Engaging students in mathematical thought

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Abstract

Many early childhood educators believe that students learn by doing. This belief is Mathematical Thought 2 often translated into having students engage in activities. Play is viewed as fundamental to learning, and children engage in using manipulatives. Students actively construct knowledge through play and by interacting with others and manipulatives. This is based on Piagetian philosophy which emphasizes fixed and invariant developmental stages through which children progress. There is an emphasis on keeping from pushing students before they are ready.

Engaging Students in Mathematical Thought

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Chapter 1

Introduction

The overall goal of mathematics instruction is to develop in students a sense of mathematical power. When students are empowered mathematically, they demonstrate the ability to freely explore mathematical ideas and problems. They are able to conjecture and reason logically when problem solving.

Communicating about and through mathematics, especially by means of discourse, is empowering. The process of explaining personal ideas and perceptions and defending them through debate and discussion leads to a more powerful knowledge of mathematics. Students who exhibit mathematical power possess a disposition to seek a clear understanding of a given problem and to persevere in discovering a reasonable solution. They may in the process confidently make use of a repertoire of strategies learned informally or through formal instruction, or invent new strategies as they solve problems.

A classroom climate which encourages spontaneous conjecture and exploration is most conducive to the development of mathematical power. Students must feel comfortable in posing their own questions about mathematics and in pursuing their own inventive strategies for solving problems. Students should be encouraged to reflect upon their thinking and to convince their peers of the validity of their findings through discourse.

Purpose of This Study

Many early childhood educators believe that students learn by doing. This belief is often translated into having students engage in activities. Play is viewed as fundamental to learning, and children engage in using manipulatives. Students actively construct knowledge through play and by interacting with others and manipulatives. This is based on Piagetian philosophy which emphasizes fixed and invariant developmental stages through which children progress. There is an emphasis on keeping from pushing students before they are ready.

The mathematics education field is moving from a linear and hierarchical curriculum of content which breaks knowledge into small parts to a curriculum which stress is activity and discourse as tools for promoting mathematical power. The curriculum goes beyond content and activities to reflective thinking about mathematics through language communication. This position has been fueled by two documents of the National Council of Teachers of Mathematics (NCTM); Curriculum and Evaluation Standards for School Mathematics (March 1991) and Professional Standards for Teaching Mathematics (March 1991).

This paper will explore the mathematics learning of young students with particular emphasis on emerging trends and directions. The early childhood field and the mathematics education field are examining new ideas about the capabilities of young students, how they learn, the role of language in learning, instructional strategies, and the content and emphasis of curriculum.

Young children possess a great deal of informal mathematics knowledge when they enter school. They also possess a variety of effective strategies for solving problems.

Instruction that builds on these strengths causes students to become mathematically powerful.

In the following paper, the author will address worthwhile mathematical tasks in which students may engage, the roles of the teacher and the students in mathematics learning, and the tools which enhance discourse, all with the goal of developing in children a sense of mathematical power. The author proposes to answer the question: What factors influence the effect of engaging students in worthwhile mathematical tasks on mathematical understanding and the development of mathematical power?

Young students possess a great deal of informal mathematics knowledge when the enter school. They also possess a variety of effective strategies for solving problems. Instruction that builds on these strengths causes students to become mathematically powerful.

Need for This Study

Historically, mathematics education has been an ever changing and dynamic process. During the 1930's through the 1950's, the mathematics education field experienced significant reform. This is known as the "meaningful arithmetic" era. This era began with the assumption that the successful learning of arithmetic by students was bade possible through rote learning. The focus of instructions was on learning skills through the practice of drill. Increases in rate and accuracy were considered proof of learning. This method was called systematic instruction.

Concurrently, the child development movement advocated the notion that students learn arithmetic best by encountering situations that require the learning of arithmetic incidentally within the context of daily living in the classroom. Instruction organized in activity-oriented or experience-oriented units based on broad topics provided the impetus for motivation. The meaning of arithmetic was viewed as of greatest importance in contrast to skill development. Skills were to be learned incidentally through the study of broad topics. Those who supported this philosophy were known as incidentalists.

As the 1950's drew to a close, it was finally resolved that systematic, sequenced instruction of arithmetic skills was required. It was further resolved that learning must be built on experiences familiar to the students with consideration of student needs, interests, and developmental level. Both extreme views then combined to form "meaningful arithmetic" which included both the systematic learning of skills and the application of those skills within the context of a purposeful situation. A balance between skill learning and arithmetic understanding was created.

The 1960's demanded that mathematics curriculum meet the nation's scientific and technological needs. This promoted educators to examine the content of the curriculum as well as effective methodology. The mathematical validity of a program was based on the emphasis on meaning that the program provided.

There was interest in building the curriculum on broad, unifying themes rather than on isolated topics. This allowed students to build on and expand mathematical ideas. Connections between mathematical concepts could be facilitated within this structure. Skills could easily be placed within a meaningful conceptual framework.

The idea of "problem solving" was introduced in part as a solution to the need for applying mathematical skills and concepts to real world situations. Problem solving could take place entirely within the setting of mathematics. It did not require a broad topic taken from another subject within which real-world situations that called for mathematical applications would need to arise. Application of skills and concepts could be contained within the subject of mathematics.

Concerns of the 1970's and 1980's included the learning of computational skills, the role of applications and problem solving experiences in mathematics, and ways of organizing instruction to provide for individual differences. Most educators believed that proficiency in computational skills was an important goal in elementary education. The previous "meaningful arithmetic" era lead to some confusion about the measure of emphasis that should be placed on computational skills. The end product became the focus of learning with a level of disregard for computational skills.

During the 1970's and 1980's, some educators felt that systematic instruction should be entirely abandoned and replaced by application and problem solving experiences. Educators were uncertain about how to incorporate skill development within the structure of the problem solving experience.

The mathematics standards, published by the NCTM in "Professional Standards for Teaching Mathematics" (1991), have greatly influenced the 1990's. This document suggests that teachers go beyond computation to focus on a broad range of content. The teacher is viewed as a facilitator to student learning. The teacher must meet the needs of students with diverse learning styles and levels of mathematical understanding.

The NCTM mathematics standards place different cognitive demands on students. The standards require students to pose their own questions and problems about mathematics. They require that students use a variety of tools for discovering sensible solutions and for explaining their findings to others.

Students must justify their theories by relying on mathematical evidence. Students apply skills learned in a meaningful context in their attempts to discover answers. They actively construct their own mathematical meaning as they seek reasonable solutions to problems.

These cognitive demands lead to the need for changes in mathematics instruction today. Teachers must provide an environment in which students are invited to investigate methods of solving problems that make sense to them. Teachers must create worthwhile tasks from which students construct meaning and apply skills within a purposeful context. Manipulatives are used as tools for constructing meaning and defending proposed solutions. Teachers are encouraged to model and instruct students in proper social discourse in an effort to guide students in meaning construction.

This kind of reform is needed today, because students need the opportunity to understand mathematical concepts in greater depth. Students need to pose their own mathematical questions, discover the answers, and construct their own knowledge. This is done through cooperative learning, engaging in discourse with other students, defending their theories, and using tools which enhance the construction of mathematical understanding.

Limitations

In reviewing the literature, the author found no conflicting information, thus limiting the study. There was only one Kindergarten teacher available whom the author could observe. Prior knowledge of student personality, achievement, and aptitudes was not known by the author. A restricted sample was used due to the author's schedule and opportunity. Some lessons were not observed in full due to the author's schedule. This was the author's first experience in observation and detailed note taking, thus her skills in this area were limited.

Definition of Terms

conjecture - When a student makes a conjecture, s/he offers an opinion or judgement based on an educated guess or by examining the mathematical information presented or constructed.

discourse - When students engage in discourse, they explain ideas and perceptions and defend them through argument.

manipulatives - Real, concrete, relevant objects used to construct mathematical meaning or used to illustrate a conjecture during discourse.

Mathematics Their Way (1976) - mathematics program available to schools possessing strong components in worthwhile mathematical tasks.

strategies - methods students use to construct knowledge.

Chapter II

Review of the Literature

The following literature review was conducted to explore factors that influence the development of mathematical power. The areas of worthwhile mathematical tasks, teacher and students roles, and tools for enhancing discourse were reviewed. Implications for curriculum and developmental appropriateness were also reviewed.

Worthwhile Mathematical Tasks

The NCTM (1991) defines worthwhile mathematical tasks as those tasks which involve constructions, applications, or exercises. They may also be projects, questions, or problems. Worthwhile mathematical tasks provide the intellectual context for mathematical development. They provide a focus for opportunities for learning and are the stimulus for students to think about mathematical concepts and procedures. Through engagement in such tasks, students are lead to make connections with other mathematical ideas and to apply concepts and procedures to solving a problem set in a real-world context. Worthwhile mathematical tasks invite students to experiment with mathematics as mathematicians do. Students' ability to solve problems and to make mathematical connections are enhanced through tasks which require students to reason and communicate mathematically.

Worthwhile mathematical tasks are based on mathematical concepts and procedures that are acceptable within the mathematics community (NCTM 1991). Students' understandings and mathematical developmental level, interests, experiences, and misconceptions should be considered when planning tasks. Individual differences in learning styles must also be considered. Tasks need to engage students' intellect, develop their mathematical understandings and skills, and stimulate students to make mathematical connections.

According to the NCTM (1991), some worthwhile tasks may be chosen through student questions or conjectures. Tasks can be solved in more than one way and may result in more than one reasonable solution. Such tasks stimulate discourse about different possible strategies and outcomes. Students are inspired to go beyond producing the right answer to engaging in speculation, pursuing alternative and inventive methods of solving problems, and considering the validity of their approaches. Tasks require students to gather, summarize, and interpret data. Students view mathematics as an ongoing life activity when worthwhile mathematical tasks are planned.

