problems. These scores were used to measure student achievement in problem solving.

In early November the ITBS battery of tests (Form H Test 11) was administered to all subjects. The mathematics subtest scores (M-1 Concepts, M-2 Problems, and M-3 Computation) were used as pretest scores for basic skills in math (regular instruction). The alternate test form of the mathematics section, Form G of the ITBS, was administered to the 91 subjects as a posttest in May to measure student achievement gain in basic skills.

In early fall, the CAT (Form 3 Level C) was administered to all fifth grade subjects. The scores for the quantitative subsection were used to determine ability levels. The median score of 47.0 was determined for all subjects. A student was classified either as high ability if the score was above the median or as low ability if that score was below the median. Ten students were eliminated from the analysis because their scores were at the median.

Instruction. Teachers of the control groups provided regular textbook mathematics instruction. No formal problem-solving instruction was presented in addition to the textbook.

The experimental-group teachers implemented the 4-step problem-solving plan (i.e., 1. understand the problem, 2. select a strategy, 3. solve the problem, 4. look back). They provided instruction beyond the textbook and presented problem-solving problems to their students during allotted class time (about 60 minutes per week). In the course of the school year, these teachers instructed their subjects in the following problem-solving strategies: (a) draw a picture, (b) make an organized list, (c) make a table, (d) look for a pattern, and (e) guess and check. Sample problems are enclosed. See Appendix A. These teachers used problem-solving exemplars they accumulated as part of their participation in the CFPS Project. Individual problems used by the teachers varied, but instruction of strategies remained a common factor.

The subjects were involved in both individual and small-group activities during problem-solving sessions. A small-group cluster had two to four students. Types of problems included multi-step and process, with the emphasis on process (higher level of thinking skills) problems. When working these problems, students have no predetermined format or plan of solution; they may discover several possible strategies whereby they can find a solution.

Each teacher was observed by a member of the problem-solving team during the year. Each one was video-taped in the spring to enable that teacher to observe and evaluate the problem-solving session.

Specific, printed instructions to the students were provided for the teachers for administration of pretests and posttests of math achievement.

Data Analysis

Data was compiled and analyzed after the posttests had been administered. A 2 ability (low versus high) by 2 teaching method (regular versus problem solving) by 2 time of testing (pretest versus posttest) mixed multivariate analysis of variance (MANOVA) was employed to determine whether differences existed for the six achievement scores (three from the ITBS and three from CFPS). The between-subjects factors were ability and teaching method, and the within-subjects factor was time of testing.

Results

Multivariate Tests

First a series of multivariate analysis of variance (MANOVA) tests were conducted. An F-ratio was obtained for the main effects and interactions of the independent variables. Results of these tests indicated that only four effects had an overall significant F-ratio. The teaching method effect was significant, multivariate $F(6, 72) = 4.19$, $p < .0011$. The ability effect was significant, multivariate $F(6, 72) = 11.34$, $p < .0001$. The teaching method by ability interaction was not significant, multivariate $F(6, 72) = 1.38$, $p < 0.24$. The time of testing (pretest and posttest) effect was significant, multivariate $F(6, 72) = 16.39$, $p < .0001$. The teaching method by time of testing interaction was <u>not</u> significant, multivariate $F(6, 72) = .79$, $p < 0.58$. The ability by time of testing interaction was significant, multivariate $F(6, 72) = 2.74$, $p < .0188$. The teaching method by ability by time of testing interaction was <u>not</u> significant, multivariate $F(6, 72) = 0.54$, $p < 0.78$.

<u>Univariate Tests</u>

The significant effects for teaching method, ability, time of testing, and the ability by time of testing interaction were further analyzed by conducting follow-up analysis of variance (ANOVA) tests. See Table 1. Only these four effects could be investigated further; the other effects were not further analyzed because of the overall nonsignificance of the MANOVA for those effects. To determine the source of the differences that were significant on the MANOVA, univariate ANOVAs were calculated for each dependent measure. Means and standard deviations for the dependent measures are presented in Table 2.

<u>Concepts</u>. For the between-subject effects only ability was significant, $F(1, 77) = 41.60$, $p < .0001$. The mean grade equivalent (G.E.) for the low-ability group was 5.77, whereas the mean G.E. for the high-ability group was 7.18. The effect for teaching method was not significant, $F(1, 77) = 1.24$, $p < .27$. For the within-subject effects, the time of testing effect was significant, $F(1, 77) = 48.53$, $p < .0001$. The mean for the pretest was 6.10 as opposed to 6.87 for the posttest mean. The ability by time interaction was also

significant, $F(1, 77) = 6.27$, $p < .02$. The low-ability group's performance increased approximately one-half a grade level, whereas the high-ability group's performance increased approximately one grade level.

