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A constructivist instructional project on developing geometric problem solving abilities using pattern blocks and tangrams with young children

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This instructional project planned and implemented mathematical instruction for young children based upon recommendations by both the National Association for the Education for Young Children and the National Council for Teachers of Mathematics. Specifically, the project presented children with a variety of geometric problem solving tasks that involved spatial reasoning. Two kinds of geometric materials, pattern blocks and tangrams, were used together with a set of frames designed to provide problem solving tasks. Examples of children's work and insights into the knowledge they constructed are presented. The tasks appeared to be interesting and appropriate. The children engaged in the tasks purposefully, and demonstrated a high level of problem solving strategies and mathematical reasoning.

A CONSTRUCTIVIST INSTRUCTIONAL PROJECT ON DEVELOPING GEOMETRIC PROBLEM SOLVING ABILITIES USING PATTERN BLOCKS AND TANGRAMS WITH YOUNG CHILDREN

A Graduate Instructional Project
Submitted to the
Division of Early Childhood Education
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by
Christie Sales
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This Project by: Christie E. Sales

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has been approved as meeting the research requirement for the Degree of Master of Arts in Education.

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Abstract

This instructional project planned and implemented mathematical instruction for young children based upon recommendations by both the National Association for the Education for Young Children and the National Council for Teachers of Mathematics. Specifically, the project presented children with a variety of geometric problem solving tasks that involved spatial reasoning. Two kinds of geometric materials, pattern blocks and tangrams, were used together with a set of frames designed to provide problem solving tasks. Examples of children's work and insights into the knowledge they constructed are presented. The tasks appeared to be interesting and appropriate. The children engaged in the tasks purposefully, and demonstrated a high level of problem solving strategies and mathematical reasoning.

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INTRODUCTION

The purpose of this instructional project was to plan and implement mathematical instruction relating to spatial reasoning and problem solving, and to examine the extent to which mathematical concepts, particularly geometric and spatial reasoning, developed as a result. The project involved two types of mathematical learning materials, pattern blocks, and tangrams. Sets of frames were developed to use in conjunction with pattern blocks and tangrams, and to assist the children as they solved spatial reasoning problems. The frames provide boundaries within which children can work as they solve the problems.

Using pattern blocks and tangrams allow children to have concrete experiences as they reason about spatial problems and may help them develop their spatial reasoning abilities. For example, in one task, a trapezoid or a parallelogram can be placed with its long side down, while in another, the short side is down. Children may begin to understand that the shape has not changed, only its position in space. Over time, they may begin to generalize that regardless of its spatial orientation, a shape never changes.

By using these geometric problem solving tasks, children may begin to acquire an understanding of geometric shapes. They may learn that two or more shapes fit together to create a new shape. Such actions provide an insight into the relationships among the shapes. Solving these problems may help children begin to develop an intuitive sense about area, perimeter, shapes within shapes, parts of a whole, geometric relationships, and number.

Pattern Block Description

A set of pattern blocks includes 250 three-eights-inch-thick pieces in six geometric shapes: 25 yellow hexagons, 50 red trapezoids, 50 blue parallelograms, 50 green triangles, 25 orange squares, and 50 tan parallelograms, see figure 1.

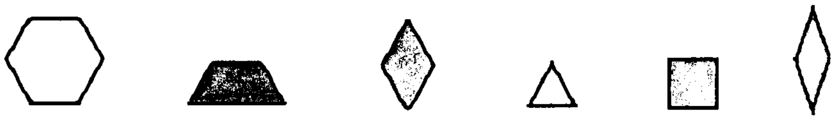


Fig. 1. Pattern Block Shapes

All shapes are closely interrelated, but four shapes, the triangle, the blue parallelogram, the trapezoid, and the hexagon, are more closely related than the others. For example, two green triangles combine to replicate a blue parallelogram. Three green triangles combine to replicate a red trapezoid. Two red trapezoids combine to replicate a yellow hexagon, as do three blue parallelograms, and six green triangles. Also, one can combine different shapes to replicate a shape. For instance, when they are combined, one blue parallelogram and one green triangle replicate a red trapezoid, see figure 2.

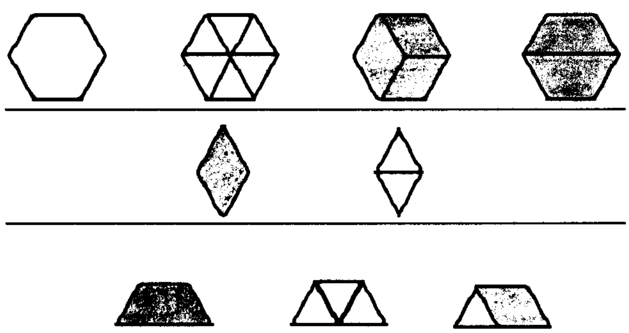


Fig. 2. Pattern Block Relationships
Examples of Relationships Among Four Shapes

The relationship between the two remaining shapes, the square and the tan parallelogram is more subtle. Two tan parallelograms occupy the same area as one square, but they cannot be manipulated to create the same shape, or fit into the same space. When combined with the triangle, however, the combinations can be used interchangeably, see figure 3.

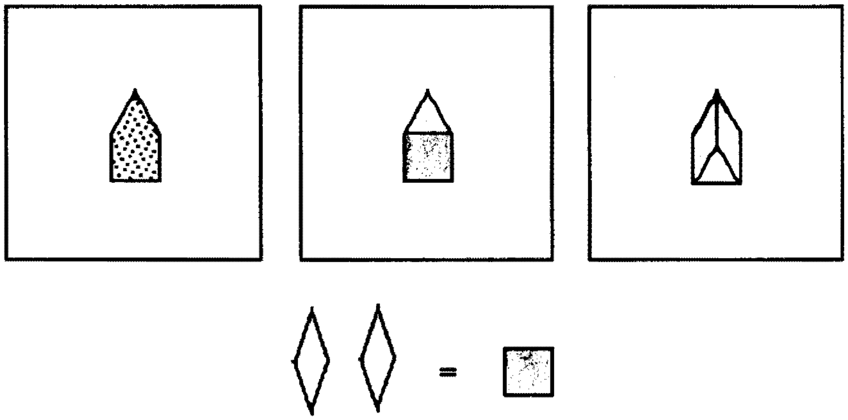


Fig. 3. Pattern Block Relationship Between the
Square and the Tan Parallelogram

The area relationship must be deduced by moving the pieces around, or solving a problem more than one way. Since two tan parallelograms and a triangle, and a square and a triangle, can create an identical shape, when the triangles are removed from both, the remaining pieces occupy the same area. This abstract concept is, no doubt, beyond the reasoning of young children. However, they can learn that one combination can replace the other.

The sides of the pattern blocks are all the same length, with the exception of one side of the trapezoid that is twice the length of the others. Because the sides are the same length, children come to expect that pattern block pieces will fit together. They need only consider the shape of the pieces they are using, not the size. When filling the frames, the blocks are nested together, which encourages children to explore relationships among shapes, including congruence, similarity, symmetry, area, and perimeter.

Tangram Description

The tangram is thought to be an ancient Chinese puzzle consisting of seven, one-quarter inch thick, geometric shapes cut from a single square. The pieces, which are all the same color, include one square, one parallelogram, and five similar triangles, two small, one medium, and two large, see figure 4.

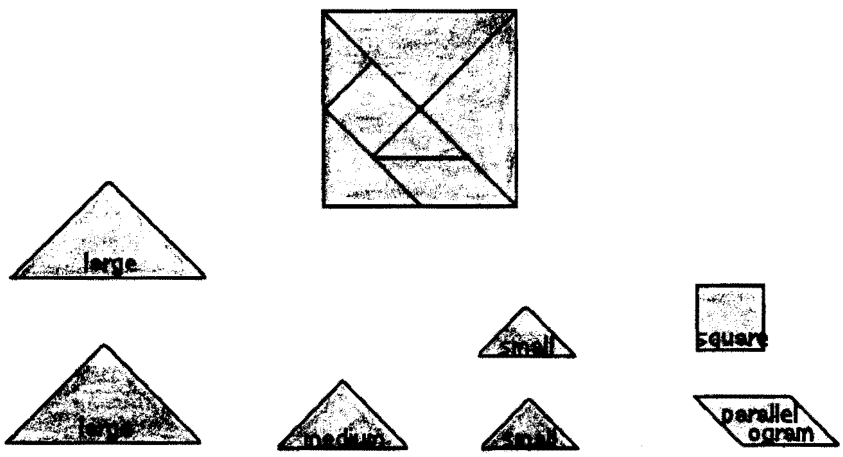


Fig. 4. Tangram Pieces: Seven Pieces in A Square Shape and Separated

The tangram shapes are interrelated, but in a different way than the pattern blocks. The medium triangle, the square, and the parallelogram have the same area. One can turn and flip the two small triangles to construct facsimiles of each of the aforementioned shapes, thus proving the equality of area, see figure 5.

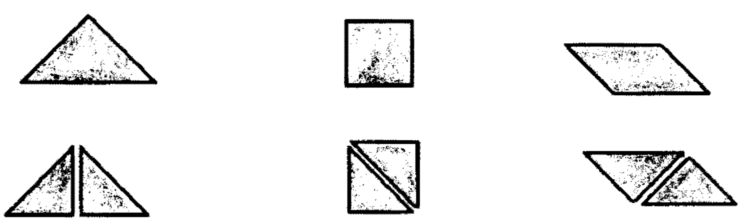


Fig. 5. Tangram Relationships: Examples of the Relationships Between the Two Small Triangles and the Medium Triangle, the Square, and the Parallelogram

The five small pieces have the same area as the two large triangles and can be combined to construct almost any shape made by the large triangles, as shown in figure 6.

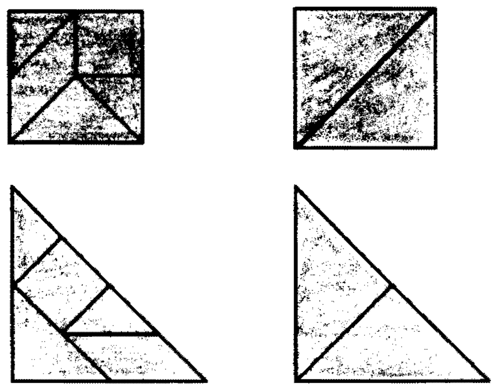


Fig. 6. Examples of the Relationship Between the Five Small Tangram Pieces and the Two Large Triangles

As with the pattern blocks, there is a relationship among the sides of the tangram pieces. However, the sides are not all the same length. For example, the hypotenuse of the small triangle is equal to one-half the length of the hypotenuse of the large triangle, the long side of the parallelogram, and the side of the medium triangle, see figure 7.

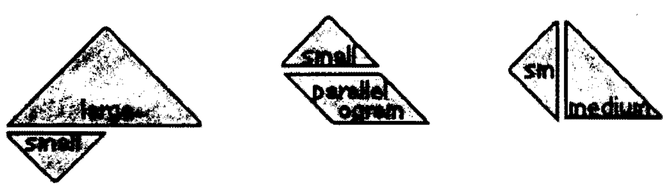


Fig. 7. Examples of Tangram Relationship Between the Hypotenuse of the Small Triangle and the Large Triangle, the Parallelogram, and the Medium Triangle

The side of the small triangle is equal to one-half the side of a large triangle, one-half the hypotenuse of the medium triangle, the short side of the parallelogram, and all sides of the square, see figure 8.

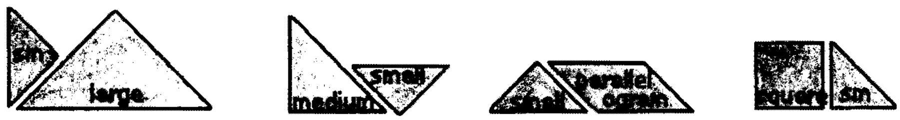


Fig. 8. Examples of Tangram Relationship Between the Side of the Small Triangle and the Large Triangle, the Medium Triangle, the Parallelogram, and the Square

When the pieces are rotated or flipped, their appearance is changed, see figure 9. This adds to the complexity of the problem solving tasks. If the pieces are not in the correct position, they will not fit into the frames. There is also the complication of having only 7 pieces to work with instead of 250.

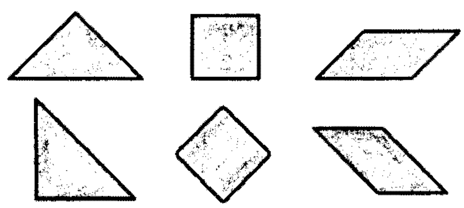


Fig. 9. Examples of Changes in Appearance When Tangram Pieces Are Turned and Flipped

When the seven pieces are used together, they can make a multitude of shapes. Because one can use the same seven pieces of the tangram to make many different shapes, children can form the foundation for understanding that different frame designs appearing to have no relation to one another have the same area.