In contrast, some teachers often use practice and drill in hopes that students will memorize the facts. Mathematics is not simply a set of skills to be memorized. Memory tasks inhibit reflective thought about mathematics. When such methodology is employed, students develop the perception that mathematics is not a subject that requires thought, but rather it is a subject which consists of memorizing isolated facts which cannot be used to find the answer to another problem.

Thorndike (1922), an associationist, suggested that students engage in drill and the practice of correct mathematical facts in order to strengthen correct mental bonds. Related mathematical concepts were to be studied far apart in order to avoid the formation of incorrect mental bonds.

Skinner (1943), a behaviorist, also advocated drill and practice activities in mathematics. Reinforcement by method of reward was given for correct answers. Punishment was given for incorrect answers.

Piaget (1970), a constructionist, claimed that children make sense of their environment in very different ways than adults. Children learn through manipulating their environment.

Peterson and Knapp (1992) assert that students need to be able to do more than simply recall facts. They need to think about mathematical problems and discuss their thought with other students. They need to construct mathematical knowledge for themselves.

Peterson and Knapp (1992) view mathematical algorithms as being open to interpretation during worthwhile mathematical tasks. There exist many different ways in which students may view a given problem. They need to be given the opportunity to share their strategies for solving problems, to articulate their thoughts, and to defend their answers.

Knowledge is viewed as a dynamic process during worthwhile mathematical tasks, according to Peterson and Knapp (1992). Knowledge is perpetually constructed and reconstructed.

Becoming a good mathematical problem-solver involves the acquisition of the disposition for interpretation and sense-making. This does not discount the need to acquire specific skills, strategies, and knowledge of fundamental mathematical concepts. Lauren B. Resnick (1992) invites us to look beyond the curricular scope and sequence of skills to developing the disposition for problem-solving.

Resnick (1992) suggests a method of organizing the class study of problem solving. The teacher begins by posing a question to the entire class. Students are encouraged to try to work the problem on their own. Then they work with one partner and then in groups of four, all the time sharing and comparing their work. Next students are drawn back to a whole class discussion. Finally, each individual student is held responsible for the solution. The teacher's role is to help students organize ideas and suggest alternative strategies. Such problem-solving sessions should be based on fundamental mathematical concepts that students have already mastered a year or two earlier.

Resnick (1992) discusses the success of group work among students. Several students who are not experts at a given task may work to scaffold each other. Scaffolding refers to the support of a child's cognitive activity during a given task. Such interaction results in a high level of cognitive performance for all.

Specific mathematics activities with students require social interaction and discourse among students, in Resnick's (1992) opinion. They need to be allowed to manipulate real, concrete materials and to discover strategies that work for them in answering their own mathematical questions. The teacher is there to clarify answers and to present fundamental mathematical concepts that help students to reach sensible solutions.

Lave, Smith, and Butler (1989) argue that mathematics should be viewed as everyday practice in problem solving. Students need to be involved in activities which much resemble those of master mathematicians. That is to say, that students be given the opportunity to use processes for solving problems as mathematicians do.

What do master mathematicians do? Master mathematicians argue the mathematical proof of a given procedure proposed for solving a problem, according to Lave, Smith, and Butler (1989). They readily recognize when and how to use skills that have been learned. Procedures are invented spontaneously, and the mathematician is at liberty to change a problem or abandon it as well as to solve it.

Lave, Smith, and Butler (1989) compare students' problem solving activity to that of an apprenticeship. During an apprenticeship, a person studies with a master. The content of what is to be taught, the processes that are to be learned, and the product or end result are interrelated. This interrelationship is clearly understood by the apprentice.

In a classroom where problem solving is viewed as an everyday practice, problems are presented in a broad and general manner, in Lave, Smith, and Butler's view. A list of mathematical principles is not directly taught within the context of their use. Problem solving activity is naturally generated in such mathematical practice.

Students need to be provided with a means to gauge their own skill, in Lave, Smith, and Butler's opinion. The classroom environment must foster the attitude that the developing skill of the apprentice mathematician is of value. Problem solving strategies that are invented are valued and communicated among teacher and peers. Students are free to form their own mathematical problems. What students do in everyday situations that involve problem solving activity is seen as relevant and important. School is not preparation for real life, but is real life.

In addition, Lave, Smith, and Butler (1989) believe that as students are engaged in problem solving as an everyday practice, they construct theories of mathematics. Only when students are given the opportunity to construct their own meaning, is substantive learning taking place. Mathematics involves activity, argumentation, and social discourse.

Lave, Smith, and Butler (1989) propose that the goals of mathematics instruction be to generate problems or tasks and provide opportunities for invention, discovery, and understanding within the context of those tasks. They do not believe in assigning exercises on specific problem types and procedures. Students must be allowed to make choices, judge, use processes, form problems, make wrong choices, and follow through on their own hypotheses to discover and prove their own theories. The classroom is a field for mathematical action. The students are a community of mathematics practitioners.

Teacher and Student Roles

The teacher's role in engaging students in mathematical thought is the sine qua non of success for students. Teachers must create an environment in which students are free to openly consider a variety of strategies for solving a given problem. Effective teachers pose questions that challenge student thinking. They listen to student conjectures and require students to justify their answers orally and in writing.

The most complex aspect of effective teaching is deciding what student ideas that are posed during discussion should be pursued further. Deciding when and how to connect mathematical notation and language to student thought is also difficult. Effective teachers know when to provide information, clarify an idea, model mathematical behavior, lead the group in a certain direction, or allow a student to struggle through a problem. They are delicately aware of student participation during discussions and the dynamics of social discourse. Who is volunteering comments, how students are able to put thought into words, and how they respond to one another is carefully recorded.

The NCTM's "Professional Standards for Teaching Mathematics" (1991) lists teacher responsibilities when students are involved in worthwhile mathematical tasks and discourse. Teachers are responsible for provoking student reasoning. They do this by posing questions, listening to student comments, and inviting students to respond to one another. Effective teachers establish an atmosphere in which everyone's thinking is respected. Students are inspired to use logic and mathematical evidence as verification for their answers. Teachers lead discussions in which reasoning and arguing about mathematical meanings and justifications for ideas take place. A tone of civility is fostered in the classroom when the teacher models and instructs students in proper social discourse.

Peterson and Knapp (1992) believe that the role of the teacher and her/his community of learners in a given classroom is to offer a forum for mathematical discussion, present challenges, and guide mathematical construction. Authority for mathematical knowledge does not lie with the teacher and does not take the form of a "correct" answer. Authority lies within the community of learners.

According to these authors, the teacher must be interested in understanding the thinking of the children. Discussion should be sensitive to student concerns. The teacher is made an active participant in the discussion by questioning students, clarifying student comments, and modeling mathematical thinking. S/he must carefully steer the discussion in a productive manner by both respecting mathematics as an established discipline and by respecting the thought of students simultaneously.

Peterson and Knapp (1992) indicate that teachers are responsible for setting goals and creating worthwhile mathematical tasks to reach these goals. They are required to stimulate and manage classroom discourse in such a manner that students and teacher may come to understand one another's thoughts more clearly. Teachers must ensure that students are connecting mathematical ideas and application. They need to create a mathematical environment in which conjecturing, inventing, and problem solving are ongoing.

Peterson and Knapp (1992) assert that the student's role in discourse involves listening to, responding to, and questioning other students' comments. They may also make conjectures and present possible strategies for solving problems. They must attempt to convince themselves and their peers of the validity of their presentation or solution by relying on mathematical evidence.

All students can contribute to discussion orally, pictorially by sharing diagrams or charts, concretely by demonstrating a particular phenomenon with concrete materials, or representational by using conventional mathematical notation and language, in Peterson and Knapp's opinions.

According to Peterson and Knapp, as students are engaged in worthwhile mathematical tasks, they initiate problems and questions and investigate their solutions. They share with one another their findings and conundrums. Students invent strategies for solving problems and use a variety of mathematical tools. They may be invited to keep a journal in which they record mathematical successes and new insights.

Peterson and Knapp point out that students develop an attitude of what mathematics is by the everyday activities in which they are involved. Their approaches to mathematical problem solving are influenced by the context in which the problems are presented. The manner in which students go about solving problems is learned within the context of the classroom environment.

Teachers need to pose problems that both provoke mathematical thinking and necessitate the use of mathematics. An environment needs to be created in which students' curiosity is stimulated. Mathematical thinking is not developed through the direct instruction of mathematical procedures. Peterson and Knapp emphasize that students must have the opportunity to negotiate meaning. A justification for the use of mathematics must be established. Students are invited to reflect upon what is happening and why. The processes students use when thinking mathematically need to be made overt, so that the student concentrates on them and is aware of her/his thinking. It is only through such an awareness that s/he is able to use the strategy again. By such concentration, the student can later choose among a repertoire of thinking strategies.

Schoenfeld (1989) proposes the notion that the nature of the mathematics classroom environment determines the student's sense of what mathematics is. In turn, the student's sense of what mathematics is directly influences how the student uses mathematics when solving problems. He believes that the effective mathematics classroom environment fosters the view, that doing mathematics goes beyond the practice of basic facts and procedures to the act of sense-making. The facts and procedures studied are tools which can be used as a means to sense-making. The classroom teacher must stimulate interactions which both help students to master basic facts and procedures and develop the attitude that mathematics is sense-making.

Schoenfeld (1989) further asserts that students may master basic mathematical facts and procedures, but they may not be able to use them sensible. Working as a mathematician involves the ability to figure out a procedure; to take a problem apart and examine how the pieces fit together sensibly. Mathematician know that things fit together for a specific reason that makes sense. Doing mathematics is sense-making. The classroom environment should reflect this attitude.

Teachers are encouraged to help students to engage in reflective thought. Students need to think about how they come up with the solutions they do. Schoenfeld says it is helpful to students if they are required to analyze their work and the work of their peers. Statements that students make should be evaluated and negotiated.