Word Problems. The teaching method effect was not significant, $F(1, 77) = 0.21$, $p < .65$. The ability effect was significant, $F(1, 77) = 38.73$, $p < .0001$. The mean grade equivalent for the low-ability group was 5.46, whereas the mean G.E. for the high-ability group was 6.94. Additionally, the time of testing effect was significant, $F(1, 77) = 16.38$, $p < .0001$. The pretest mean G.E. was 5.96 as opposed to 6.46 for the posttest mean. The ability by time effect was not significant, $F(1, 77) = 0.50$, $p < .49$.

Computation. The teaching method effect was significant, $F(1, 77) = 9.38$, $p < .003$. The mean G.E. for the regular teaching method was 6.16 as compared to 5.81 for the problem-solving teaching method mean. The ability effect was significant, $F(1, 77) = 63.39$, $p < .0001$. The mean for the low-ability group was 5.44, whereas the mean for the high-ability group was 6.50. The time effect was significant, $F(1, 77) = 24.13$, $p < .0001$. The mean for the pretest was 5.76 as opposed to 6.19 for the posttest mean. The ability by time interaction was not significant, $F(1, 77) = 3.13$, $p < .09$.

Understanding the Problem. The teaching method effect was not significant, $F(1, 77) = 1.72$, $p < .20$. The ability effect was significant, $F(1, 77) = 28.20$, $p < .0001$. The mean for the low-ability group was 5.77 out of 10, whereas the mean for the high-ability group was 7.80. The time effect was significant, $F(1, 77) = 19.51$, $p < .0001$. The mean for the pretest was 6.21 as opposed to 7.38 for the posttest mean. The ability by time interaction was not significant, $F(1, 77) = 0.04$, $p < .84$.

Plan for Solving the Problem. The teaching method effect was not significant, $F(1, 77) = 2.03$, $p < .16$. The ability effect was significant, $F(1, 77) = 28.37$, $p < .0001$. The mean for the low-ability group was 5.80 out of 10, whereas the mean for the high-ability group was 7.84. The time effect was significant, $F(1, 77) = 20.35$, $p < .0001$. The mean for the pretest was 6.26 as opposed to 7.41 for the posttest mean. The ability by time interaction was not significant, $F(1, 77) = .49$, $p < .49$.

Answering the Problem. The teaching method effect was not significant, $F(1, 77) = .06$, $p < .82$. The ability effect was significant, $F(1, 77) = 30.43$, $p < .0001$. The mean for the low-ability group was 3.51 out of 10, whereas the mean for the high-ability group was 5.68. The time of testing effect was significant, $F(1, 77) = 11.97$, $p < .0009$. The mean for the pretest was 4.11 as opposed to 5.11 for the posttest mean. The ability by time interaction was not significant, $F(1, 77) = .94$, $p < .34$.

In summary, the results revealed that teaching method did not play a prominent role in influencing the results. Ability played an important role for every dependent variable. Time of testing results gave evidence that learning occurred for all measures, irrespective of type of instruction, in all groups. Concepts results showed that high-ability students improved more than low-ability students. See Table 2.

Discussion

Restatement of Purpose

The main purpose of this study was to examine the effects of ability and instruction on the mathematical problem-solving achievement of fifth graders. This study attempted to determine differences in mathematical achievement for both basic skills and problem-solving skills of low- and high-ability students when a program of regular textbook instruction was used in the classroom as opposed to a program of regular textbook instruction combined with problem-solving instruction.

Implications

The results of this study indicate that there was only one significant interaction of ability and instructional method upon the problem-solving performance of fifth graders. Teaching method showed a significant effect on computation, but overall there is little support for the hypothesis.

In conclusion, even though the results of this study did not support the hypothesis that problem-solving instruction would have a significantly beneficial effect on the mathematical problem-solving achievement of low-ability fifth graders, neither do the results suggest nor justify an abandonment of problem-solving instruction either as an instructional practice or as an area of research.

Limitations

One factor that may have affected the outcomes of the study was that test validity was not established for the teacher-constructed CFPS problem-solving instrument prior to its use as the pretest and posttest. Providing visual aids for using specific strategies to solve the problems and a ceiling effect may also have been contributing factors of that test.