Unlike the pattern block designs that have many solutions because of the large number of pieces and the ease with which pieces can be substituted for one another, most tangram frames, have only four or five solutions. The limited number of pieces and the specific relationships of the shapes make the tangram problems more complex than the pattern block problems.

Frame Description

The frames are square wooden boards, one-eighth-inch thick, each with a different geometric design cut from the center. There is a set of frames for pattern blocks and a set of frames for tangrams. Both sets contain a large variety of designs. Each frame functions as the outline of a puzzle allowing students to manipulate the pieces within the constraints of the frame. Additionally, the defined outline of the frames assists children in solving the problems; for at times they can, after adding one or two pieces, determine where other pieces will fit to complete the problem.

There are both basic and complex frames. Basic pattern block frames have smaller designs cut from the center and almost exclusively use only the four closely interrelated shapes, the triangle, the blue parallelogram, the trapezoid, and the hexagon, see figure 10.

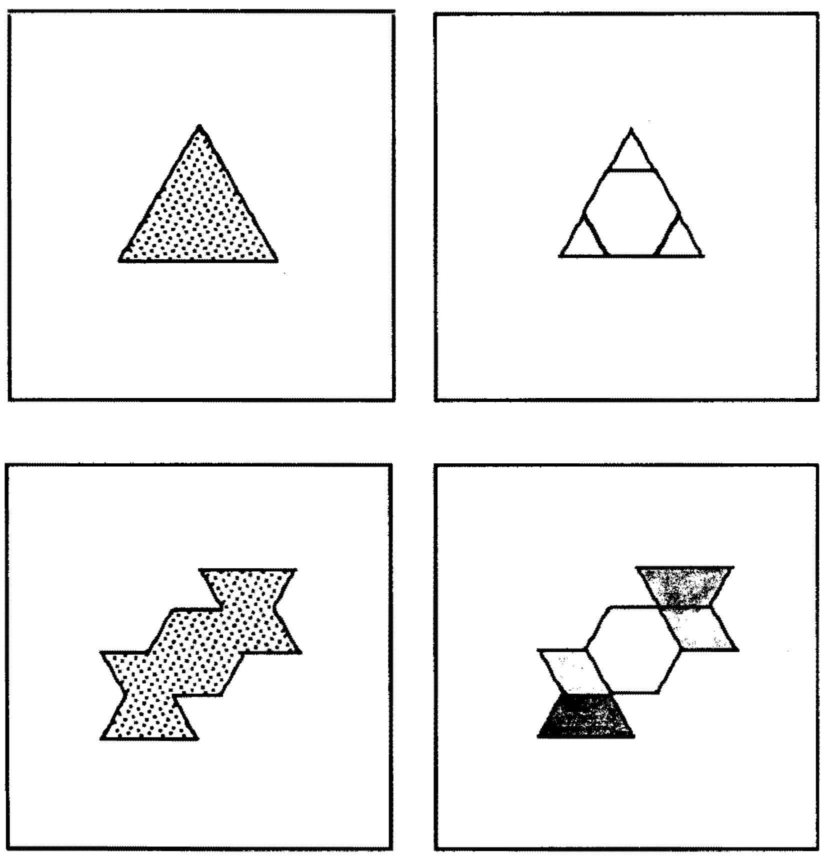


Fig. 10. Sample of Basic Pattern Block Frames

Complex pattern block frames have larger designs requiring the use of more blocks. Many require use of either the orange squares, the tan parallelograms, or both, along with one or more of the other four shapes, as shown in figure 11.

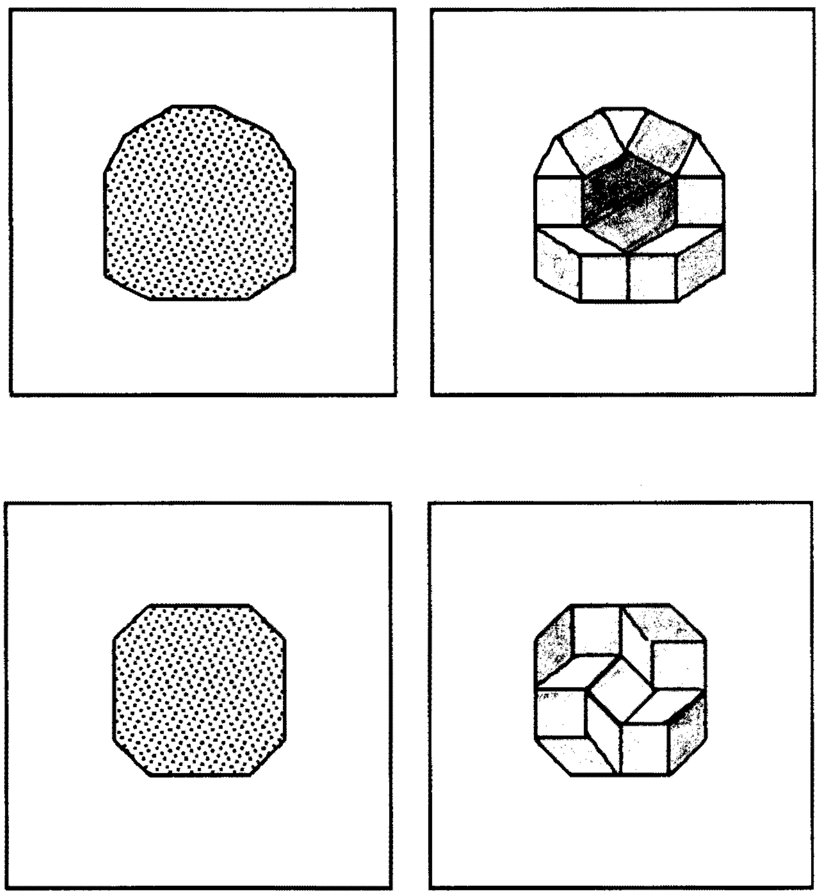


Fig. 11. Sample of Complex Pattern Block Frames

Like the pattern block frames, basic tangram frames have smaller designs and use five or fewer of the seven tangram pieces, see figure 12.

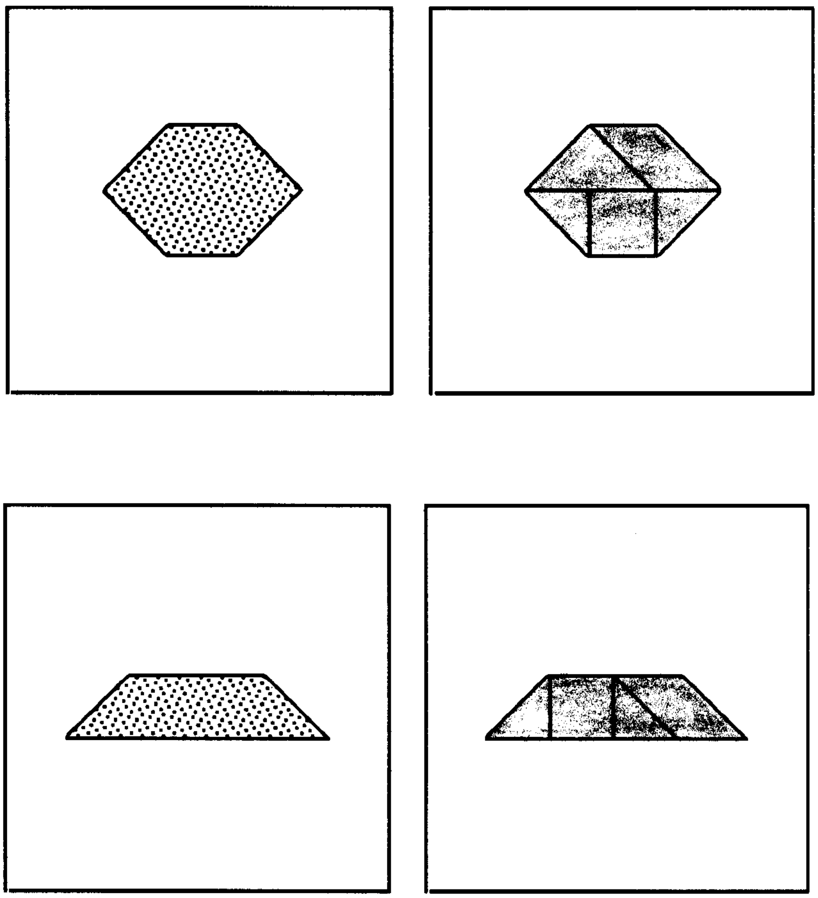


Fig. 12. Sample of Basic Tangram Frames

Complex tangram frames, on the other hand, have larger open designs requiring the use of all seven pieces, as shown in figure 13.

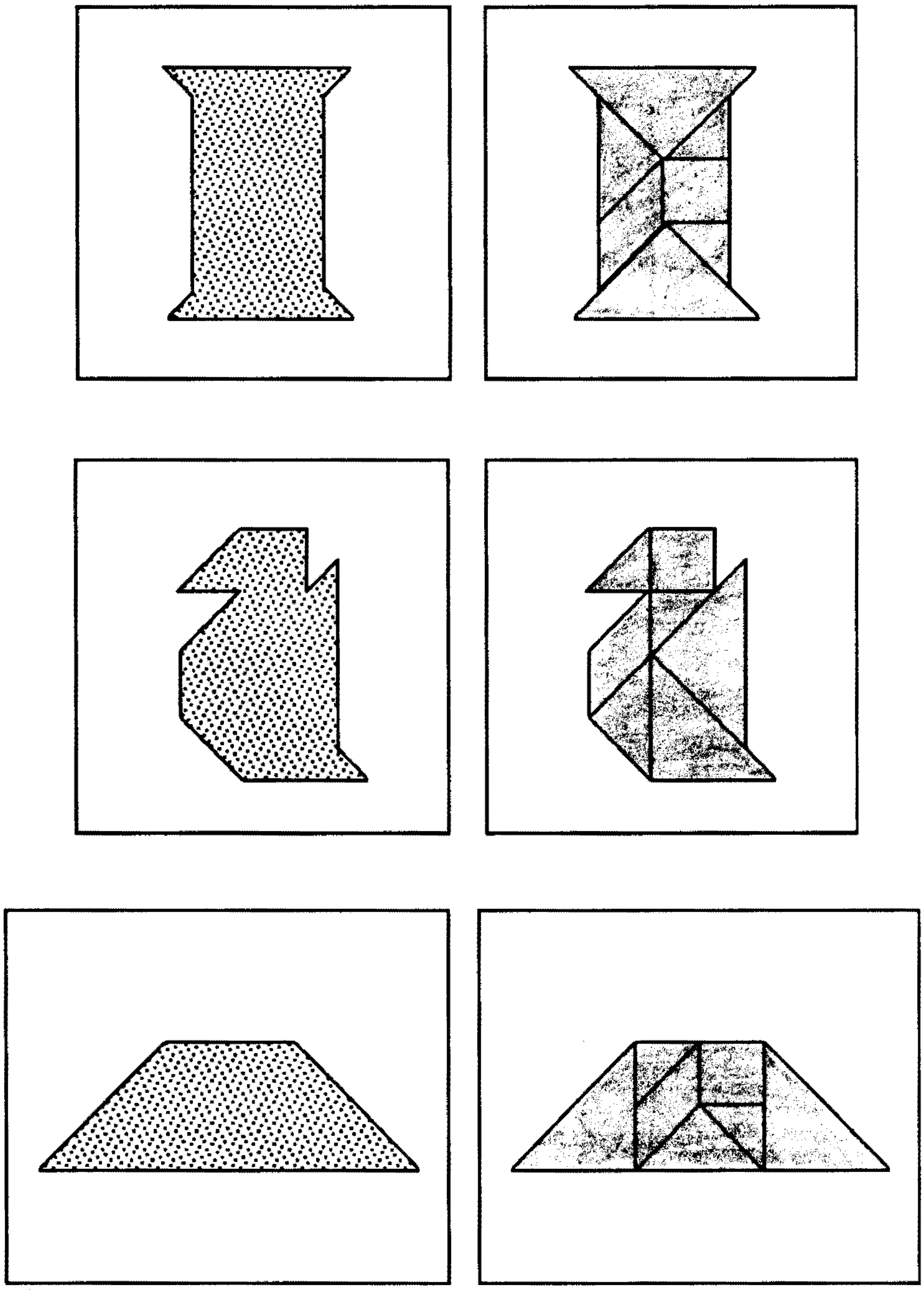


Fig. 13. Sample of Complex Tangram Frames

Particularly when using tangrams frames, children can use the knowledge constructed in solving basic frames to help solve larger, more complex frames, see figure 14.