Resnick (1992) suggests that mathematics be treated as a discipline in which possible strategies for solving problems are not restricted to the traditionally acceptable methods. Teachers must often regard mathematics as a discipline in which there is no room for questioning established "truths." This attitude results in a lack of discussion between children about mathematics. The goal is to use the established rules to find the "correct" answer to a given problem. This atmosphere provokes the study of a set of rules for solving problems in a book or on a worksheet.

Teachers must view learning as a process of interpretation and meaning construction. Mathematical statements may have more than one interpretation. Students must be invited by the teacher to argue and debate their individual interpretations. This process leads students to realize new patterns and relationships that they did not observe before, according to Resnick (1992).

Mathematical expressions should be presented with reference to real things. Resnick compares the developmental process of natural language development with the process of mathematical language understanding. Students need to argue about the meaning of mathematical expressions. The conflict of opinions encourages the constructive learning process. The teacher must honor multiple interpretations and support answers on the basis of their sensibility rather than on the basis of being "correct".

Students tend not to draw upon their informal background knowledge about mathematics, which they have developed before entering school. They do not draw upon this knowledge when attacking mathematical problems, because most math activities in school do not relate to real situations. Teachers can encourage children to draw upon this knowledge by focusing instruction on the interpretation of real-world mathematical situations, in Resnick's (1992) view.

Resnick (1992) believes that the teacher is expected to model problem solving processes during class discussions. This may be done by "thinking aloud" while solving a problem and by possibly pretending to be puzzled in order to allow the students to come to a conclusion.

If students do not engage in mathematical dialogue, they will be denied the opportunity to learn about other ways of looking at a problem. The teacher gains great insight about a student's thinking process and is better able to diagnose where the child is in her/his development. A classroom environment in which students are encouraged to discuss various points of view is most conducive to the development of mathematical power.

Tools for Enhancing Discourse

The National Council of Teachers of Mathematics (NCTM) (1991) encourages teachers to guide their students in using a variety of mathematical methods and tools when doing mathematics. Manipulatives are a popular tool that is used in contemporary mathematics activities. They may be a valid tool when used to clarify an idea or solve a problem. Manipulatives are most effectively used within the context of a worthwhile mathematical task. They should not be the focus of the lesson. For example, the teacher may do a lesson on connecting blocks. The blocks have become the focus of the lesson. They should be used as one possible tool to solve a problem, making the problem the focus of the activity.

Computers, calculators, concrete materials used as models, pictures, diagrams, tables, and graphs are also valid tools for working worthwhile mathematical tasks and for enhancing discourse. Other tools, which are not concrete but which aid in effective mathematical communication, include metaphors, analogies, stories, written hypotheses, explanations, arguments, oral presentations, and dramatizations. (NCTM, 1991)

Students are encouraged to use the above mathematical tools for constructing meaning by the NCTM. These tools are helpful during discourse which is focused on exploring mathematical ideas. Students are able to explain or defend an idea more clearly when a model, presentation, etc. accompanies an oral conjecture or argument. The tool may also serve to prove a hypothesis.

Invented mathematical procedures, language, and notation have been advised by the NCTM when students are engaged in worthwhile mathematical tasks. This allows students to construct their own meaning and therefore leads to a deeper understanding of the mathematical concepts being studied. Conventional mathematical notation should be introduced following the development of a given concept in a meaningful context. In this manner, students experience a need to know and an interest in learning the conventional form.

Thompson (1980), in his article, "Piaget and Kindergarten Mathematics," summarizes the results of Piaget's research about how children learn and examines the implications of that research for kindergarten mathematics instruction. He addresses the concept of mathematical tools and discourse.

Thompson also contends that current practices in kindergarten mathematics require students to push pencils or crayons across workbook pages. There exists an overwhelming emphasis on numbers that may lead students to the idea that mathematics is knowing how to manipulate numbers.

Thompson address the value of social interaction. Piaget states that students need to interact verbally with other students in order to develop the ability to consider viewpoints other than their own. In mathematics such interaction occurs when students are asked to verify their answers among their peers through dialogue. In this manner, they are confronted with ideas which conflict with their own, and this motivates them to reflect on and revise their ideas or to further argue them. The teacher's role is to help them resolve the differences. Students develop confidence in their abilities to solve problems through this process. Students are often better able to understand another student's explanation than the teacher's.

Thompson emphasizes the use of real, concrete materials within the context of relevant problem solving. Workbooks and worksheets are not included in the definition of real concrete material. They do not enhance the student's ability to explain or argue a mathematical idea during discourse. They are not an effective tool for solving problems or for recording mathematical understandings.

Implications for Curriculum

What curriculum model best suits itself to the engagement of students in mathematical thought? Surely the traditional curriculum which follows the scope and sequence of the math text purchased by the school district is too limiting to encompass the method of teaching which the author suggests is vital for students' mathematical development.

Finkelstein (1993) offers an early developmental education curriculum model which requires that teachers begin by carefully considering how students develop and learn. She urges teachers to study Howard Gardner's multiple intelligences theory which celebrates students' varied and unique intellectual capacities. She believes that teachers should spend a greater amount of their time listening, engaging in discourse, and observing students than directly instructing students. Teachers need to perceive themselves as learners in the educative process and as guides of students' learning.

Parental support and societal influences effect curriculum. Parents, teachers, and students must work in cooperation to reach success.

Finkelstein stresses the importance of placing the student at the center of schooling and not at the established curriculum. Topics must be chosen which allow for in-depth study and which incorporate the student's existing knowledge. The process of learning should be emphasized more than the final product. Assessment and evaluation are viewed as ongoing activities in the teaching-learning process.

Developmental Appropriateness

Walsh (1991) makes the claim that the concept of developmental appropriateness needs to be expanded. He questions the notion of broad, universal stages through which students are presumed to pass, and argues that the majority of educators in fact do not all embrace the same theory concerning child development.

Walsh (1991) examines the definition of developmentally appropriate practices published by the National Association for the Education of Young Children (NAEYC) and concludes that the consensus which the NAEYC claims exists in fact does not. There appears to be a confusion among educators about what this definition means. Walsh places practitioners into three categories. He labels those who view development as a factor exclusively of biological maturation, maturationists. Environmental influences and experience are entirely ignored by this group.

Another category of educators are labeled Vulgar-Piagetians who may state that they "follow Piaget" but in fact have never actually read Piaget. They appear to know nothing of the constructivist theory that Piaget espoused.

The third category Walsh calls Pagetian. These practitioners do have a basic understanding of Piaget. Within this group exists differences in the interpretation of Piagetian theory thus resulting in a lack of consensus.

Walsh (1991) contends that the universal stages defined by Piaget are assumed to be universal and invariant. He has found that this theory has not stood up well under empirical test. He cites the work of Vygotsky who proposes that a child's social and cultural influences profoundly effect the child's development. Development is viewed as a social process.

According to the definition of developmentally appropriate practices, learning is dependent on development. Development is seen as the prerequisite for learning. Walsh presents an interesting perspective on the established definition of developmental appropriateness and raises some noteworthy concerns.

Summary of Literature Review

The development of mathematical though described above must be fostered in the classroom. It is proposed that such development is best fostered in a classroom where the students are encouraged to explore strategies for figuring out answers to their own mathematical problems as well as teacher-posed problems. It is felt that students must be allowed to work cooperatively in search of possible solutions. It is further felt that teachers must facilitate discourse between students about mathematics.

This literature review indicates that students' learning is uniquely individual. Students construct their own knowledge, and their individual experiences and perspectives create their unique view of mathematics. It is imperative that teachers assess how students are thinking so that they can provide effective and appropriate feedback. The teacher can determine the reasoning processes and correct or reinforce the processes, not just the answers. The following action research study was conducted to explore these ideas.

Chapter III

Action Research

The purpose of the following action research was to determine the extent to which teachers' practices fit those which are advised by the NCTM standards and developmentally appropriate practices as defined by the NAEYC. In order to do this, the author first created a list of criteria in three areas: worthwhile mathematical tasks, teacher and student roles, and tools for enhancing discourse.

After developing the checklist, 11 lessons were observed during which the author recorded teacher-student interactions, conversations, and behaviors. The lessons were selected according to the author's schedule. Each lesson was 25 minutes long.

During these lessons, the author scripted by hand in order to capture all student and teacher comments for later review and analysis. After the observations, the author used the list of criteria developed in the three areas above to tally the frequency of each component present in each of the 11 lessons. Not all components were expected to be present in all lessons. Teachers tend to reflect patterns of strengths in their teaching. Also, certain lessons may emphasize particular components over others. Barriers to and supports of effective mathematics practice were also delineated. The author noted the barriers and supports that were present in the observed lessons in an effort to analyze the reasons for the methodology used.

The study involved the observation of a kindergarten teacher's practices in a midwestern school and their alignment with professional recommendations for teachers.

Sixty-five percent of the student population receives free or reduced lunch. The teacher teaches a half day kindergarten program. The afternoon class, which the author observed, consisted of 18 students, 8 of who are girls, and 10 of whom are boys.

The teacher has 23 years of experience and is dedicated to the teaching profession. She has a Master of Arts degree in early childhood education and has attended workshops in developmentally appropriate practices. The observed teacher has training in activity based learning and believes she offers a rich developmentally appropriate curriculum. She uses Mathematics Their Way (Baratta-Lorton, 1976) for her instructional lessons. She has received training in using this program. The emphasis of this program is on guiding students' mathematical conceptual development through the use of manipulatives. By giving students the opportunity to explore the manipulatives, they can more easily create in their minds a representation for mathematical concepts and symbols.