Moreover, implementing an instructional program with experimental and control groups may have inherent problems. The control group may have been influenced by the John Henry effect (an endeavor to perform as well as another group). On the other hand, the experimental group may have been influenced by attitudes, quantity, or quality of time of instruction in problem solving. In the present study, the amount of instruction was not carefully monitored. Hence, variability in the amount of instructional time may account for the results.

This study may also have been subjected to the particular dilemma of using measures of student performance which did not adequately differentiate whether or not that performance depended on domain-specific knowledge or on knowledge of problem-solving strategies. It is possible that students who did not receive instruction in problem-solving strategies would be able to perform well on that test simply by using their mathematical knowledge. Moreover, students might have developed their own problem-solving strategies which they employed on the CFPS test.

Another plausible explanation for the lack of differences for the two teaching methods would be the amount of instructional time. Based on the results of this study, it appears that one hour of instruction per week is not sufficient to increase problem-solving performance.

Research Recommendations

Research relative to problem solving represents an incomplete field of study at best. Further efforts in research might be directed toward explaining the apparent small differences between the experimental and control groups' mathematical performance. Why did the problem-solving groups fail to make more significant gains on problem-solving performance measures?

Much more classroom research is needed to determine the effectiveness of current instructional problem-solving programs. Are important components of successful problem solving missing or simply lacking adequate attention? The instructional problems are compounded by difficulties in measuring problem-solving outcomes. How can the measurement of problem-solving processes be improved? It seems that much more time and attention should be given to problem solving at all grade levels. Further research might not only explore the longitudinal effects of problem-solving approaches on mathematical performance, but also explore its effects in relation to the following areas: across-the-curriculum performance, social interaction, attitudes, and thinking skills.

Though a great deal of information has been compiled about factors that affect and influence problem solving, a comprehensive structure which accounts for individual differences, instruction, and achievement has not yet emerged. However, the challenge remains for educators to implement available research implications which promote learning situations in which problem-solving skills and achievement can be nurtured and enhanced.

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Table 1

Summary of Effects in Univariate Analysis of Variance on Six Dependent Variables

	Effects			
Dependent variables	Teaching method	Ability	Time of testing	Ability by time of testing
Concepts	NS	S	S	S
Word problems	NS	S	S	NS
Computation	S	S	S	NS
Understanding	NS	S	S	NS
Planning	NS	S	S	NS
Answering	NS	S	S	NS

Note. S (Significant); NS (Not significant).

Table 2

Pretest and Posttest Means and Standard Deviations by Teaching Method and Ability

Teaching Method		Regular		Problem Solving	
Ability		Low (n=21)	High (n=19)	Low (n=19)	High (n=22)
	Dependent variables				
PRETEST	CONCEPTS	5.54 (0.90)	6.90 (0.94)	5.51 (1.29)	6.45 (0.95)
	WORD PROBLEMS	5.20 (0.94)	6.73 (1.00)	5.12 (1.81)	6.74 (1.22)
	COMPUTATION	5.47 (0.64)	6.30 (0.94)	5.12 (0.61)	6.14 (0.72)
	UNDERSTANDING	4.71 (2.71)	6.90 (2.18)	5.63 (2.59)	7.55 (1.71)
	PLANNING	4.71 (2.65)	7.00 (2.31)	5.58 (2.55)	7.68 (1.70)
	ANSWERING	2.86 (2.67)	5.11 (2.21)	3.47 (1.61)	5.00 (2.18)
POSTTEST	CONCEPTS	6.08 (1.05)	7.91 (1.13)	5.95 (1.08)	7.53 (1.44)
	WORD PROBLEMS	5.91 (0.94)	7.16 (1.17)	5.58 (1.25)	7.14 (1.19)
	COMPUTATION	5.86 (0.45)	7.10 (0.76)	5.28 (0.70)	6.52 (0.89)
	UNDERSTANDING	6.05 (2.18)	8.53 (1.26)	6.79 (1.62)	8.18 (1.92)
	PLANNING	6.00 (2.12)	8.53 (1.31)	7.00 (1.52)	8.14 (1.91)
	ANSWERING	3.52 (1.86)	6.79 (1.87)	4.26 (2.21)	5.91 (2.67)

Note. Standard deviations are in parentheses.

Appendix A

PROBLEM:

Johnson's cat went up a tree,
Which was sixty feet and three;
Every day she climbed eleven,
Every night she came down seven.
Tell me, if she did not drop,
When her paws would reach the top.