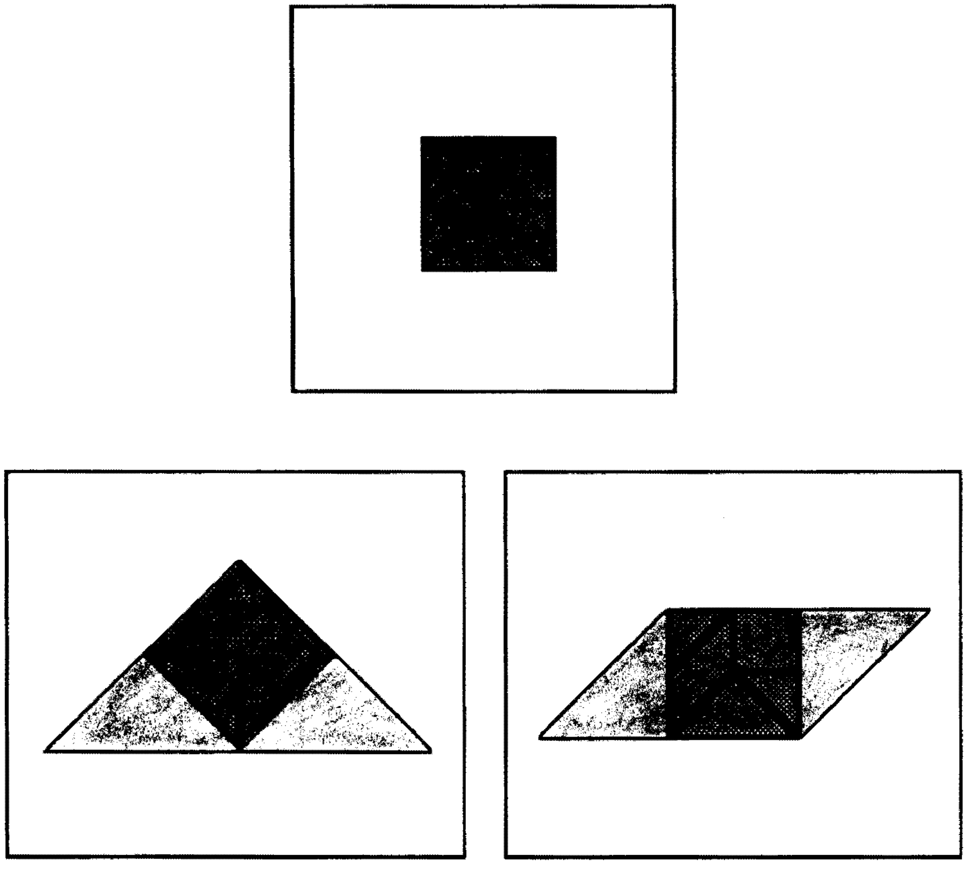


Fig. 14. Example of Basic Frame Incorporated Into Complex Frame

A special feature of the frames is that once they are filled, they may be lifted off the design without disturbing the solution. This permits the children to find many solutions to the same problem, while keeping a record of their previous work. Keeping a record of their work offers children the opportunity to reflect upon the many solutions they have found, as illustrated in figure 15.

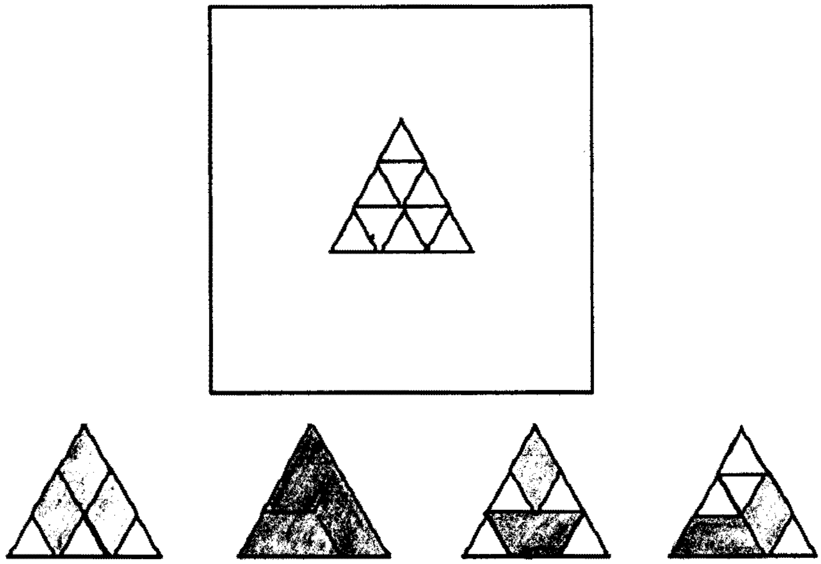


Fig. 15. Examples of Solutions to Triangular-Shaped Pattern Block Frame

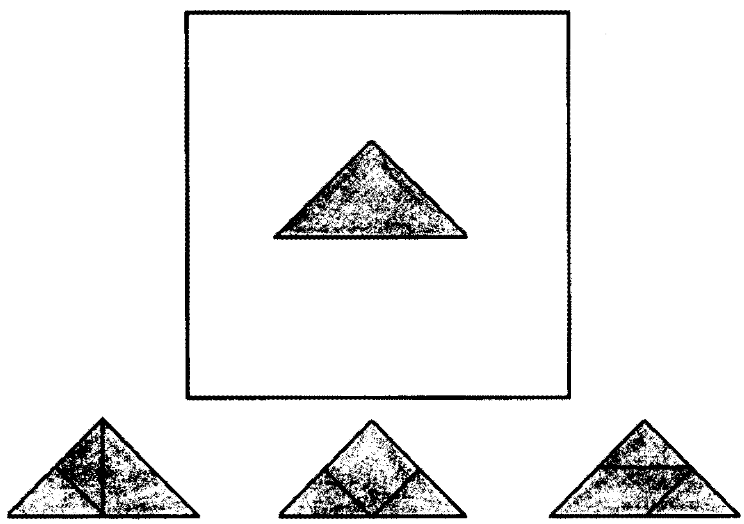


Fig. 16. Examples of Solutions to Triangular-Shaped Tangram Frame

Due to the complex nature of the tangrams, very few early childhood educators expose young children to these materials. It is more common to find pattern blocks among the materials selected.

REVIEW OF LITERATURE

The use of manipulatives in teaching mathematics has increased as educators have learned more about how children learn mathematical concepts. Traditional mathematics teaching involves memorizing procedures and formulas. Although memorization provides students with the means to produce correct answers, it does little to facilitate understanding (Karp, 1991). Piaget stated that young children think in qualitatively different ways from that of older children and adults (Piaget, 1969/1970). They draw conclusions that make sense to them, not from what they are told or required to memorize, but from what they see. Thus, using what is observable to them, they form many understandings, often erroneous according to adult ways of thinking.

According to the National Council of Teachers of Mathematics (1989), mathematics can be viewed as the process of constructing patterns and relationships. In fact, mathematics has been characterized by many as the science of patterns. The use of computers has enabled this digression from the traditional view of mathematics as a collection of facts and procedures. Steen (1989) noted that such relationships have become the foundation of "lasting mathematical structures," (p. 616) which serve to clarify and predict natural phenomena that fit the pattern, and expand its applications to linking mathematical subdivisions into more complex patterns with greater explanatory power.

Spatial sense is one component of constructing patterns and seeing their relationships and is an integral part of mathematical reasoning and work with number. We use spatial sense and spatial language to describe relationships, communicate about position, and give directions for finding locations. We use it when we visualize the relationship of objects and their parts, and when those objects are combined, divided, or moved in space (Del Grande, 1990; Kosslyn, 1983; Wheatley, 1990).

Spatial sense is an elusive notion that defies precise description or comprehensive definition, a characteristic it shares with problem solving. Nevertheless, educators attempt to describe some of its characteristics. It is variously described as intuitions or notions about one's environment and its objects, insights about two and three dimensional shapes, interrelationships of shapes and their properties, and recognition of similarities and differences. It is sometimes expressed as experiences that focus on geometric relationships of direction, orientation, perspective of objects in space, and the relationship of shape and size, how a change in shape relates to change in size, and symmetry. Other aspects or components of spatial sense include spatial visualization, spatial reasoning, spatial perception, visual imagery, and mental rotations (National Council of Teachers of Mathematics,1991; Del Grande, 1990; Wheatley, 1990).

Spatial sense is also conceptualized as self-generated imagery. According to Kosslyn (1983) self-generated imagery consists of construction, re-presentation, and transformation. Constructing an image is more than the mental process of taking a picture. It is influenced by previous knowledge, unique to each person, and can be concrete and restricted or dynamic and abstract. After construction, an image is stored. When a need arises for the image to be re-presented, it may no longer be the same as when previously constructed. Transformation is changing the image, for instance, changing shape or rotation in space.

For young children, examples of spatial sense would include looking at a puzzle and picking appropriate pieces, recognizing that two or more of one shape combine to create a different shape, or recognizing existing patterns and creating new ones.

This project utilized tasks or problems that are geometric in nature and have a strong spatial sense component. Students filled frames using one of two kinds of blocks. Because filling designs with geometric pieces involves many components of spatial sense, these were spatial tasks or problems that children were solving.

The instruction in this project differed from a standard view of teacher-given explanations or direction of students' actions and activities. Rather, the instruction consisted of making the tasks (the frames) available to students. First, students engaged in solving these spatial problems independently, and second, they were asked to discuss their insights and ideas about their work.

Support for this view of instruction comes from a variety of sources. According to the National Association for the Education of Young Children position statement on developmentally appropriate practice, young children learn by being actively engaged in meaningful activities in the context of their experience and development (Bredekamp, 1987). Young children are motivated by a desire to make sense of their world and construct knowledge by adapting new experiences to previous knowledge. They do this through playful interaction with objects and people.

The work of child development teachers and theorists has demonstrated that young children construct knowledge through a complex process of interaction between their thinking and their experiences as they develop and begin to see points of view of others. According to Piaget (1969/1970), not only do young children think in qualitatively different ways than older children and adults, they construct knowledge only through interaction with their environment.

From his research, Piaget (1936/1952, 1937/1954) concluded that action is the source of knowledge and intelligence. As children develop mentally, they gradually free their thoughts from action, and reason abstractly. However, for young children, action is still closely connected to reasoning.

DeVries and Kohlberg (1990) reviewed Piaget's distinction between the two types of psychological experiences. The physical experience, when one acts on an object by touching, pushing, pulling, dropping, squeezing, and so forth, results in physical knowledge about the object. The source of knowledge is in the object. The other psychological experience is logico-mathematical. This is when one assigns characteristics to objects or puts objects into relationships. For example, when one talks about objects being similar, *similar* is not in the objects themselves, instead, the relationship takes place in the mind of the knower. After pointing out these significant distinctions, Piaget noted that, in actuality, these types of experience, action, and knowledge, are indissoluble.

The best way for children to acquire physical and logico-mathematical knowledge is through experimentation with activities that interest them. According to DeVries and Zan (1994), when children are motivated by interest, they exert the effort necessary to reason and make sense out of those activities.

This thinking drives constructivist education and the view that methods for teaching young children must be different than those for older children and adults. Young children require problems that both interest them and elicit reasoning. They need to hypothesize and test answers that make sense to them. If what makes sense to them doesn't work, they experience disequilibrium that causes intraindividual conflict. This internal conflict, causes children to rethink, weigh their conclusions, and form new hypotheses to test. It is the teacher's job to plan new experiences where children will be confronted with their erroneous ideas.

In the constructivist approach, teachers of young children should be companion guides, or facilitators, who create environments within which children learn. These environments should provide stimulating, challenging materials and activities. Teachers observe and stimulate children by asking questions, providing materials, and engaging them in group discussions. Observations provide information about children's levels of understanding and development which, in turn, is used to pose new challenges that stretch children to higher levels of understanding. According to Kamii and DeVries (1978), teachers must keep in mind the importance of fostering an experimental attitude within the community of children and encourage the exchange of observations and ideas.

This same notion is fundamental to the reform in mathematics teaching being advocated by The National Council of Teachers of Mathematics. A major goal of the reform is to help children develop mathematical power, including such skills as the ability to explore, conjecture, reason logically, and justify their thinking; to solve nonroutine problems; to communicate, to develop personal self-confidence and positive dispositions about mathematics.

The Professional Standards for Teaching Mathematics explicitly address the concept of viewing instruction in terms of the teacher acting as a facilitator who creates climate and provides tasks for children working both independently and in small groups. In fact, the mathematics teaching standards are built around the notion of tasks (projects, questions, problems, exercises and activities in which students engage), discourse, (ways of representing, thinking, talking, and agreeing and disagreeing that students and teachers use), environment, (the unique interplay of intellectual, social, and physical characteristics, such as materials and space, that shape the classroom atmosphere). Finally, teachers analyze, by systematically reflecting and monitoring their classrooms in terms of the development of every student's mathematical literacy and power. This closely fits the view of instruction advocated by the National Association for the Education of Young Children. (National Council of Teachers of Mathematics,1991; Bredekamp, 1987)

This vision for content and instruction in mathematics challenges teachers to determine what materials, strategies, and instruction facilitates and support the aforementioned learning environment. In this instructional project, I attempted to provide tasks, opportunities for discourse, and a classroom environment conducive to interaction, interplay and reasoning by using pattern blocks and tangrams in concert with the problems posed by the frames. During the project, I observed how young children interacted with these materials and to what extent mathematical concepts, particularly geometric and spatial reasoning, developed as a result.