The teacher is deeply concerned with building students' self-esteem and scholastic self-confidence. Students and their ideas are highly valued and she works hard to make her classroom a place in which students feel comfortable to take risks and grow. The teacher believes in respecting students' ideas and allowing them to explore their environment freely. A student centered classroom prevails, and often lessons and discussions are student lead. The following list of criteria was used to evaluate the effectiveness of worthwhile mathematical tasks, teacher and student roles, and tools for enhancing discourse.

Criteria for Evaluation of Classroom Practice

Worthwhile Mathematical Tasks

Component #	Effective Practice	Ineffective Practice
1	Projects, questions, or problems that involve constructions or applications set in a real world context that are purposeful and meaningful to the intended group.	Projects, questions, or problems that are hypothetical and are not meaningful or purposeful
2	Projects that utilize manipulatives a tool for enhancing meaning construction, application, or discourse.	Projects that allow play with manipulatives that do not lead to meaning construction or are not useful for enhancing discourse.
3	Problems that require students to reason and communicate about mathematics.	Problems that require student to do tasks for the sake of doing the task. The problem is an end in and of itself.
4	Tasks based on significant mathematics, knowledge of student understandings, interests and experiences, and the range of diverse ways in which students learn.	Tasks that are not based on significant mathematics, do not consider student understandings, interests or experiences and are limited in the ways in which students may learn.
5	Engage students' intellect, stimulate students to make connections within mathematics, and develop students' mathematical understandings and skills.	Students are not encouraged to make connections within mathematics, and students' mathematical understandings and skills are not developed.
6	Call for problem formulation and problem solving	Call for the "correct" answer.
7	Present mathematics as an ongoing life activity.	Present mathematics as a set of skills to be learned and a subject separate from all other subjects.
8	Capture students' curiosity and need to know.	Ignore students' natural curiosity and need to know.

Component #	Effective Practice	Ineffective Practice
9	Tasks that can be solved or approached in more than one way.	Tasks that require one method for solving.
10	Tasks that grow out of student conjecture.	Tasks which dismiss student conjecture as irrelevant.
11	The gathering, summarizing, and interpretation of data based on a need to know.	The gathering of data for no purpose that is real and meaningful tot he students.
12	Tasks that require students to consider the validity of their approaches and findings.	Tasks that do not require students to justify their findings, but only to find the "correct" answer.
13	Nest skill development within the context of problem solving.	Isolate skills and concepts.

Teacher and Student Roles

Component #	Effective Practice	Ineffective Practice
1	Orchestrate classroom discourse in ways that promote the investigation of mathematical ideas.	Classroom discourse managed in ways that inhibit the investigation of mathematical ideas.
2	Students use tools to pursue mathematical investigations.	Students use tools for play.
3	Teachers help students make connections between prior knowledge and new information.	Teachers ignore the importance of connecting prior knowledge with the new.
4	Students are involved in constructing their own mathematical knowledge.	Students are told by the teacher an explanation for phenomenon.
5	Students impose their own interpretations on what is presented to create a theory.	Teachers impose upon students their interpretation of what is presented.

Component #	Effective Practice	Ineffective Practice
6	Each student's understanding of mathematics is considered uniquely personal.	All students are required to come to the same conclusion.
7	Authority on mathematical knowledge lies within the community of learners.	Authority on mathematical knowledge lies with the teacher.
8	The teacher poses questions that engage and challenge student thinking.	The teacher poses questions that do not challenge or engage student thinking.
9	The teacher requires students to justify answers by relying on mathematical evidence.	The teacher does not require students to justify answers and may only look for the "correct" answer.
10	The teacher is delicately aware of student participation and ensures that all participate by providing opportunities that consider the diverse learning styles of students.	The teacher allows only a select few who understand the activity in the manner the teacher intended to participate without regard to including all students.
11	The teacher is acutely aware of when to lead the students in a different direction, especially when a particular student may be off track.	The teacher is too permissive in allowing students to lead discourse or involvement in tasks.
12	Students are invited to propose a variety of different methods for solving problems.	Students are restricted to one method or teacher selected methods for solving problems.
13	The teacher is delicately aware of what student questions or conjectures to pursue in depth.	The teacher either chooses to follow an irrelevant comment that leads to student confusion, or dismisses comments that could lead to significant meaning construction.
14	The teacher provides mathematical information when needed to guide student mathematical construction.	The teacher provides mathematical information than inhibits mathematical construction.

Tools for Enhancing Discourse

Component #	Effective Practice	Ineffective Practice
1	Students use tools for constructing meaning.	Students use tools for play.
2	The introduction of conventional mathematical notation follows the development of a concept in a meaningful context.	The introduction of conventional mathematical notation is done either in isolation or not within a meaningful context.
3	Tools are used for defending student formulated theories that have been developed within a meaningful context.	Tools are not used for defending student formulated theories.
4	Students use tools for explaining a personal conjecture to other students in a problem solving situation.	Students do not use tools for explaining a personal conjecture.
5	Students use tools to accompany a presentation of their conclusion in a personally meaningful way as a means of proving their theory.	Students do not use tools to accompany a presentation of their conclusion.

The literature review indicated distinct barriers to and supports of effective practice. The barriers to and supports of effective practice in creating worthwhile mathematical tasks, in teacher and student roles, and in using tools for enhancing discourse are listed below.

Worthwhile Mathematical Tasks

Supports	Barriers
Tasks were prompted by student conjecture.	The teacher not skilled in the delivery of the lessons.
Students naturally went about investigating their own questions.	Some lessons in Mathematics Their Way were not effective in providing worthwhile mathematical tasks.
The students' curiosity and "need to know" were captured.	The ability to capture the students' curiosity and "need to know" was not present.

Teacher and Student Roles

Supports	Barriers
Developmentally appropriate practices and guidelines have been published as a resource for teachers by the NAEYC in "Developmentally Appropriate Practices in Programs Serving Children Ages birth to 8" (1991).	Respect for student centered activities and discourse prevents the teacher from establishing a clear focus or goal for a given lesson.
Standards for the effective teaching of mathematics have been published as a resource for teachers by the NCTM in "Professional Standards for Teaching mathematics" (1991).	Knowledge of how to effectively elicit from students justification for conclusions prevents opportunities for meaning construction.
	Knowledge of how to encourage all students to participate in discourse or mathematical tasks is limited and therefore prevents opportunities for meaning construction.

Tools for Enhancing Discourse

Supports	Barriers
Designated sessions are given for students to engage in free play with manipulatives.	The teacher does not distinguish between tools used for enhancing discourse or meaning construction and free play with manipulatives.
Worthwhile mathematical tasks are provided out of which naturally arise the need to know conventional mathematical notation and language	The introduction of conventional mathematical notation land language is not presented within a meaningful context.
Effective tools and their use for developing meaning construction are defined by the NCTM in "Professional Standards for Teaching Mathematics" (1991).	Discourse is not focused on the exploration of mathematical ideas and meaning construction.

Examples of Teacher Lessons

Typically, the teacher's lessons follow a four step plan. She begins by introducing the concept to be studies. Much discussion among teacher and students ensues at this point. Very limited discussion among students may occur. The second step is to set the students to work on manipulating materials for the purpose of "discovering" the concept. In the third step, the students record their findings pictorially and may attach conventional mathematical symbols to the pictures or may choose not to. The final step involves discussing students' findings and recordings. Students share with the group and interact verbally with the teacher or an occasional student who may make a comment. The following are two examples of lessons taught. Other examples can be found in Appendix B.

Date: April 12, 1992

Lesson Topic: What is 4?

Concept: Fourness

Procedure: The kindergarten teacher was observed teaching the concept of fourness. She began the lesson by writing the number four on the board and followed her Math Their Way guide quite closely looking at her manual from time to time. The following discussion ensued:

Teacher: Who can tell me what you see in your mind when I write this symbol? (writes 4)

Student 1: A four.

Teacher: Okay, you see a four.

Student 1: I like school.

Teacher: So, when I write this, you thought about liking school. How did you think about liking school?

Student 2: I like (teacher's name).

Teacher: When you look at this four, you think about me. Did you know about four before you came to school? What did you think about before you knew me?

Student 3: An electric train.

Teacher: How does this make you think of an electric train?

Student 3: I have one.

Student 4: I'm thinking about my birthday.

Teacher: Your fourth birthday, maybe. Tell us about it.

Student 4: It's after April.

Next, students were invited to roam about the room looking for things in groups of four. When they found something, they were directed to the tables to draw what they found. The teacher then took a dictation of what the child had to say about the picture. Some students had difficulty finding a group of four objects, wandered around a bit, and began drawing a picture entirely unrelated to the concept of fourness. The teacher directly told these students what example of four they could use. For example, one student was told to look at the chair legs under him. He counted four and excitedly set to work on drawing the legs. Other students who clearly understood the concept of fourness, drew accurate pictures but did not really follow the direction of finding something in the room. For example, one drew a picture of her fourth birthday. Another drew four flags, though there were not four flags in the room. These students' drawings reflected some definite understanding of fourness.

Finally, it was time for sharing. Students were directed to sit in a circle. Volunteers were invited to stand next to the teacher and read the sentences that the teacher had written during dictation. Occasionally, students were asked to point in the room where they found the four items they drew or asked why they chose to draw what they did. Questions and comments from other students were very limited. Each student simply shared the page s/he drew and sat down waiting for the next.

Date: April 14, 1992

Lesson Topic: Three plus two equals ?

Concept: Addition

Procedure: The kindergarten teacher was observed teaching the concept of addition with 3+2=5. She again used her Math Their Way manual to guide her instruction. She began in the following manner:

Teacher: How could you picture this in your mind? (writes 3+2 on board)

Student 1: 3+2

Teacher: What do you see in your mind?

Student 1: Three dogs.

Teacher: And then what do you see for the 2?

Student 1: Two horses.

Student 2: I know what I see! Four!