SKILLS REQUIRED (readiness)--

Addition and subtraction

TIME ALLOTMENT-- 15 minutes

PROBLEM TYPE-- Complex translation

STRATEGY--Make a list
Draw a picture

SOURCE: Amusing Problems, Row Peterson p. 5

TEACHING ACTIONS

before--

1. How high was the tree?
2. How high did the cat climb each day?
3. How far did she come down at night?
4. What was the distance she gained each day?
5. Can you draw a picture to show what happened each day?

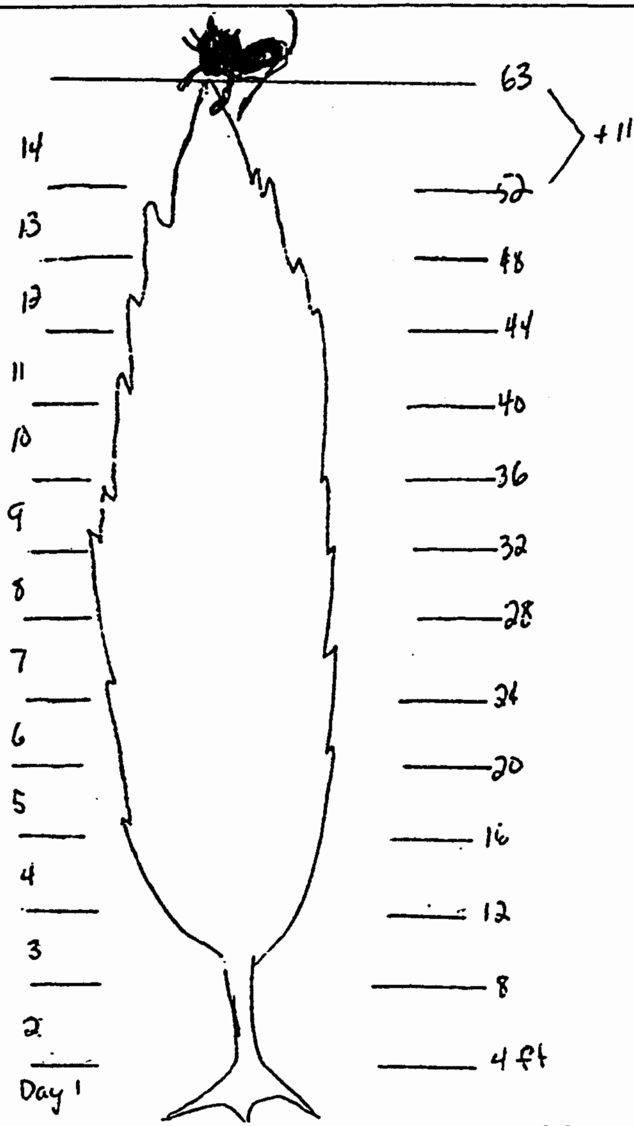
during--

1. How far had she gone by the end of the second day?
2. What might happen on the last day?
3. Make sure you carefully see what happens each day.

after--

1. What might cause a person to miss this problem?
2. Why is it important to check each step carefully?

SOLUTIONS



Each day the cat moved up 4 feet. In 13 days she had climbed 4 feet x 13, or 52 feet. Then on the 14th day her paws reached the top, since 52 +11 = 63.

COMMENTS

PROBLEM:

Crossing the River

A man (M), fox (F), goose (G), and some corn (C) are on one side of a river. The man wishes to get himself, the animals, and the corn across the river using a boat (B) which will carry him and only one other thing. The fox will eat the goose if left alone together, and the goose will eat the corn if left alone with it. How can the crossing be made?

Source: Problem Solving.. A Basic Mathematics Goal Book 2 page 72

SKILLS REQUIRED (readiness)--

No mathematics skills needed

TIME ALLOTMENT-- Will depend on strategy used

PROBLEM TYPE-- Applied

STRATEGY--
Act it out
Diagram
Logical thinking

TEACHING ACTIONS

before--

Who must be on the boat at all times?

How many things need to be taken across the river?

What things can not be left alone?

during--

Hint: (Could give it away)
Do you think something could be taken across, but then taken back to the other shore on later trip, then returned on another trip?

after--

What was the first thing to be taken across? Why?

EXTENSION: Use this as a lead into the Missionary and Cannibals problem found on page 75 of this same book.

SOLUTIONS	COMMENTS
M F G C B ----------- ORIGINAL POSITION F C ----------- M G B TRIP 1 M F C B ----------- G TRIP 2 C ---------- M F G B TRIP 3 M G C B ----------- F TRIP 4 G --------- M F C B TRIP 5 M G B --------- TRIP 6 F C ------------ M F G C B TRIP 7	