METHODOLOGY

The instructional project was divided into two phases. The first phase used pattern blocks and the second, tangrams. The pattern blocks were introduced first because I felt that children's initial experiences with pattern block tasks would enable them to engage successfully in the more complex tangram work.

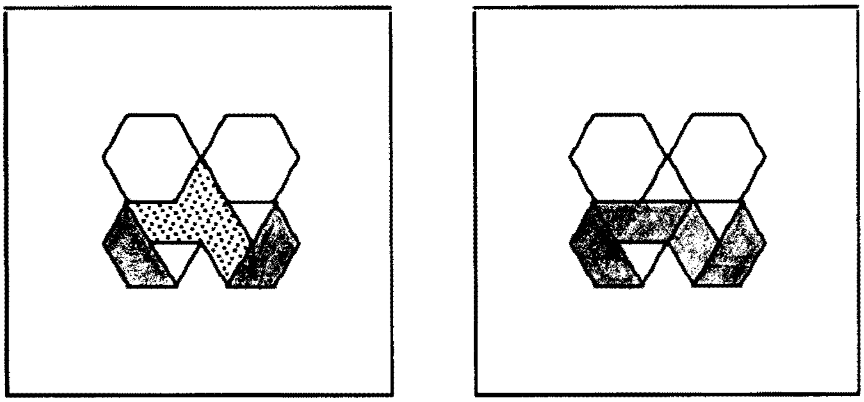
Within each phase of the project, there were three segments. The first segment consisted of placing the materials without the frames on the table in the center of the classroom during the activity period. At various times each day, I sat at the table and made simple designs to encourage children to join in the activity. Three or four children came to investigate and participated for short periods of time. Only one child stayed for more than five minutes. It appeared that the children were not sure what to do with the blocks. Since the blocks alone did not seem to stimulate interest in most children, after only one or two days of exploration, I decided to introduce the frames.

In the second segment of the instructional project, I placed the frames on the table with the pattern blocks or tangrams. I began by introducing only the basic frames. At several points during the project, I added more complex frames when, the children, no longer finding the simple frames challenging, asked for others. The new frames immediately stimulated the children's curiosity and interest.

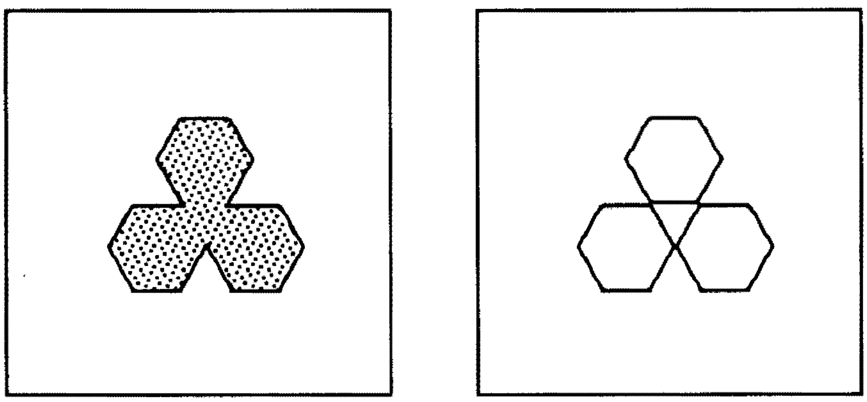
The children generally worked individually, although several often sat together and chatted while they worked. Sometimes when they asked for help and sometimes to probe their thinking, I participated in the activity. Occasionally, the children took part in small and whole group discussions. I encouraged all the children to experiment with the blocks and frames. As they explored and solved the tasks, I watched for appropriate times to interject questions. There were two types, those that attempted to identify what and how children were thinking, and those that were intended to foster children's thinking and reflection.

I observed and recorded children's reactions and learning, their language, and any other creative activity in which they engaged. This was done by keeping a teaching journal and by video taping each session in which the materials were used. I considered this the ongoing assessment phase of the work.

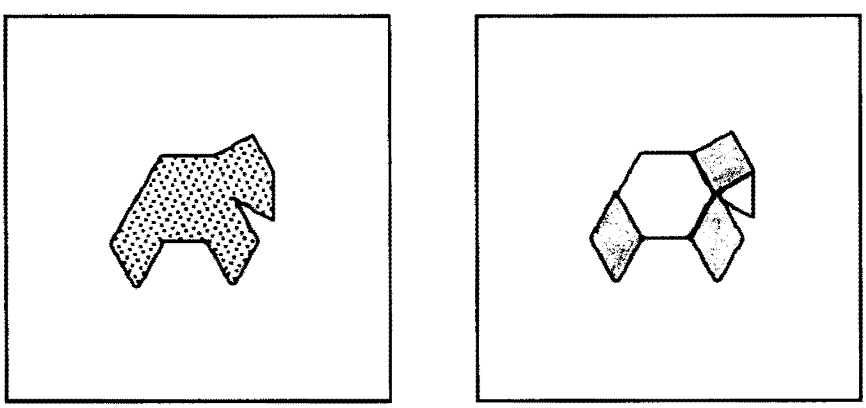
After all of the children had an opportunity to participate in the tasks, I implemented a short summative assessment by conducting small and large group discussions. During these discussions, I attempted to stimulate reflective thinking by posing questions designed to help the children analyze and synthesize information. Next, I introduced new frames. For the pattern block phase, I presented one partially filled frame, leaving a space for the placement of three or more blocks, depending on how the children chose to fill it. The second was a basic frame and the third was a more complex frame. The tangram assessment involved only two frames, one basic and one complex, see figures 17 and 18. I observed and documented the students' work to assess its value by paying particular attention to the reasoning taking place, and the knowledge children had constructed.



Partially Filled Frame and One Solution

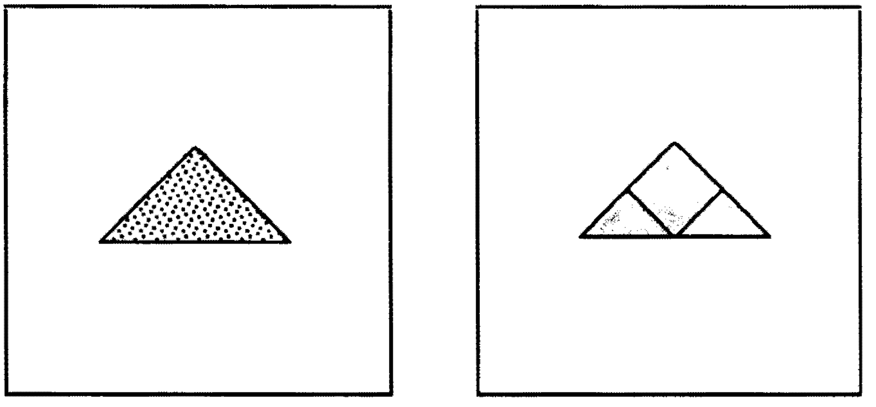


Empty Basic Frame and One Solution

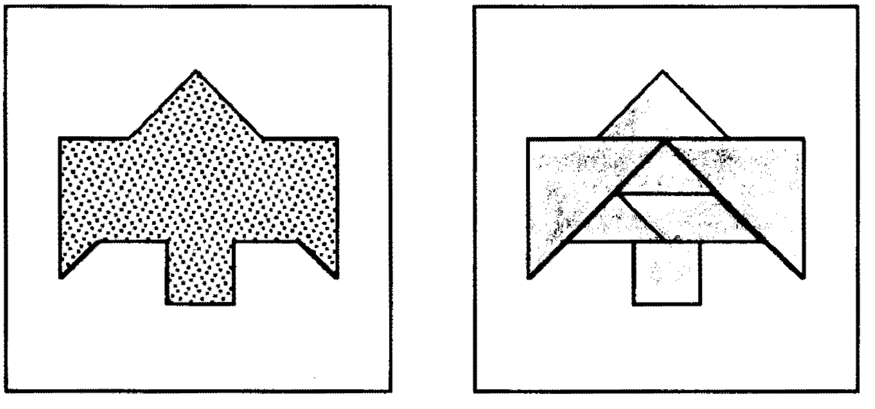


Empty Advanced Frame and One Solution

Fig. 17. Pattern Block Assessment Tasks



Empty Basic Frame and One Solution



Empty Advanced Frame and Solution

Fig. 18. Tangram Assessment Tasks

PROJECT

This instructional project was carried out with four and five year old children in two half-day prekindergarten programs in a small rural Midwestern community, in which I was the teacher. The prekindergarten met three times a week during the 1993-1994 school year. The children ranged in age from four years five months to five years six months. The instructional project began at the end of January 1994, and continued at intervals until the beginning of June 1994.

The instructional project consisted of two phases, first pattern blocks, and then tangrams. Each phase had three segments. The first segment involved exploration of the materials without the frames. I made this a relatively brief time as children did not seem to need much time become acquainted with the materials. The second phase involved presenting problem solving tasks along with the pattern blocks and tangrams and continued for approximately six to seven weeks. I added new tasks as the children asked for more problems to solve. Third, there was a short summative assessment phase. During this phase, I asked the children to show me what they had learned.

In June, we had a final group discussion during which the children had an opportunity to state their preferences and discuss the similarities and differences between the pattern blocks and the tangrams.

Prior to this project, I had had extensive experience using pattern blocks and tangrams with children in my prekindergarten classroom. These earlier experiences indicated that the materials appeared to engage children's interest, and persistence. Thus, this pilot work provided abundant evidence that these tasks were reasonable, appropriate, and children engaged in them productively.

Data were collected during all aspects of the instructional project in the following ways: video tapes and photographs of the children were made while they were involved in the problem solving tasks, anecdotal records and journal notes were recorded, and third, a brief oral assessment I conducted with the children at the end of each of the two major phases of the project. These have been reviewed and were the basis for identifying the key points that I will discuss under each of the questions.

FINDINGS

Pattern Block Findings

The children's work with pattern blocks extended over approximately a two month period. I was interested in discovering the degree to which young children would become engaged with these materials, and to which they would construct intuitions and knowledge about space, shape, and their relationships. To guide my inquiry, I addressed three questions. Did the tasks or problems appear to be appropriate and interesting? Did the children engage purposefully in the tasks? Did the children demonstrate spatial sense and an understanding of spatial relationships and mathematical reasoning?

Did the tasks or problems appear to be appropriate and interesting?

As one would expect, there was a wide range of engagement by the children. However, all of the preschool children chose to engage in problem solving tasks more than once and were successful in completing several tasks. At least half the children solved the more advanced frames, which were truly complex tasks.

Even children whose participation tended to be relatively brief seemed to enjoy and be actively engaged in the problem solving tasks. Clay is an example of this type of child. In one six-minute period, he began with an advance frame that required the use of squares and tan parallelograms and was unable to complete it. Next, Clay chose an easier frame which he filled quickly. He lifted the frame and filled it a second time, using a different solution. He proceeded to construct five different solutions and seemed to enjoy the challenge of creating multiple solutions to a problem. I found it interesting that he was not discouraged by his first task and seemed to know that there were others he could solve. He was very involved and stimulated enough that his discouraging experience did not prevent him from continuing to solve the problems.

Several children spent a great deal of time working on the problems, and they chose to work with them almost every school day during February and March. On occasion, they would spend 20 to 30 minutes with the pattern block tasks. The advanced frames took most children about four minutes to solve, while basic frames usually took less than a minute. Sometimes they did the same frame more than once, trying, like Clay, to find additional solutions. Other times, they worked with many different frames. For example, Krista spent 32 minutes one day filling almost every frame, some of them two or three times.

Elizabeth was perhaps the strongest example of a student who found these geometric problem solving tasks appealing and challenging. She spent at least part of every day working with the pattern blocks and frames, frequently for 25 to 40 minutes. She often told me proudly, "There's nothing I can't do." Once she worked 45 minutes trying to fill a large and very difficult frame, quitting only because activity time was over. She saved her work and continued during the next session for another 25 minutes until she completed the task.

Children were always pleased and proud of their accomplishments. They demonstrated their pleasure in a variety of ways. Sometimes they wiggled their bodies and smiled at their creations, sometimes they merely looked at other children or me and smiled, while at other times they pointed to their finished work and made comments like, "Teacher, teacher, lookit here! Look what I made!" or, "Hey, look at this. Do you like my design?" or simply, "Ta Da!"

I believe the children wanted and enjoyed challenge. For example, after using the first set of basic frames for several days, they asked if I had other frames. I added new frames on three different occasions. Each time, the new frames were at least as complex as the previous frames. Throughout the project, I noted that adding new frames and new complexity stimulated renewed interest in the tasks by the children. Some children chose to work only on the new frames.

The amount of time most children spent on the problem solving tasks, the fact that all of them completed some frames, their interest in challenging tasks, and their demonstrations of feelings of accomplishment suggest that the problems were appropriate and interesting to them.