Teacher: You see four all together. 3+2 makes you think about 4? Do you see four of anything special?

Student 2: Four cats.

Teacher: I'm not sure how you got four out of that. Three cats first? (draws 3 cats)

Student 2: Then four.

Teacher: Then you thought about four cats? (draws a fourth cat) I'm trying to understand how Student 2 thinks about 3 and 2. He thinks about 4. I'm trying to think how Student 2 thinks. Could you help me a little bit more? I've got 3 cats. And what do you think next?

Student 2: Two horses.

Teacher: Two horses for the two? What happened when you were thinking about the 4 cats?

Student 2: (shrugged) I don't know.

The teacher then distributed buckets of objects that children could manipulate and form into groups of three and two. Some initially chose to play with the objects. Others put them into groups according to attribute. Still others made patterns. About half the group sorted objects into piles of three and two.

The teacher circulated among the students asking them to tell her what they were doing. They very eagerly shared what they were doing and why. A few of the students who were playing, making different kinds of groups, or making patterns were guided toward representing 3+2 with their objects.

After a short period, students were lead to the tables to create pictorial representations of the work they had done with the manipulatives. The illustrations the students made were most interesting. Student 2, the student with whom she engaged in discussion during the introduction of the lesson, drew 3 cats, 4 cats, and then 2 horses. He also wrote 3+2. Student 3, who possibly still had his mind on his train at home, drew his train and railroad track. The teacher encouraged him to draw 3 cats and 2 cats on his train picture. Student 5 drew 2 rabbits and 3 grapes. He wrote 3+2=5, 3+3=6, and 3+4=7.

Sharing time began when students were seated on the floor in a circle and progressed in much the same fashion as the first lesson. Students were invited to stand next to the teacher, show their picture, and tell about it. The teacher asked most of the questions and made most of the comments.

All lessons appeared to follow the same procedure and method as the above examples. Additional observations that further substantiated the author's findings may be found in Appendix B. Examples of student work may be found in Appendix A.

Chapter IV

Results

Results will be discussed in two parts. The first part will be an analysis of the two lessons described in Chapter 3. The second part will be the result of the scoring of the 11 lessons that were observed against the criteria for effective practice in the three areas upon which the author focused.

Researcher Commentary and Analysis of the Two Lessons

The observed teacher had training in activity based learning and believed she offered a rich developmentally appropriate curriculum. Despite her belief in beginning students' conceptual development with the exploration of manipulatives, this teacher began lessons with the presentation of symbols. The lessons focused more on the symbol rather than the concept. This method of teaching indicates a lack of fit with the criteria that were written for this study. It does not seem to fit the recommendations of the Mathematics Their Way program, nor does it seem to fit other careful conception of effective instruction.

In analyzing the lesson on the concept of four, it appeared that the objective of the lesson was not made clear to the students. It is questionable whether this presentation was helpful for students in understanding the concept of fourness. When certain students were clearly off track, it might have been to their benefit if they were guided back by the teacher to thinking about fourness. When the teacher was asked why she conducted the discussion in this manner, she responded by saying that she wanted students to come up with their own unique personal association with fourness. What seems to be off track to adults, she believes, may be a clear connection for students. This belief may stem from her deep regard for each student's ideas and train of thought. Research does not support this notion. Lave, Smith, and Butler argue that students be encouraged to use processes for solving problems as mathematicians do. The students' thinking must be clearly on track for them to make accurate connections. The NCTM standards indicate that teachers must guide students to make mathematical connections that make sense.

The author questions whether this lesson indeed involved students in a worthwhile mathematical task. Questions were not posed that stimulated intellectual though or that involved constructions and applications. The introduction did not inspire a need to know in students. The pictures students had drawn were not effectively used to enhance discourse. Students were not required to explain their reasoning or justify their drawings. Students did not engage in discourse at all. They merely showed their pictures and read their sentences with the help of the teacher.

During the 3+2 lesson, it appears that the teacher caused the class to focus on Student 2's misconception that 3+2 could equal 4. This may have been a result of the teacher's effort to validate all student comments. The teacher needed to provide an experience in which this student proved or disproved his conjecture that 3+2=4. This was not done, so he, and possibly other classmates perhaps, went away believing that 3+2=4.

Student 1 appeared to have a good start when she said 3+2 made her think of 3 dogs and 2 horses. Perhaps this could have been elaborated upon by giving this student the opportunity to prove or disprove her conjecture. Student 2 rudely interrupted Student 3 and the teacher's attention was given to him. This does not promote a climate of appropriate social discourse. Rather than trying to elicit from a student what he is thinking by putting words in his mouth as when the teacher said, "Three cats first?", an open ended question might have been asked or no comment at all. This would allow other students to contemplate Student 2's assertion and perhaps a comment from a classmate might help to clear up this student's misconception.

A study that bares some relation to this study carried out by Cohen (1991) is an observational study of a second grade teacher who again believed she was teaching in an exemplary fashion, but whose beliefs and practices seemed not to fit. Although she used manipulatives during instruction, she had a tendency to focus on the manipulating of the materials rather than the quality of thinking that was taking place. The teacher whom Cohen observed also accepted any answer or comment from students without regard to their relevance to the given problem.

Scoring the Lessons

The content of the 11 lessons was analyzed using a 0 to 5 scale. Zero indicates that the criteria examined was not present. One indicates that the teacher was least effective in meeting this criteria. Five indicates that the teacher was most effective in meeting this criteria. This scale was used because there were criteria which were not present in the observed lessons that are considered effective in promoting the development of mathematical power. The lessons were reviewed and tally marks were recorded for each component. If a particular component was not present in any of the lessons a score of zero was assigned to that component. Table 1 shows the assignment of tally marks to each score. Barriers to and supports of effective practice are listed, also. The observations were analyzed using these lists. Preceding each component of effective practice is a score given to the observations based on the 0 to 5 scale. The scorer repeated the process twice to strengthen reliability.

Table 1. Lesson Scores

Number of Tally Marks	Score
0	0
1-2	1
3-5	2
6-8	3
9-11	4
12-15	5

Worthwhile Mathematical Tasks?

Worthwhile mathematical tasks were previously defined as those tasks which engage the learner in constructing knowledge and which provide the intellectual context for mathematical development. Through such tasks, students are lead to reason as mathematicians do and to communicate mathematically. It appears that the tasks in which the teacher engaged the students were not of this nature. Objectives were not made clear to the students. A need to know was not provoked within a relevant and meaningful context. For example, there was no real need to understand the concepts of fourness or 3+2. Problems were not posed which stimulated mathematical thought and communication. Students were allowed to make comments or pursue investigations which were not mathematically related. Students' sense of mathematical power was not enhanced by these methods.

Score	Component #	Effective Practice	Ineffective Practice
1	1	Projects, questions, or problems that involve constructions or applications set in a real world context that are purposeful and meaningful to the intended group	Projects, questions, or problems that are hypothetical and are not meaningful or purposeful
1	2	Projects that utilize manipulatives as a tool for enhancing meaning construction, application, or discourse	Projects that allow play with manipulatives that does not lead to meaning construction or are not useful for enhancing discourse
3	3	Problems that require students to reason and communicate about mathematics	Problems that require student to do tasks for the sake of doing the task. The problem is an end in and of itself
1	4	Tasks based on significant mathematics, knowledge of student understandings, interests and experiences, and the range of diverse ways in which students learn	Tasks that are not based on significant mathematics, do not consider student understandings, interests, or experiences and are limited in the ways in which students may learn
1	5	Engage students' intellect, stimulate students to make connections within mathematics, and develop students' mathematical understandings and skills	Students are not encouraged to make connections within mathematics, and students' mathematical understandings and skills are not developed
1	6	Call for problem formulation and problem solving	Call for the "correct" answer
0	7	Present mathematics as an ongoing life activity	Present mathematics as a set of skills to be learned and a subject separate from all other subjects
0	8	Capture students' curiosity and need to know	Ignore students' natural curiosity and need to know

Score	Component #	Effective Practice	Ineffective Practice
1	9	Tasks that can be solved or approached in more than one way	Tasks that require one method for solving
5	10	Tasks that grow out of student conjecture	Tasks which dismiss student conjecture as irrelevant
0	11	The gathering, summarizing, and interpretation of data based on a need to know	The gathering of data for no purpose that is real and meaningful to the students
0	12	Tasks that require students to consider the validity of their approaches and findings	Tasks that do not require students to justify their findings, but only to find the "correct" answer
0	13	Nest skill development within the context of problem solving	Isolate skills and concepts

The following table compares the highest possible score and the observed teacher's score on each of the above components of effective practices in worthwhile mathematical tasks.

Table 2. Comparison of the Highest Possible Score and Teacher's Score

	Component #												
	1	2	3	4	5	6	7	8	9	10	11	12	13
Highest Possible Score	5	5	5	5	5	5	5	5	5	5	5	5	5
Teacher Score	1	1	3	1	1	1	0	0	1	5	0	0	0
Percent Effective	20	20	60	20	20	20	0	0	20	100	0	0	0

The barriers to and supports of effective practice in creating worthwhile mathematical tasks are listed below.

Supports	Barriers
Tasks were prompted by student conjecture	The teacher was not skilled in the delivery of the lessons
Students naturally went about investigating their own questions	Some lessons in Mathematics Their Way were not effective in providing worthwhile mathematical tasks
The students' curiosity and "need to know" were captured	The ability to capture the students' curiosity and "need to know" was not present

The teacher was not effective in planning worthwhile mathematical tasks nor skilled in selecting lessons from Mathematics Their Way that were worthwhile. This resulted in a lack of curiosity from the students. A need to know was clearly not stimulated by the lessons the observed teacher chose nor by the manner in which she presented them.