Did the children engage purposefully in the tasks?

I noted several aspects of purposeful engagement in the children's activity, as the above examples suggest. Most of the children appeared to take ownership of the problems by setting goals for themselves. They seemed to care very much that they were able to solve the problem. Occasionally, I helped them set goals by posing problems that they chose to act upon. In the following three examples, both types of goal setting occurred.

In the first example, Glenn was working to find solutions to the triangular-shaped frame. He first solved the problem by using only triangles, as shown in figure 15 (p. 15). When I asked him if he could tell me about his work, he announced that he had made a triangle out of triangles. It was clear that he deliberately chose to use only triangles. He then set another goal and exhibited purposefulness by repeating the task, this time using the red trapezoids to fill the triangular-shaped frame.

Glenn next chose a more advanced frame. At one point he picked up a blue parallelogram. He looked at the block and at the design, and then dropped the block back into the box. When he had completed the task, I asked him how he knew which pieces to use. His response was, "I looked at the shape," When I asked if he could do it another way, he nodded and very deliberately selected tan parallelograms, squares, and triangles to use.

As I reflect on Glenn's work, it does not seem that his goal was as methodical with the advanced frame as it had been initially. Rather than having a preconceived plan, his goal was to fill the frame any way he could. In searching for the particular pieces he had in mind, he would focus on each angle of the design and seemed to know precisely where he would place each piece. He worked deliberately, making sure that each block fit a specific place before he selected another one. Only once was he unable to insert a block that he selected. He was absorbed with this problem for 32 minutes, an indication of the deliberate and focused way he selected blocks. Finally, his decision to try the same task again, suggests that he chose the more complex goal of finding another solution.

The second example involves Krista, who chose the same triangular-shaped frame as Glenn saying, "This is easy." After successfully filling the frame several different ways, she spent approximately five minutes experimenting with tan parallelograms, moving them back and forth, trying to find a way to make them fit. When she was unable to do so, she removed the parallelograms and filled the frame with a hexagon and three triangles, as shown in figure 10 (p. 10). After 28 minutes, Krista had filled almost all the available frames, announced she was finished, and left.

I noticed that Krista, like Glenn, set goals for herself. Not only did she work purposefully, but it is obvious from her comment, that she was confident in her ability to accomplish tasks she set for herself. Her experimenting with tan parallelograms suggests that she was curious, willing to take a risk, and persistent. She not only tried something she wasn't sure would work, but spent several minutes experimenting and trying different ways of placing the blocks before deciding it was not possible to make them fit.

Finally, Samantha was so confident of her abilities and interested in the activity that she needed no encouragement from me to set goals for herself. She chose seven frames and placed them in a pile in front of her. She said, "I'm going to do all of these," and began filling one of the frames. When she had only a small space left, she selected a trapezoid, which was too big to fit, and held it above the space. She turned it around, compared the trapezoid with the space, and then dropped it back into the box. She then selected two triangles and placed them into the empty space.

I believe that this example suggests that Samantha used her spatial sense, including knowledge about orientation, shape, and space. One possible interpretation of her decision to use two triangles in the last empty space is that she understood that three triangles are the same as one trapezoid. Since the trapezoid was too large for the empty space, she may have reasoned that two triangles might fit.

Many of the children were very persistent. Krista did not quit until she had tried the tan parallelograms in every possible way. Clay was persistent when he spent 43 minutes creating many solutions to two small frames. He stayed with the task he had created for himself until he had exhausted every solution he could devise. Not only was he persistent, he was deeply engaged in thought. During the entire problem solving task, he was analyzing and comparing the solutions he had already found with the new and different ones he was trying to create.

When finished, Clay felt a great sense of accomplishment. Because he wanted to save his designs, we took a photograph. When he realized that the photograph did not come out of the camera and he would have to wait to see the picture, he insisted that we save his work. We placed all the designs on a board and stored them in a safe place on a high shelf. I knew he was proud of himself, but I had no idea how deep this feeling was until the next day, when he brought his mother to school to see his designs.

These examples were representative of the children's ability to take ownership of the problem solving tasks and set goals for themselves. They indicate that the children engaged purposefully in the tasks.

Did the children demonstrate spatial sense and an understanding of spatial relationships and mathematical reasoning?

As the above examples indicate, children also engage in reasoning about mathematical and spatial concepts. I will elaborate on their understanding and reasoning in this section.

Children frequently formed and tested hypotheses as they attempted to solve the problems. I often heard them making statements such as, "I'll make it a different way, all green," or, "I know, I'll use all diamonds." Krista proposed a solution involving the tan parallelograms. She wondered if something she hadn't tried would work, tested her guess, and found that it didn't. Samantha was hypothesizing when she held the trapezoid above the empty space and compared the shape with the space.

Many children looked at the space they were planning to fill before they chose a piece. They made a decision about which piece they wanted and searched for that specific piece. Depending on the child and the complexity of the frame, some children, like Glenn, were able to fill almost an entire frame using every piece they selected. Occasionally, their choices were wrong, and they had to remove the pieces from the frames and start over. Sometimes children chose a piece and turned it around to look at it from different perspectives, as Samantha did.

Some children seemed to be able to predict what the finished product would look like. After Jami filled a large parallelogram-shaped frame with several different shapes, as shown in figure 19, I asked her if she could make it all one color.

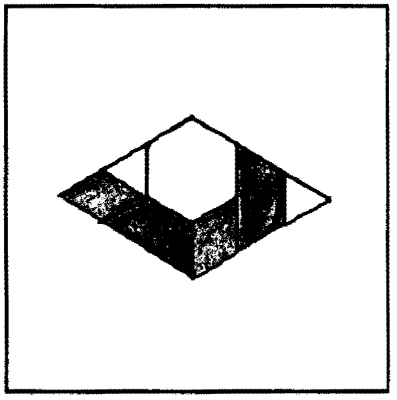


Fig. 19. Jami's First Solution to the Parallelogram-Shaped Frame

She lifted the frame from her first solution, placed a hexagon inside, and then added a trapezoid as figure 20 shows.

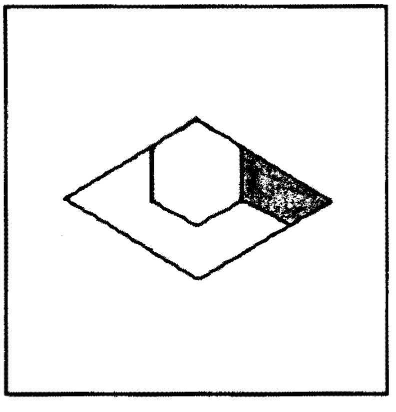


Fig. 20. Hexagon and Trapezoid in Parallelogram-Shaped Frame

"Can you make it all yellow?" I asked, whereupon she looked at the hexagon and at the parallelogram she had already finished, and shook her head no. I asked her if she could make it all red and she responded by taking the hexagon out of the frame and filling it with trapezoids. As she added the trapezoids, she turned them different directions to make them fit as shown in figure 21.

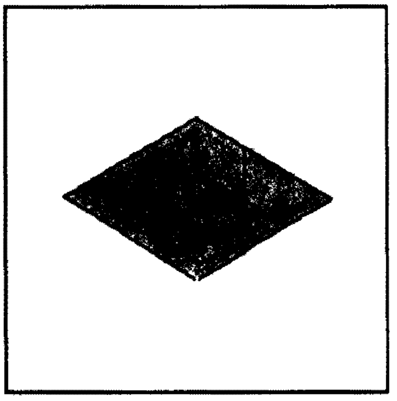


Fig. 21. Jami's Red Solution to the Parallelogram-Shaped Frame

When she finished, I asked her if she could make it a different color. "Yup," she said, and filled the frame with blue parallelograms (see figure 22). Elizabeth, who was sitting next to Jami filling frames said, "A diamond out of a diamond." Again, I asked her if she could make the parallelogram-shaped frame using blocks of another color.

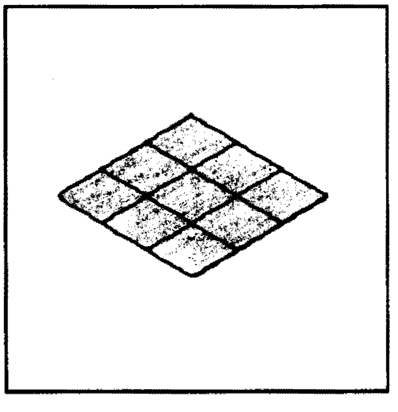


Fig. 22. Jami's Blue Solution to the Parallelogram-Shaped Frame

She selected another hexagon, inserted it in the frame, pondered it, and tossed it back. Finally, she chose several triangles and solved the problem by making it all green as shown in figure 23.

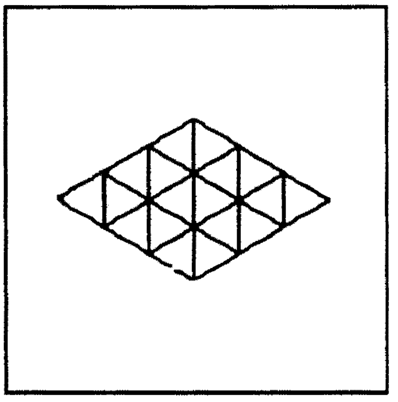


Fig. 23. Jami's Green Solution to the Parallelogram-Shaped Frame

Later, she chose an arrow-shaped frame saying, "I'm going to make it all one color," and began by placing a hexagon in the arrow-shaped frame (figure 24).

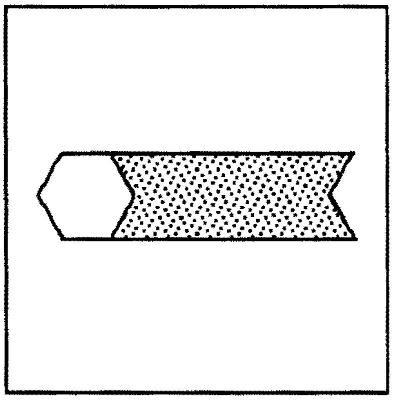


Fig. 24. Hexagon in the Arrow-Shaped Frame

She looked at the hexagon, then returned it to the box, and said, "Maybe I'll (pause) Ah Ha!" She then filled it with trapezoids. When she was about half finished, I said, "Is that going to work? Are you going to be able to make it all red?" Jami nodded and finished her design (see figure 25).

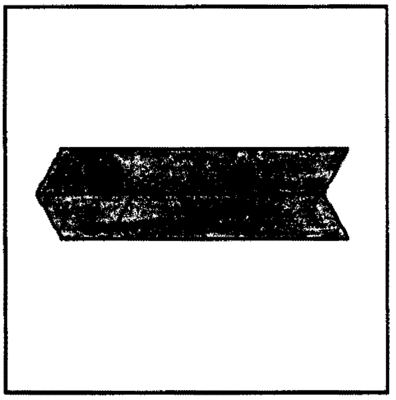


Fig. 25. Jami's Red Solution to the Arrow-Shaped Frame

Not only was Jami setting goals for herself, she was also predicting and reasoning about the problem. Apparently, when she looked at the yellow hexagon, inside the parallelogram, she was able to visualize and predict what would happen if she added more hexagons. Comparing it to the solution she had created earlier, she apparently could determine that the hexagon had no angles to fit the angles of the parallelogram, while hypothesizing that the trapezoid did. To make the trapezoids fit, she set them in at different angles. To do this, she had to think about the shape of the trapezoid in relation to the space available and be able to rotate the trapezoids to fit in them.

When she chose to make the parallelogram-shaped frame with the trapezoids, she made no comment, but simply went ahead and tried it. She may not have been sure that she could make it all red, but to find out, she had to test her hypothesis (see figure 21, p. 35). She was much more confident when she started filling the parallelogram-shaped frame with blue parallelogram. Perhaps it was because she saw the relationship between the small blue parallelograms and the large parallelogram-shaped frame with which she was working (see figure 22, p. 35). We know that Elizabeth saw the relationship, because she announced, "A diamond out of a diamond."

Later, Jami checked again to see if the hexagon would fit within the parallelogram-shaped frame. She concluded, as she had before, that it would not, and proceeded to use triangles. This indicated that she saw the relationship between the triangles and the parallelogram-shaped frame as shown in figure 23 (p. 36).

Jami was also considering similarities and differences among the pieces. She searched for like pieces and learned that the trapezoids, the blue parallelograms, and the triangles combined to create a large parallelogram, while the hexagons could not.

One could almost "see" her mind reasoning when she selected the arrow-shaped frame. First, she set a goal for herself, to make it all one color, and then inserted a hexagon and looked at it (see figure 24, p. 36). I believe she was thinking about how many more hexagons would fit into the arrow. Perhaps she imagined them sitting next to each other and determined that they would leave triangular spaces. She may have reasoned that no hexagon would fit into the end, because the end was concave rather than convex.

Upon removing the hexagon from the arrow-shaped frame, she apparently thought about the shapes that would fit together to fill the arrow. She may have been seeing the relationship among the pattern block pieces, the shape of the frame, and the pattern the blocks made as they fit together. She also may have imagined rotating the pieces in space to see how they would fill the frame. She must have even visualized how they would fit together to complete the design before she finished, because when she was only half finished, I asked her if what she was doing would work, and she knew that it would (see figure 25, p. 37).

In the following examples, children appear to be demonstrating other mathematical reasoning that involves an understanding of number, an awareness of similarities and differences, an awareness of the relationships among the blocks, and knowledge of the geometric names of shapes.

Many children knew the geometric names for some of the shapes. I noted that Ethan, although he had not yet learned the color names, knew the geometric names for the triangle and square. Some children continued to refer to the pieces by color even when they knew the geometric names. For example, when Emily initially needed a triangle for her frame she said, "I need one more green," but later in the day, when Lance asked her for some triangles, she picked out a handful and gave them to him. Vince was close to completing a frame, but was unable to find the final piece. He pointed to the space left in his pattern and said to Lance, "Lance, give me one of them little triangle ones."

Children often made comments indicating their understanding of number. As they described their work to me, they made statements such as, "There, I only need one more (piece)," "Teacher, teacher, look, one, two, three, four, five, six, seven," " I used just one red, and four blues," and "Five there, and two there."

As Stacy lifted the frames off several designs and said, "I'm making a hundred designs." Kory filled a frame with blue parallelograms, looked across the table at Vince and said, "Vinnie, you need all green, I need all blue." Then filled another frame, lifted it, and replicated his design two more times. While he was working, Stacy chose an identical frame and copied Kory's design. When she finished she said excitedly, "Kory, look. Kory, we match!" She pointed back and forth to the two designs and said, "Match, match." Kory replied, "Yup, I made three and you made two." Stacy responded, "Count them. One, two, three, four, five."

Stacy was excited about solving the frames and wanted to solve many tasks. Even though she obviously had not yet constructed the actual meaning of a *hundred*, she obviously equated it with many solutions. However, both she and Kory demonstrated an understanding of number and one-to-one correspondence as they discussed the number of solutions each had devised.

Stacy was comparing and analyzing when she replicated Kory's design and showed it to him. He, too, was comparing and analyzing when he recognized that Vince was filling a frame using a different color. It seemed that he also noticed that he and Vince were creating similar solutions and that they were both using only one color. Kory employed this process when he confirmed Stacy's observation that their designs matched.

I became particularly interested in Elizabeth's work because she spent more time on the tasks than any other child. In addition, she provided fascinating insights into her thinking. She quickly constructed an understanding of the relationships among the blocks and their substitution properties. My impression was that she had a better understanding of these relationships than the other children. To my surprise, as I studied the video tapes of the project, it became clear that many of the children had constructed the same knowledge. Elizabeth was just more verbal about it. Therefore, I present these examples, not solely to describe what Elizabeth understood, but as representative of the understandings of many of the children.

The first example begins as Elizabeth, upon finishing a frame, said, "Look what I am doing, two yellows, two reds, two blues, and two greens. I'll make it a different way, all green." After placing six triangles in the frame, she folded her arms, looked up and said, "See I did this, just like the yellow." She selected a yellow hexagon and placed it upon the green hexagon, to confirm they were the same shape. She added more green triangles to her design and said, "Lookit, I made two just like the yellow!" She continued working and a few minutes later said, "I made a blue just like a yellow." Later still, she said, "I only used two reds like this to make a yellow, Mrs. Sales, lookit!" Vince, who was working next to Elizabeth, said, "Look at me! I made a diamond green one." I looked at the pattern blocks in front of him and said, "You made a green diamond just like a blue one?"

He smiled and nodded (see figure 2, p. 5).

Elizabeth set a goal for herself when she decided to make her frame all green, and when she placed the six triangles together, she saw the relationship between the triangles and a hexagon. To confirm her supposition, she placed a yellow hexagon on top of the green hexagon to see if her prediction was really true. Once she began to reason about the relationships among the blocks, she soon established that three blue parallelograms or two trapezoids fit together to replicate a yellow hexagon. Vince ascertained the relationship between two triangles and one blue parallelogram. Both children immediately began to substitute one set of blocks for another as they filled the frames.

When Elizabeth had completed a duodecagon, as shown in figure 26, I saw her place her hand across the center of her design and look on both sides.

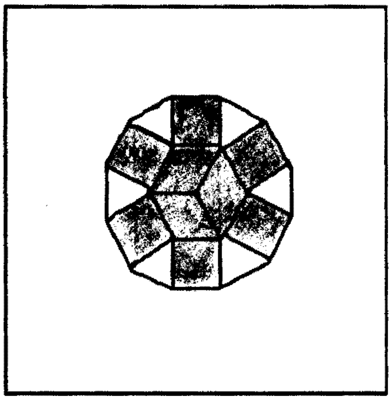


Fig. 26. Elizabeth's Solution to the Duodecagon-Shaped Frame

When I asked her why she did that she told me it was so she could see how many greens she had. "How did you do that?" I asked. She placed her hand across the center of the design and said, as she looked on both sides, "There are three greens on both sides and three and three make six." Then she whispered in my ear, "There are six oranges, too, and three blue ones." Curious to understand the strategy she had used, I asked her to explain again. She placed her fingers on the two orange squares across from one another and told me there were three on each side and that, too, made six. Then she touched the center of the design where she had made a blue hexagon and said, "And three blues."

Elizabeth was reasoning with numbers. I believe that she possessed previous knowledge about the number combination, $3 + 3 = 6$. When she placed her hand across the design and divided it into two halves, she was able to prove what she suspected was true. As I watched Elizabeth place her hand across her design, I assumed that she was contemplating the symmetry and expected her to say that it looked the same on both sides, but was surprised when she told me she was adding the triangles together. Yet, she really was looking at the symmetry. She saw that the design was the same on both sides of her hand and knew it had the same number of blocks on each side of the dividing line.

The children devised various strategies for solving the pattern block frame tasks. When they worked on small frames, they generally started at the top and worked down, or at one side and worked across. Occasionally, when using the hexagons, they began in the middle and worked toward the edges, but only with small designs. When children worked with larger frames, they often started at the outer edges and worked toward the center. Children, truly engrossed in the problem solving tasks, often explored several possible strategies.

On occasion, children became frustrated while working with very advanced frames. Their problem was often caused by thinking about only the four most closely interrelated shapes (the triangle, the blue parallelogram, the trapezoid, and the hexagon). This may have been because the initial frames only required those shapes. Consequently, the children often tended to exclude the square and the tan parallelogram from their thinking. Usually, they simply needed to be reminded that there were six shapes, not just four, with which to work. I accomplished this by asking questions and engaging them in discussions designed to help them focus on the type of angles requiring the use of the squares and tan parallelograms.

This happened to Stacy when she chose a very advanced frame and filled the edges, leaving a square in the center. To fill the square, she systematically and patiently tried every pattern block shape except the hexagon and the square. After four minutes of moving pieces back and forth, she walked away. Four days later she selected the same frame and employing the same strategy, again filled the edges and left a square in the center. I suggested she look at one corner of the center square and try to find a block that would fill it. She found a square, inserted it in the corner of the design, looked up, smiled, and completed the frame using three more squares. Her success encouraged her to choose another frame and begin again, in contrast to her initial experience four days earlier.

In this example, Stacy acted upon the objects by inserting, moving, and exchanging pieces. She employed previous knowledge and pondered the relationships among the shapes and the empty spaces. Later, she incorporated new knowledge, the awareness of all six shapes rather than four, into solving the more complex task.

The evidence above leads to the conclusion that the children did, indeed, demonstrate an understanding of spatial sense, shapes, and other mathematical reasoning.

Summative Assessment

I began the summative assessment phase of the pattern block work with a class discussion. The discussion lasted for approximately fourteen minutes and involved fifteen of the eighteen children. The three children who did not participate in the group discussion listened attentively.

I set the box of pattern blocks on the floor and asked the children what they knew about them. They said there were six different shapes and identified them by color name. Most children knew the geometric names of the triangle and the square, but referred to parallelograms as diamonds. One child distinguished between the two parallelograms by referring to the tan parallelogram a diamond and the blue parallelogram a big diamond. When I held up a trapezoid and a hexagon, and no one commented, I told them the geometric names. Vince said, "You can make hexagons, like this," holding up a yellow hexagon. I asked him to show me how, and he selected two trapezoids and made a replica of the yellow hexagon. Elizabeth told us she used one green, one red, and one blue. Katie said she could make a blue one and did so. Krista held up two triangles together and said, "See." When I asked her what she made, she held up a blue parallelogram, and then showed us a trapezoid she had made out of one blue parallelogram and one triangle. Samantha made a hexagon by placing six triangles together. As she held up a square, Jami said, "Mrs. Sales, there's no way you can do this." Emilie held up a blue hexagon she had made, and Ethan showed us how to make a tree by placing trapezoids on top of one another. I look on the floor and saw that Katie had made a large square out of four small squares.

It appeared from this discussion that some children, used the pattern blocks as symbols representing pictures they visualized. Ethan made a design and pretended it was a tree. His use of the blocks reminded me that children engage with materials in multiple ways. They enjoyed making interesting abstract pictures, as well as using them to solve problems and construct knowledge. Many of the children constructed knowledge about the similarities and differences between and among shapes. Most were able to identify, a triangle, a square, a parallelogram or diamond, a trapezoid, and a hexagon, even though they did not yet label them. They were interested enough in this activity to expend effort in reasoning about the problems. Finally, they understood that there was more than one way to solve a problem.

Next I interviewed each child in both classes individually. Of the 18 children in my classroom, I have detailed records for 15. One child did not wish to participate in the assessment, and I have incomplete records for the other two.

During the assessment, I presented the children with three new pattern block frames (see figure 17, p. 18). The first frame was partially filled so that they could use as few as three blocks or as many as seven, depending on the pieces they chose. The second frame, completely empty, was a fairly easy basic frame and required use of only the four closely interrelated shapes. The third frame was small but more complicated. It dictated the use of the square and the tan parallelogram in conjunction with the other four shapes. Along with presenting the frames to the children, I asked them how many ways they could make a hexagon.

All fifteen children I assessed were able to fill the three frames. Two children had some difficulty with the advanced frame, but after I suggested they look at the corners, they were able to complete it. All children could make a hexagon two ways, and seven could make it four or more of the seven possible ways.

During the summative assessment, I noted one event of particular interest. After Elizabeth demonstrated that she could replicate the yellow hexagons in all seven possible ways, I handed her the second assessment frame (see figure 16). I asked her what was the largest piece she could fit into the design. She selected a yellow hexagon. I said, "If you use hexagons, how many pieces will it take to fill it?" She didn't answer so I suggested she try it. After placing only one hexagon, she touched the center and said, "It's going to have a triangle." She finished the frame by adding two more hexagons and a triangle. When one places two hexagons together, a V shape forms between them. After adding two hexagons, several children recognized that a triangle would fit into the center, but a single hexagon provides no such hint.

Tangram Findings

The children's experiences with tangrams paralleled their experiences with pattern blocks. There was approximately the same level of engagement, the children were curious, persistent, looked for a challenge, and were pleased and proud of themselves when they completed the tasks. They assumed ownership of the problems and set goals for themselves. This section will focus on the complexities and questions arising from the additional demands the tangrams made on the children's spatial perceptions and reasoning capabilities. As previously discussed, the tangrams are a more perceptually complex set of materials, and solutions to the problem solving tasks are not as clear as those with the pattern blocks. The same three questions will be used to discuss insights related to the tangram materials.

Did the tasks or problems appear to be appropriate and interesting?

A few interesting differences occurred concerning the children's involvement in the tangram tasks. Three of the 18 children, for example, chose not to participate in this work, and one child who spent little time with pattern blocks became very involved with the tangram problems.

Interested children spent approximately 20 to 30 minutes each session working with the tangram tasks, but spent about seven minutes on individual advanced frames. On one occasion, two children spent an entire 50 minute activity period solving tangram problems. All children who participated were successful in completing several basic frames.

Most students initially found the tangrams and accompanying frames more difficult. I frequently heard such comments as, "I can't figure it out," and "What can fit here?" When I heard such comments, I tried to direct their attention to specific features of the tasks, such as angles, by asking such questions as, "What will fit in that space over there?" or, "Look at the bottom of your frame. Can you fill that space?" Sometimes I suggested that they take their pieces out and begin again, or that they use the largest pieces first.

As I reviewed my journal notes and the video tapes, it became obvious that I was providing more suggestions and specific hints, as well as asking more questions than I had intended. This may have been due to my excitement about the project and my desire to have the children enjoy and work with the materials. In addition, with the end of the school year approaching, I wanted the children to exit the project feeling successful.