Some students did use the manipulatives as a tool for meaning construction of their own volition. Some students invented problems or followed the problem given by the teacher that required them to reason and communicate about mathematics. Many students naturally conferred with those around them as they were working. The teacher did pursue some tasks that grew out of student conjecture. Most of the problems though, were not set in a real world context and did not capture the students' need to know.

Manipulatives are a valid tool used for constructing knowledge. Students need the opportunity to freely investigate new manipulative materials before they are required to be used as tools for a specific task or lesson. In one lesson, Mathematics

Their Way jewels were used. The students had not handled these manipulatives before. They were very colorful and interesting in shape. Many students were distracted from the tasks because of their curiosity about the jewels.

Lack of interest in the task and play with manipulatives distracted students from the problem and from engaging in meaning construction. Data seemed to have been gathered for not meaningful purpose. Skills were isolated and not connected to significant concepts.

Teacher and Student Roles

The National Council of Teachers of Mathematics advises that teachers should model questioning and commentary styles which lead to effective social discourse between students. This is accomplished first by creating a classroom in which students feel comfortable sharing their ideas and conjectures with their peers. It is the author's belief that the teacher had successfully provided this for her students. Effective teachers also pose questions that challenge student thinking and require the justification of student answers. The teacher did this to some limited extent, as when she was trying to elicit from student 2 an explanation for why he thought of four when he saw 3+2. But in the instances in which students were off track, the teacher made comments that did not relate to the problem at hand, or that distracted others from the problem. It may have been helpful for the teacher to make no comment or ask the class to respond to that student's idea respectfully and get them back on track. This was not done and it is feared many students left the experience confused.

Score	Component #	Effective Practice	Ineffective Practice
0	1	Orchestrate classroom discourse in ways that promote the investigation of mathematical ideas	Classroom discourse managed in ways that inhibits the investigation of mathematical ideas
2	2	Students use tools to pursue mathematical investigations	Students use tools for play
0	3	Teachers help students make connections between prior knowledge and new information	Teachers ignore the importance of connecting prior knowledge with the new
2	4	Students are involved in constructing their own mathematical knowledge	Students are told by the teacher an explanation for phenomenon
4	5	Students impose their own interpretations on what is presented to create a theory that makes sense to them	Teachers impose upon students their interpretation of what is presented
5	6	Each student's understanding of mathematics is considered uniquely personal	all students are required to come to the same conclusion
5	7	Authority on mathematical knowledge lies within the community of learners	Authority on mathematical knowledge lies with the teacher
1	8	The teacher poses questions that engage and challenge student thinking	The teacher poses questions that do not challenge or engage student thinking
1	9	The teacher requires students to justify answers by relying on mathematical evidence	The teacher does not require students to justify answers and may only look for the "correct" answer

Score	Component #	Effective Practice	Ineffective Practice
0	10	The teacher is delicately aware of student participation and ensures that all participate by providing opportunities that consider the diverse learning styles of students	The teacher allows only a select few who understand the activity in the manner the teacher intended to participate without regard to including all students
0	11	The teacher is acutely aware of when to lead the students in a different direction, especially when a particular student may be off track	The teacher is too permissive in allowing students to lead discourse or involvement in tasks
3	12	Students are invited to propose a variety of different methods for solving problems	Students are restricted to one method or teacher selected methods for solving problems
0	13	The teacher is delicately aware of what student questions or conjectures to pursue in depth	The teacher either chooses to follow an irrelevant comment that leads to student confusion, or dismisses comments that could lead to significant meaning construction
1	14	The teacher provides mathematical information when needed to guide student mathematical construction	The teacher provides mathematical information that inhibits mathematical construction

The following table compares the highest possible score and the observed teacher's score on each of the above components on effective practice in teacher and student roles.

Table 3.

	Component #													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Highest Possible Score	5	5	5	5	5	5	5	5	5	5	5	5	5	5
Teacher Score	0	2	0	2	4	5	5	1	1	0	0	3	0	1
Percent Effective	0	40	0	40	80	100	100	20	20	0	0	60	0	20

Barriers to and supports of effective practice in teacher and student roles in mathematical tasks and discourse are listed below.

Supports	Barriers
Developmentally appropriate practices nd guidelines have been published as a resource for teachers by NAEYC in "Developmentally Appropriate Practices in Programs Serving Children Ages birth to 8" (1991).	Respect for student centered activities and discussion prevented the teacher from establishing a clear focus or goal for a given lesson.
Standards for the effective teaching of mathematics have been published as a resource for teachers by NCTM in "Professional Standards for Teaching Mathematics" (1991).	Limited knowledge of how to effectively elicit from students justification for conclusions prevents opportunities for meaning construction.
	Limited knowledge of how to encourage all students to participate in discourse or mathematical tasks prevents opportunities for meaning construction.

The barriers to effective practice in teacher and student roles in mathematical tasks and during discourse outweighed the supports. The teacher was not able to establish a clear focus of the tasks or during discourse. Since the teacher did not effectively elicit from students justification for conclusions, opportunities for meaning construction were lost. All students were kept from participating in discourse because the teacher did not provide a variety of methods for involvement.

The observed teacher was effective in creating an environment in which each student's understanding of mathematics was respected. Emphasis was clearly placed on a student centered classroom. Students viewed themselves as mathematical authorities. Students were allowed to voice their own interpretations of a problem. The author's concern was that when some students were not focused on the problem and made interpretations or comments that were not on track, the teacher did not lure them back to effective discourse. This further prevented students from making mathematical connections.

Tools for Enhancing Discourse

Using manipulatives as a tool for promoting mathematical thought is a valid use. It is questionable as to whether the manipulatives in the above lesson were indeed used to promote mathematical thought. Some students did use them as such, but it may have been purely accidental. Creating pictorial representations of mathematical findings can be a valid tool for enhancing discourse. Presenting and explaining their meaning helps the students construct mathematical knowledge and leads to mathematical empowerment. But students were not invited to critique one another's findings. And therefore, were not challenged to defend or argue their perceptions and conclusions.

Score	Component #	Effective Practice	Ineffective Practice
2	1	Students use tools for constructing meaning	Students use tools for play
0	2	The introduction of conventional mathematical notation follows the development of a concept in a meaningful context	The introduction of conventional mathematical notation is done either in isolation or not within a meaningful context
2	3	Tools are used for defending student formulated theories that have been developed within a meaningful context	Tools are not used for defending student formulated theories
2	4	Students use tools for explaining a personal conjecture to other students in a problem solving situation	Students do not use tools for explaining a personal conjecture
0	5	Students use tools to accompany a presentation of their conclusion in a personally meaningful way as a means of proving their theory	Students do not use tools to accompany a presentation of their conclusion

The following table compares the highest possible score and the observed teacher's score on effective practice in using tools for enhancing discourse.

Table 4. Comparison of Highest Possible Score and Observed Teacher's Score

	Component #				
	1	2	3	4	5
Highest Possible Score	5	5	5	5	5
Teacher Score	2	0	2	2	0
Percent Effective	40	0	40	40	0

Barriers to and supports of effective use of tools for enhancing discourse are outlined below.

Supports	Barriers
Designated sessions are given for students to engage in free play with manipulatives	The teacher does not clearly distinguish between tools used for enhancing discourse or meaning construction and free play with manipulatives
Worthwhile mathematical tasks are provided out of which naturally arise the need to know conventional mathematical notation and language	The introduction of conventional mathematical notation and language is not presented within a meaningful context
Effective tools and their use for developing meaning construction are defined by the NCTM in "Professional Standards for Teaching Mathematics."	Discourse is not focused on exploring mathematical ideas and meaning construction

The teacher appeared to have been unaware of the importance of providing a designated place and time for free exploration of the manipulatives prior to their use as tools for investigating solutions to a particular problem. The introduction of conventional mathematical notation was done out of context and void of a need to know on the part of the students. Discourse was not focused on the exploration of the intended concept. Discourse was not effectively lead toward meaning construction.

The author found that some students spontaneously used the manipulatives for constructing mathematical meaning. Some students used the manipulatives to explain an idea or discovery to a student next to them or to the teacher as she was circulating unsolicitously. Many students played with the manipulatives and did not use them during sharing when they explained what they had found.

Chapter V

Summary, Conclusions, and Recommendations

Summary

In summary, it appeared that worthwhile mathematical tasks were not created in which to promote the development of mathematical power. Most of the observed tasks lacked the invitation for meaning construction and application of discoveries or learned concepts. They were not set in real world context which was meaningful or purposeful to the students. The observed teacher was strong in her ability to pursue tasks that grew out of student conjecture, as shown in Table 2. Some problems were presented that had the potential for requiring students to reason and communicate about mathematics, but other variables kept this from happening. The observed teacher's percent effectiveness was low for most components for worthwhile mathematical tasks.

The teacher did not require students to justify their solutions by relying on mathematical evidence. She did not effectively lead the discussion in a fruitful direction. Some students were confused by the direction of the discussion. True discourse was not present. The teacher's consideration of each student's understanding of mathematics as uniquely personal was high, as evidenced by Table 3. She also was strong in establishing the attitude in her classroom that authority on mathematical knowledge lies within the community of learners. She was effective in encouraging students to impose their own interpretations on what is presented to create a theory that makes sense to them. The observed teacher's percent effectiveness was quite low in 12 out of the 15 components for teacher and student roles.

Opportunities for students to explore the manipulatives before using them as tools for solving a given problem were not provided. As a result, the students engaged in play with the manipulatives at a time when they were required to use them as tools for solving a problem. Because of this, the students were unable to focus on the task at hand. Table 4 shows that the teacher scored low in all components for using tools for enhancing discourse.