This situation posed a dilemma for me. On one hand, the clues helped the children find solutions to the problems, but on the other hand, I believe that children learn best when they construct their own knowledge by working through problems on their own. As a result of my reflection, I worked at being less directive in my comments and questions. However, I continue to struggle with how I might have altered the instructional process. For example, I might have held group discussions at the end of each day with interested children, and allowed children to share their knowledge.

I focus on this particular issue because I believe deciding when and how to provide guidance is central to all constructivist teaching. Since all teachers guide children in some way, such as, pulling things together or engaging children in reflective thought on their actions, the key question is, how do educators best provide guidance? My project provided me an opportunity to struggle with these instructional components of constructivist teaching.

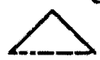
As the children became more familiar with the tangrams, they began to feel more competent and note relationships among the pieces. This happened fairly quickly, possibly because of their previous work with pattern blocks.

<u>Did the children engage purposefully in the task</u>?

Because of the characteristics of the tangram pieces, the children set different goals than they set for pattern blocks. Unlike pattern blocks tasks where children could choose to fill a frame with all one shape, or numerous combinations using more than one of each shape, goals for the tangram frames could only be derived by deciding how many or which of the seven tangram pieces could be used to fill a frame. Children could choose to fill a frame several different ways by selecting different combinations from the seven tangram pieces, as can be seen in figure 32 (p. 55).

In one situation, Emilie was stuck and asked for help with her problem. Her goal was to fill the frame but she was having difficulty making the pieces fit. I suggested she remove all the pieces and try using the large pieces first. She turned them around and around and pushed them back and forth until she made them fit. Once she inserted the larger pieces, she could see where to place the remaining smaller pieces and quickly filled the frame. She chose another frame, and again tried several small pieces, removed them, and selected a large triangle. She turned the triangle piece around considering where it would fit, suddenly she seemed to see, and quickly completed the frame. She said, "Lookit, Mrs. Sales, I'm gonna do it again," and began another frame.

Emilie was intent on filling each of the frames she had chosen. Initially, she appeared to use simple trial and error, moving pieces back and forth and around to see if she could find any way to slip them into the frame. After working for a time, she began to turn the pieces, and analyze them more closely. Apparently, she was examining the pieces to see how they would fit, rather than relying on random placement of pieces.

Many of the other children went through a similar process. Their first approach was random trial and error, almost depending on luck. Later, they began to be more purposeful by examining the pieces more closely and relating them to the spaces remaining in the frames. They began selecting pieces, rotating them, and flipping them over to obtain different perspectives of the shapes. This makes sense, because the same piece looks quite different when seen from diverse perspectives. As can be observed, the same triangle in this position  looks quite different than it does in this position  .

When I noted that children were becoming discouraged, I provided encouragement by reminding them this was hard work, and that they needed to be patient. Sometimes a question I asked helped them think about their problem in a new way. When Vince was frustrated trying to fill his frame with the five small pieces I said, "If you put your parallelogram in like that, can you find a place for the square?" The question appeared to help him see his problem more clearly or from a different perspective, because he located where to place the square and finished quickly saying, "What did I tell ya, easy, easy, easy."

On one occasion, when I reminded the children that sometimes they might need to start over several times, Emilie said, "This is hard work, I'll have to start over 50 times." When tasks are beyond children's capabilities, they feel defeated and quit. However, there appears to be a fine line here. When a task makes sense to them, and they believe that their efforts have the possibility of leading to success, they will attempt challenging tasks. Emilie did not feel discouraged or defeated when she made that comment. She was light hearted and involved with her work. Buoyed by her earlier successes, I believe she felt challenged and inspired by the difficulty of the task. Recognizing it was hard work, nevertheless, she believed she was equal to the challenge and persisted.

<u>Did the children demonstrate spatial sense and an understanding of spatial relationships and mathematical reasoning</u>?

Eventually the children began to see both the relationships among the pieces and the relationships between the empty spaces and the pieces they had to fill them. This was particularly true of Emilie. Having just completed a frame that she found fairly simple, she chose a more complex frame, saying, "I'll bet this is easy, too." She found, however, that her last piece did not match the remaining space. She removed the pieces, saying to herself, "Take that all apart." She began again by inserting a triangle along the bottom edge. She could easily determine from the remaining empty spaces within the frame where the unused pieces belonged. As she put in the last pieces, she began to wiggle and bounce, looked up, and smiled. She chose another frame and started to insert the pieces, stopped, looked at it, and said, "Oh yeah, put the largest pieces in first," then easily filled the frame. Emily had, I believe, created a new strategy that had not been necessary when using pattern blocks, that is, using the large pieces first.

Emilie verbalized other concepts she had constructed during the tangram phase of the project. For example, I heard her say, "I made a square out of two triangles, and I needed a square." Later, she made a parallelogram shape using the two triangles. She placed the first triangle into the parallelogram and said, "Mrs. Sales, watch, one-half," and then dropped in the second triangle. Emilie understood that each triangle was half of the parallelogram.

I noted many other instances of children constructing similar relationships. For example, Krista made a large triangle out of two smaller ones. Samantha filled the same triangular frame using two, four, and then five pieces. After each solution she reported how many pieces she had used. Other children also demonstrated an understanding of number, as well as an understanding of relationships that permitted one or more pieces to be substituted for other combinations of pieces, or larger pieces to be constructed from smaller pieces. These relationships and the ability to create multiple solutions for the same frame, although this was more difficult with the tangram pieces, are similar to the relationships the children constructed when working with the pattern block problems solving tasks.

In the children's minds, the work with tangrams may have simply been an extension of the work with pattern blocks. It is noteworthy that the more they worked with geometric shapes, the more naturally they began to use the traditional mathematical names. During the tangram phase, one even noted instances of the use of the word, "parallelogram." For example, as Jami, set each tangram piece into the frame, she said, "Square. Parallelogram. Triangle."

In the following two examples, Katie and Elizabeth employed previous learning to help with their tasks. Katie demonstrated recognition of numerals as she reasoned to solve her problem. She selected an abstract frame and mentioned that it looked like a two, and, after turning it around, she decided maybe a five, or a six, or a seven (see figure 27).

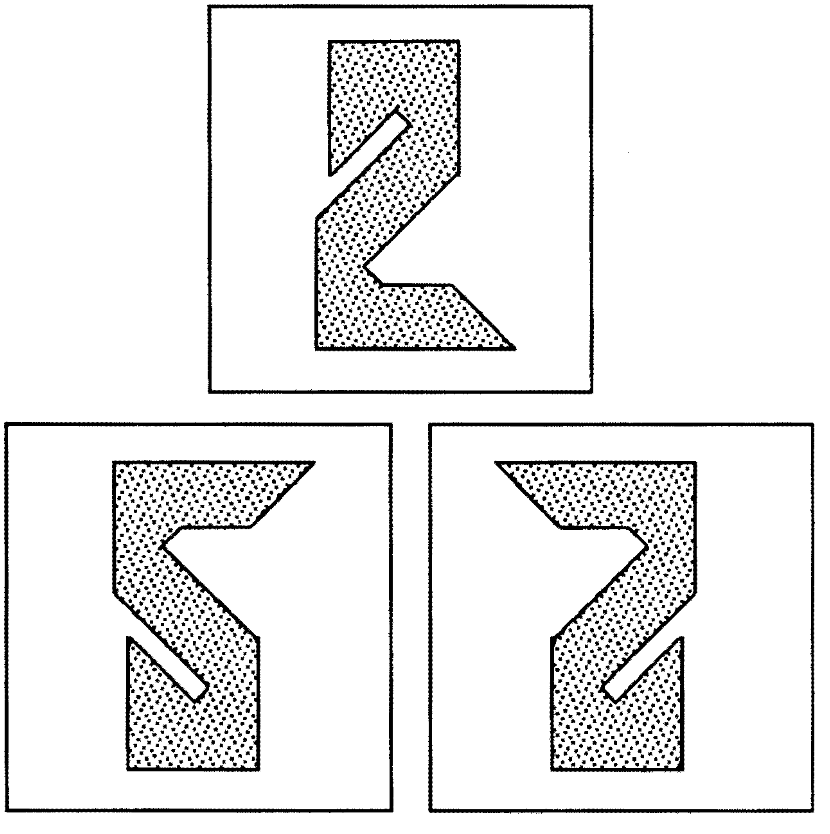


Fig.27. Katie's Imaginary Numbers

She then chose a small triangular frame to work on, and tried to insert the two smallest triangle pieces. After several attempts, she successfully turned the pieces so that they would fill the frame. Next, she chose the large triangular frame similar to the one she had just finished and selected one of the two large triangle pieces. She placed the piece into the frame several ways without solving the problem. Finally, she pointed to the small triangular frame she had just completed and said, "Hum, have to think about how I did this one," and easily replicated her earlier solution. When Katie went from the small to the large triangle, she saw that her earlier work would be helpful in solving of a new problem (see figures 28, 29, and 30).

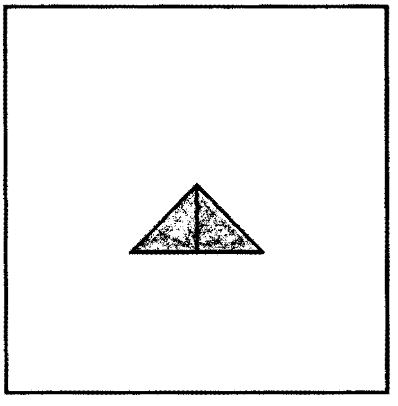


Fig. 28.Katie's Solution to the Small Triangle

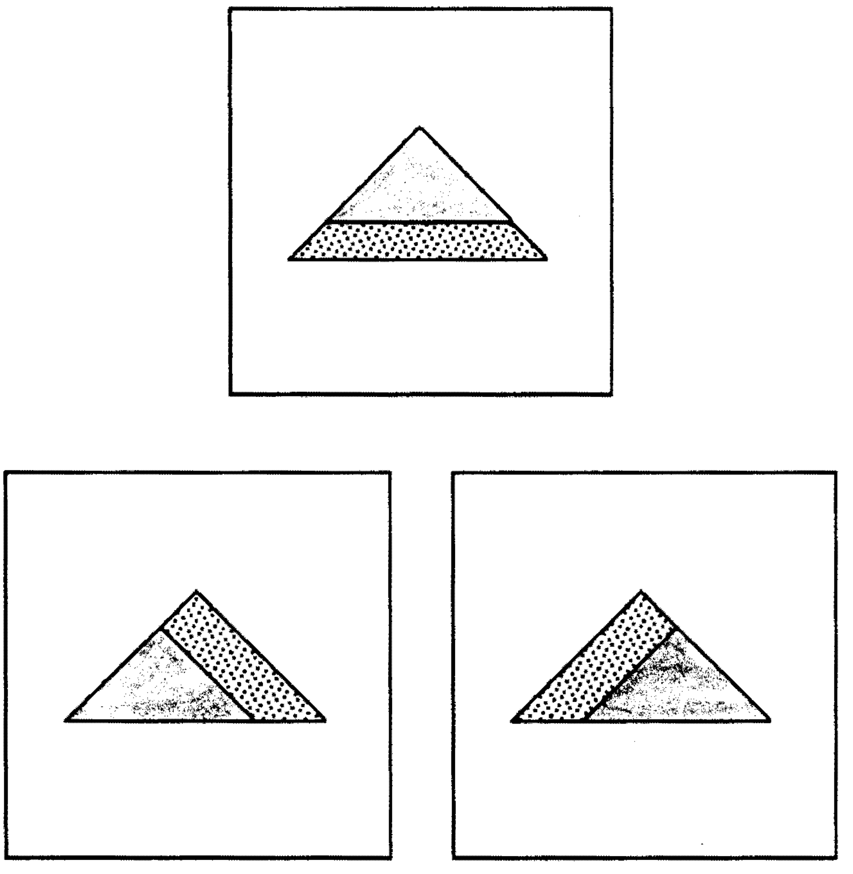


Fig. 29. Katie's First Three Solutions to the Large Triangle

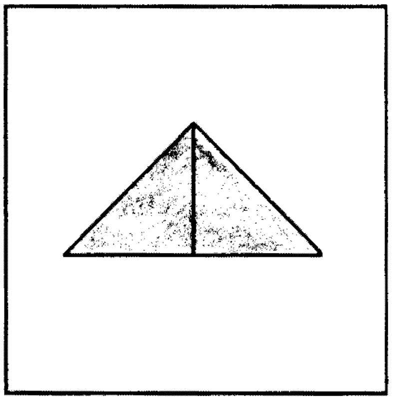


Fig. 30. Katie's Final Solution to the Large Frame

Elizabeth filled the square frame with the five small tangram pieces. Later, she was working on a frame with a chevron design requiring all seven tangram pieces, and unsuccessfully tried the pieces several different ways. Eventually, she put the two large triangles in the bottom points of the chevron leaving a square space tilted 45 degrees so that it appeared to be a diamond. She tried several times to put the five pieces into the diamond space. Finally, she said to no one in particular, "I need help." She glanced around the room and then across the table at the work she had done earlier. She pulled the square close to her, rotated it to match the square she was working on, and replicated the rotated square inside the chevron frame (see figure 31).