Conclusions

It is the author's opinion that the observed teacher had only limited knowledge and ability in the above components which lead to the development of mathematical power. It would appear that a teacher's rhetoric may not always match her/his practices. While this teacher in her own mind believed she was being extremely child-centered, as the author examined her teaching it was found that her teaching practices did not reflect her beliefs. Within the limits of the 11 observed lessons, several components that one would expect to find in such practice, were either poorly lacking or substantially missing from her instruction. The result of the observed instruction on the development of mathematical power in the students was evaluated by using the NCTM standards. The following was discovered:

- Worthwhile mathematical tasks were not presented during instruction.

- Tasks in which students were engaged did not involve constructions and applications in mathematics.
- Students' need to know was not enhanced by the questions the teacher posed.
- Intellectual thought about mathematical concepts and procedures was not stimulated by this method.
- Established teacher and student roles did not lead to meaning construction.
- Questions were not posed which challenged student thinking.
- The teacher did not exhibit a sense of when to provide information or lead the group in a fruitful direction.
- Only a few students dominated the discussion.
- A variety of choices for making contributions to discourse were not provided.
- The manipulatives and pictures were not used effectively or validly as tools for enhancing knowledge construction or discourse.
- The activities lacked focus and direction, and therefore, prevented students from making mathematical connections.
- Students were given only one choice of tools for participating in discourse.
- Even well trained teachers may have difficulty orchestrating effective classroom discourse.
- The action research method is valuable for evaluating a program.
- Teachers can easily conduct action research by teaming with another teacher who can observe and take notes.

- Such research causes the teacher to think reflectively about her/his instructional methods and effective practices.
- It provides evidence of the effects of the classroom environment on student development.

Recommendations

It is in the interest of the author to repeat the action research process with herself as the teacher. Her skills in observation and detailed note taking have increased. The author recommends that video tapes be taken to more accurately record instruction and student-teacher interactions. Other teachers or parent volunteers may be asked to video tape. In this manner the author can continue to reflect about her teaching and develop effective practices.

Mathematics Their Way (1976) is considered by many teachers to be an effective program in developing mathematical power in students. It should include worthwhile mathematical tasks, examples of effective teacher and student roles in developing mathematical power, and suggestions of tools and their use for enhancing discourse. It is recommended that teachers critically analyze all materials and programs available for their use. The NCTM publication, "Professional Standards for Teaching Mathematics" (1991), would be a helpful guide in this process.

It is further recommended that teachers study methods for effectively eliciting from students coherent responses. Proper discourse takes place when one person makes a comment and the next person responds with a comment contingent to the first comment, and so on. When a student makes a comment that does not refer to the previous comment, proper discourse is not taking place. Teachers need to model appropriate discourse so students understand what it is. Teachers need training on effective techniques for enhancing discourse so they are not afraid to indicate respectfully that a comment is not appropriate, and so they are able to guide students in a fruitful direction. Part of the teacher's role is to record data and information with words or pictures so students are provided with a reference during discourse.

In conclusion, the teaching of mathematics to students in a meaningful manner involves not only exemplary materials but also interest on the part of the teacher to encourage students as participants. When this occurs, students are the beneficiaries as they are empowered mathematically.

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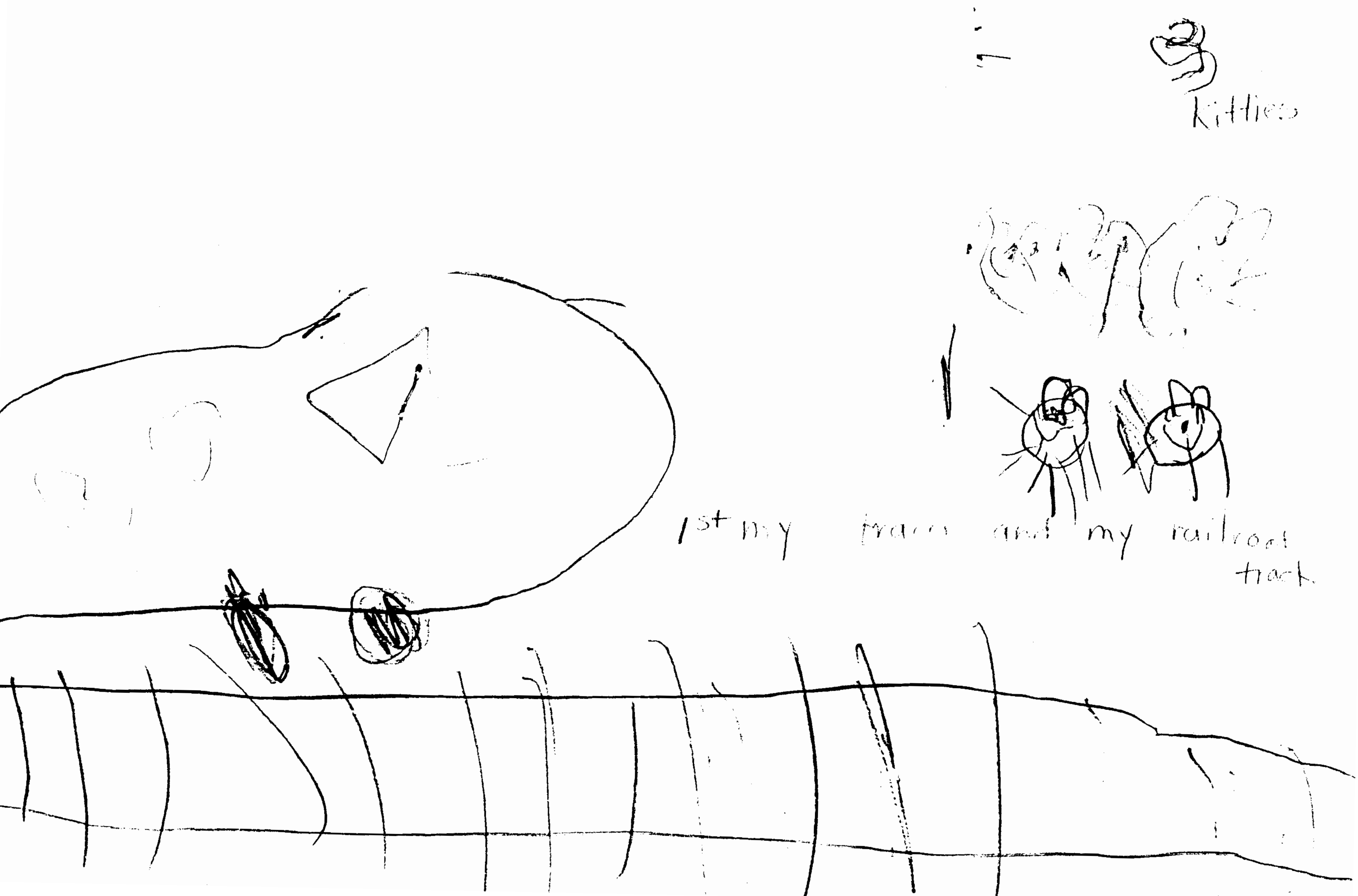
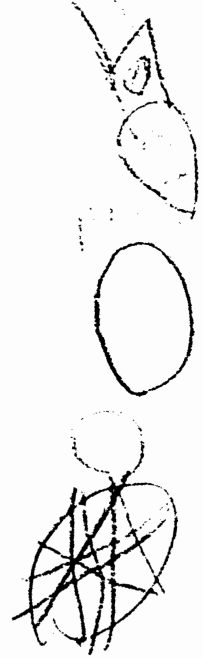
Appendix A

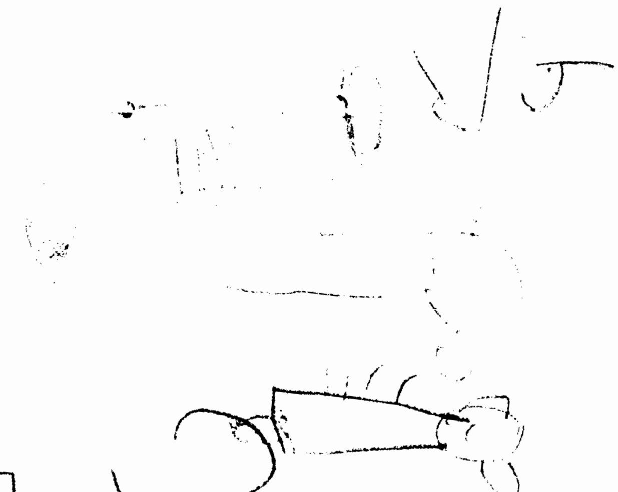
Samples of Student Work



3
kitties
1st my train and my railroad track

cats
4 cats
2 horses

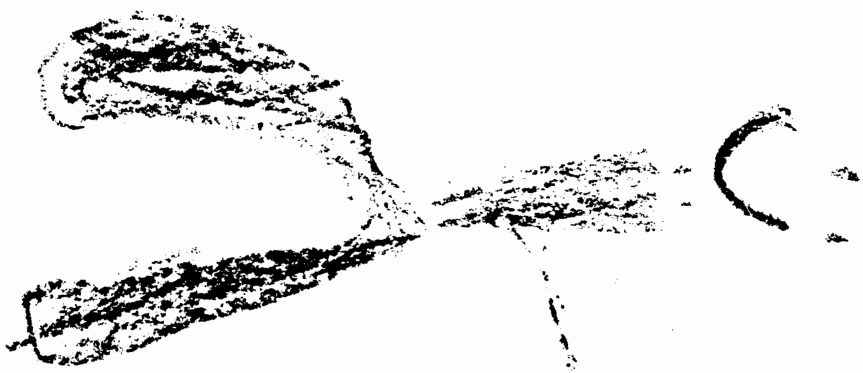




7 = 4 + 3

6 = 3 + 3

4 is 4 flags



Appendix B

Example Lessons

Date: February 22, 1992

Lesson Topic: Graphing

Concept: Reading a graph, comparing numbers

Procedure: The teacher began the lesson by announcing that they would be working with the cloth graph. She layed out a large cloth graph on the floor. She told them that they would be graphing girls and boys. Cards labeling the girls and boys columns were placed on the graph.

The students were directed to stand in a space on the graph under girls or boys according to their gender. They were asked which column had more and which had less. The students were easily able to answer these questions correctly.

Next, each student was told to draw a picture of herself/himself on a card. They placed their cards on the graph according to gender. Students discussed the remaining spaces. One student indicated that if one boy who was absent had been there, the boys column would have more in it. The teacher agreed and remarked that the graph would be changed if he were there.

Date: February 23, 1992

Lesson Topic: Graphing

Concept: Comparing numbers in columns

Procedure: The teacher began the lesson in much the same manner as described above. The cloth graph was spread out on the floor. She announced that they would be doing graphing again, and that they would be graphing hair color.

Each student was given a card on which s/he drew her hair and colored it yellow, brown, or black. The teacher placed cards at the top of each column indicating yellow, brown, and black.

The students were invited to place their card in the appropriate column. They did so accurately. Numbers were compared in each column. Questions such as, Which column has the most?, Which has the least?, and How many more does this column have than that? were asked. A few students who dominated the discussion were able to answer these questions easily. Most students did not respond. It is difficult to tell if they knew or not.

One student suggested graphing brown eyed, brown haired people and blue eyed, yellow haired people. The teacher directed the students to turn their cards over and draw their eyes. She then called students with the above combinations to place their cards on the graph. The students did so easily.

The teacher asked how the graph had changed. One student noted that not everyone put their card on the graph, so there were fewer cards. Another said that their were only two columns this time. Still another told that one column had more than the other and how many. Again, the discussion was dominated by the same few who seemed to understand the activity.

Date: February 26, 1992

Lesson Topic: Graphing

Concept: Reading a graph, comparing numbers

Procedure: The teacher announced that they would be doing graphing and placed the cloth graph on the floor. She told the students that they would be graphing the number of people who liked each of four Winnie the Pooh characters. Their choices were Piglet, Kanga, Tigger, and Pooh. Each of these characters' pictures were used as labels for the columns.

The teacher asked the students to predict which character would be most liked and which would be least liked. Several students gave responses. She then passed out milk cartons on which the students were instructed to draw a picture of themselves. She explained that these would be used to represent their choice on the graph.

The teacher called each character one at a time. The students brought their milk carton up and placed it on the graph. After each character was called and graphed, the number wa counted and compared to the next column. Questions such as, Which has more?, How many more?, and Which has the least? were asked.

The teacher asked the students if there was anything else they could say about the graph. One student suggested, "How many did Kanga and Tigger get together?" The teacher pursued this question and asked the student how many she thought they had together. The student replied correctly. The teacher asked how the student got that number. The student replied, "I remembered this was 5 and so I said, "6, 7, 8, 9, 10, 11, 12, 13. There are 13." She did this by pointing to the column with five and then counting on. The teacher responded by nodding her head in agreement.

Date: March 1, 1992

Lesson Topic: Graphing

Concept: Reading a graph

Procedure: The teacher began the lesson by spreading the cloth graph on the floor and placing real apples, oranges, lemons, and grapefruit as headings for four columns. She told the students that they would be graphing which fruits were most liked in the class. The students were told to place their milk carton on the fruit they liked the best.

Teacher: Which column has the least? (pause, no response from students) Which column is the smallest? Which has the smallest number?
Student 1: Lemons have zero.
Teacher: Yes, you are right. Which has the next biggest number after zero?
Student 2: Apples. Thirteen. I like apples.
Teacher: There are more apples, but which has just a little bit more than lemons? Which is just a little bit more than lemons?
Student 3: Oranges has three.
Teacher: Let's put the fruit in order from littlest to biggest number. (Students did so correctly.)
How many more people liked oranges than liked lemons? (pause, no response from students)
There is one more person on the oranges graph than on the lemons. So, there is one more person who likes oranges than lemons. How many more people liked grapefruit than oranges?
Student 4: Two
Teacher: How did you get two?
Student 4: I covered up the one who liked oranges and counted 1, 2.
Teacher: Show us.
Student 4: (Counted)
Teacher: How many more people liked apples than grapefruit?
Student 2: Ten and there are 13 stripes on the flag, too.
Teacher: Show us how you got 10.
Student 4: I know how! It's not 10. It's 13, because grapefruit is zero, so you count the apples.
Student 5: Could we have a piece?
Teacher: There are only four pieces of fruit. How could we cut them so each person gets a piece?
Student 5: We could cut them in half.
Teacher: Okay. (cuts them in half) How many people are in our class?
Student 6: Twenty.

Teacher: Let's count how many pieces we have. (count 8) Is that enough pieces?
Students: No.
Teacher: What can we do?
Student 4: Cut some more.
Teacher: How about if I cut them in half again? Now we have 16 pieces. Is that enough?
Students: No.
Teacher: Cut some more.

The teacher cut the fruit so there were 20 pieces and distributed them to the class.

Date: March 8, 1992

Lesson Topic: What is five?

Concept: Fiveness

Procedure: The teacher announced that they were going to work on five. She counted out five toothpicks from a bucket. She made a 5 with the toothpicks. she asked students to tell her what else she could make with five toothpicks. One student suggested a house and another the letter "w". So she made a house and a "w" each out of five toothpicks.

Next, the teacher made 771 out of five toothpicks and glued them onto a piece of paper. She wrote her name on the paper. She told the students they would have a chance to make something out of five toothpicks and glue them on paper.

A bucket of toothpicks was passed around and each student counted out five toothpicks. One student made a 5. Others made a window, a house, and a fork. The teacher circulated among the students asking them to tell about what they were doing. One student had made a box with one toothpick left over.

Teacher: How many did you use to make the box?
Student 1: Four.
Teacher: And how many are left?
Student 1: One.
Teacher: So, four plus one equals . . .?
Student 1: Five.

The teacher approached another student.

Student 2: I made two forks!
Teacher: How many toothpicks did you use to make this fork?
Student 2: One, two, three, four, five.
Teacher: So, five plus five is . . .?

Student 2: Six!
Teacher: Count them
Student 2: One, two, three, four, five, six ...
Teacher with Student 2: Seven, eight, nine, ten.
Teacher: Will you make two more of the same thing?

The student made two tv antennas. When asked how many toothpicks were used all together, he counted ten.

Date: March 9, 1992

Lesson Topic: What is five?

Concept: Fiveness

Procedure: The teacher began the lesson by telling the students that they were going to work on five today. She brought a box of tiles of different colors around to the students. They each counted out five tiles. The teacher directed the students to show what they cold do with vive tiles. She demonstrated that each tile must touch a corner of another tile. The students were set to work with the tiles. The teacher circulated and asked students about what they were doing.

Student 1: I have four white one and five green ones.
Teacher: How many do you have all together?
Student 1: Nine
Teacher: Show me how you got nine.
Student 1: You said you had four white and five green. So we could write four for the white and plus five for the green. Four plus five equals nine.
Student 2: I made a side.
Teacher: How many tiles did you use to make your sidewalk?
Student 2: Ten.
Teacher: Could you make something with five?
Student 2: Okay.

Date: March 12, 1992

Lesson Topic: What is six

Concept: Sixness

Procedure: The teacher announced that they were going to work with six today and use toothpicks. She demonstrated in much the same manner as before. She made a figure with her six sticks and glued them to paper. She carried a bucket of sticks around as each student counted out six. Students were set to work and the teacher circulated asking questions.

Teacher: Tell me about what you made.
Student 1: A "F" and a "A."
Teacher: Count them for me.
Student 1: One, two, three, four, five, six.
Teacher: What else could you make?
Student 2: (interrupting) I made a "H" and a "I."
Teacher: Did you know that that spells a word? "HI" spells hi!

During sharing the class discussed different number sentences that equaled six. For example, five prongs on the fork plus one handle is 5+1=6. Another student came up with two sides of a house, plus two sides of the roof, plus two sides of the chimney is 2+2+2=6. Another student had four long sides of a rectangle, plus two short sides is 4+2=6. Yet another student had three toothpicks in "H" plus three toothpicks in "I" which is 3+3=6.

Date: March 15, 1992

Lesson Topic: What is six?

Concept: Sixness

Procedure: The teacher began the lesson by telling the students they would be working on six with the tiles. She reminded them that the tiles must touch the corner of the other tiles. She presented paper squares the color of the tiles and explained that when they finished their designs they were to glue the papers that were the same color as the tiles with which they made their design on black paper. The paper squares were meant to represent the tiles and were used to record what the students had done.

Some students made interesting patterns with the six tiles that lent themselves well to writing different number sentences. For example, one student created a letter "V" with a white, green, white pattern. The number sentences $3+3=6$ and $4+2=6$ were written by the teacher during sharing to represent the combinations the student had illustrated in her design.

Date: March 16, 1992

Lesson Topic: Number combinations with sum of four

Concept: Addition

Procedure: The teacher presented a box of "jewels" to the students. Each color of jewel represented a different group. One black jewel represented one group of one. Two green jewels connected represented one group of two. Three yellow jewels connected represented one group of three. Four red jewels connected represented one group of four. Five blue jewels connected represented one group of five.

The students were challenged to make as many different combinations that added up to four as they could. They were given five cups in which to hold each combination. Worksheets were provided on which to record the combinations. Extra jewels were dumped in the middle of the circle of students. They were allowed to get different jewels that they didn't have to complete their combinations.

Some students were called to record their combinations on the worksheets. They did so with crayon. Those who had only one combination simply drew just that one. Those that had come up with all five, drew all five.

Sharing took place on the floor in a circle. The teacher called students one at a time to show their worksheet and tell about their combinations. The teacher and the student sharing did most of the talking. Students in the audience were not called on to ask questions or make comments. Their role was to listen. (See Appendix C for examples of worksheets.)