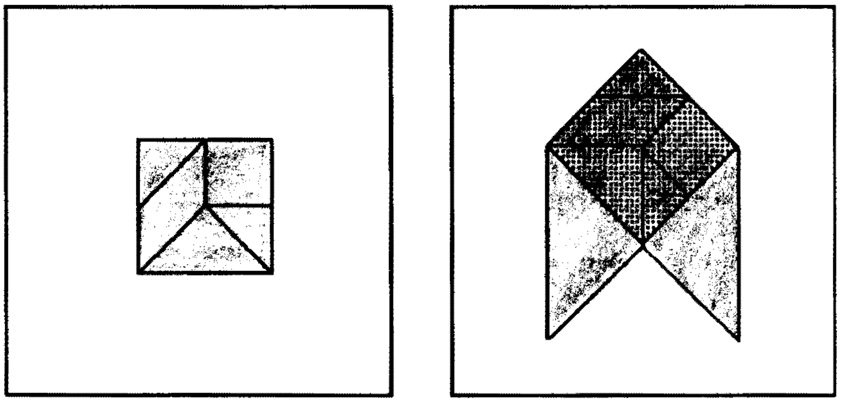


Figure 31. Elizabeth's Solution to the Chevron-Shaped Frame

Some mathematics educators claim that when young children see a triangle, it is always a right triangle or an equilateral triangle, or that they only see a square when it is in the standard position. When rotated 45 degrees, they may see only a diamond, not a square (P. R. Trafton, personal communication, July 20, 1994). It would appear from this example that working with tangrams helps young children construct a broader view of geometric shapes. Elizabeth recognized that a square was a square even though it was rotated so that it looked more like a diamond.

When she became frustrated with the diamond inside the chevron, she understood the relationship between it and the square she had made earlier. She then used her earlier work to help guide her in completing her later task.

During this instructional period, many of the children demonstrated they had constructed the same knowledge. They developed strategies for more complex frames as they learned that combinations of pieces could replace other pieces. They recognized that triangles, squares, and parallelograms can be constructed from the two smaller triangles, that a larger triangle, rectangle, or parallelogram can be constructed three different ways using the two small triangles, and a medium sized triangle, a square, or a parallelogram, and that a square or a triangle can be constructed from either the two large triangles or the five small pieces (see figure 32).

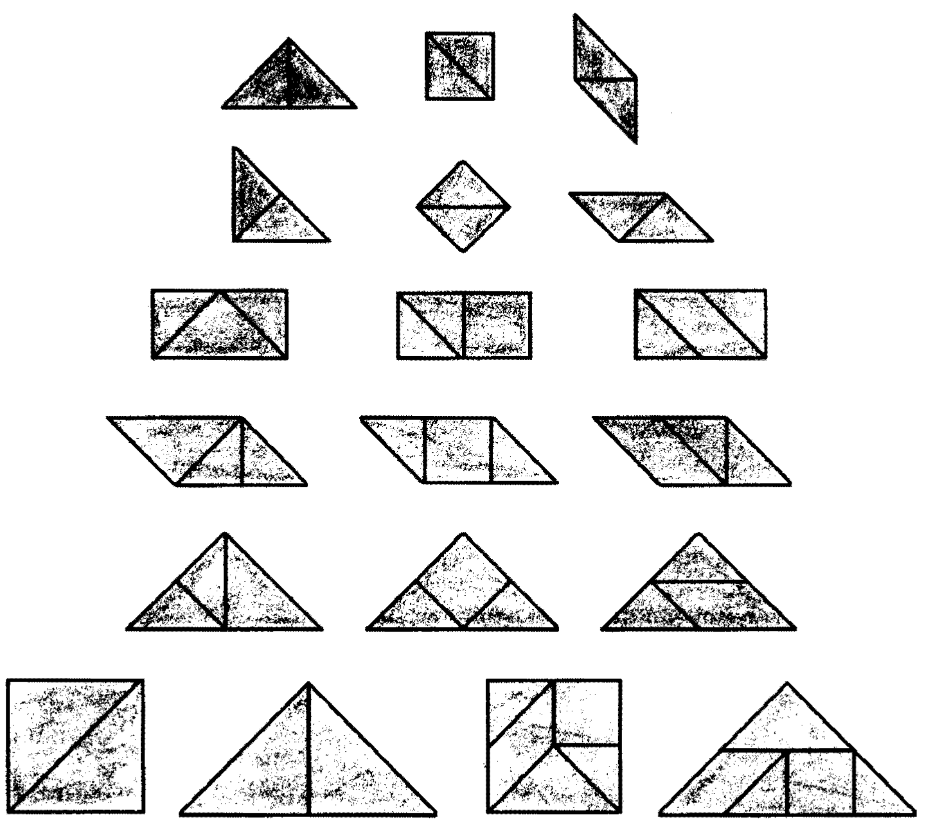


Fig.32. Identical Pieces Combined to Make Different Shapes

Once they became familiar with these strategies, the children determined ways to create those spaces within the advanced frames, the way Elizabeth had within the chevron, and fill the space they created. Mastery of this strategy enabled the children to solve almost any tangram problem (see figure 33).

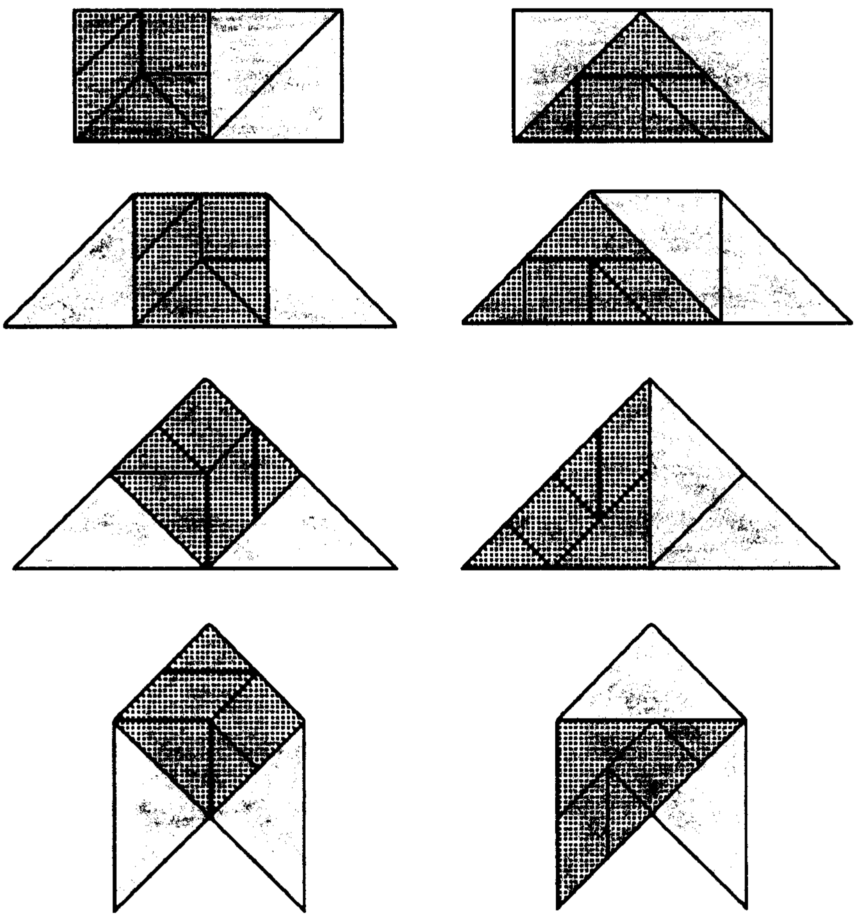


Fig. 33. Basic Tangram Shapes Within Advanced Tangram Shapes

Summative Assessment

I conducted a summative assessment with the 15 children who participated in the tangram project by asking each child to complete two frames: one a medium sized triangle, and the second a new abstract shaped frame requiring all seven pieces (see figure 17). Of the 15 children who took part in the assessment, all completed the triangular frame at least one way. Eleven completed the abstract shape. When the other four children were unable to complete the abstract frame independently, I sat with each of them and asked what might fit into the spaces. This guidance enabled them to complete the task.

Everyone participated in a group discussion, during which I asked the children what they had learned about tangrams and the pattern blocks. I showed them an orange tangram set and the box of pattern blocks. The children told me that the square pattern blocks were the same color as the tangram set, and that both sets had squares and triangles. As would be expected, since young children tend to *focus on only one attribute at a time, they commented only on the similarities.*

The children demonstrated nearly the same level of knowledge and success with the tangrams as they did with the pattern blocks. When asked to vote on which material they preferred, two children abstained, two children claimed they liked both materials equally, six children voted for the pattern blocks, and eight children voted for the tangrams. This was a bit surprising because of my perception that there was more enthusiasm for the pattern blocks than for the tangrams. It was possible that the children said they preferred the tangrams because we had just finished working with them, and they were fresh in the children's minds. Several children who were very close friends, all voted for the tangram frames. Their vote may have arisen out of loyalty to one another, rather than their interest in the tangrams. One child who did not participate in the tangram portion of the project voted for the tangrams.

CONCLUSIONS

The major conclusions from this teaching study are cited below and are supported by my review of the videotapes of class sessions, and review of my teaching journal. Numerous examples supporting these conclusions appear in the previous section.

The children's voluntary extended engagement over several weeks support the conclusions that pattern block and tangram activities were appropriate problem solving tasks. Although the tangram tasks were more complex, they were also within the capabilities and interest level of these preschool children.

The children were interested and stimulated by the challenge of the problems. They patiently struggled to find solutions to many problems, and often sought multiple solutions for the same problem. Several children seemed excited about the possibility that "hard" work requires many tries before a solution is found. Their renewed interest in the tasks each time more advanced frames were introduced, and their excitement over doing "hard work" would suggest they wanted and thrived on this challenge. They exhibited pride in their accomplishments, as can be noted from their expressions of satisfaction when they completed a problem solving task.

They developed a feeling of competence in their abilities to set personal goals, predict outcomes, exert effort to find solutions, and accomplish the tasks. They reasoned about actions and their effects. They received immediate feedback from their decisions and choices, and devised new ways of solving their problems.

Children, who appeared to visualize solutions as they worked on problems, were able to predict results. They seemed to be developing the ability to engage in spatial reasoning, including constructing knowledge about orientation of objects and knowledge of shapes and space.

The children exhibited a knowledge of several mathematical concepts and relationships and engaged in mathematical reasoning, including an awareness of similarities, differences, patterns, an awareness of the relationships among the blocks, and knowledge of the geometric names of shapes. They analyzed and compared solutions. They discovered relationships among the pieces, acquired an understanding of geometric shapes, and began to see parts of a whole as they learned to substitute one or more blocks for other blocks. As they became more familiar with the shapes, they began to use geometric names. They demonstrated an understanding of number, one-to-one correspondence, counting, and initial concepts of addition. They developed a variety of problem solving strategies. In the tangram portion, in particular, many children began to see a relationship between solutions they had found for basic frames and solutions for complex frames.

The ongoing and summative assessment of both portions of the instruction suggested that the children succeeded in solving both sets of tasks. My sense is that the children's initial experience with pattern block tasks enabled them to engage successfully in the more complicated tangram tasks, although, the design of this study did not allow me to collect any evidence on this. The children exhibited similar behaviors in solving both, for example, hypothesizing about solutions, analyzing and comparing pieces and solutions, and persistence in solving problems they chose.

As was expected, children's language was somewhat limited when describing their thought processes. When asked to describe how they solved their problems, they gave short obvious responses as, "I used these." However, they were able to verbalize their reasoning as they worked on a task, such as setting goals or describing substituting one set of pieces for another.

From this evidence, it appears that interacting with the pattern blocks, tangrams, and accompanying frames within an instructional environment recommended by the National Association for the Education of Young Children and others knowledgeable in the field of early childhood, and the National Council for Teachers of Mathematics, facilitates children's construction of mathematical understanding. Also, these tasks appear to be within the cognitive capabilities of four-year-old children. This is interesting because such tasks are typically used with children one or two years older.

RECOMMENDATIONS

This was conceived as an instructional design project to test methods advocated by early childhood educators and reformers in mathematics education. It presents students with interesting, engaging, and productive tasks that allow them to construct knowledge through interaction with the tasks, peers, and adults. However, the study also raises fascinating questions about children's learning, their understanding of spatial relationships, specific spatial ideas, and specific geometric relationships. These findings suggest the need to rethink or revise the prevailing beliefs of many educators about children's abilities and raise many questions that should be pursued further. Children's high level of success and the level of insight I was able to gain in this instructional setting were significant enough to warrant additional in depth examination of children's thinking about spatial sense and specific spatial relationships. Additional formal research studies examining growth and development of spatial reasoning and spatial sense also seem warranted.

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