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THERMAL STRESSES IN AN ISOSCELES RIGHT-TRIANGULAR PLATE WITH PINNED EDGES

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The object of this paper is two-fold: (1) to show that in certain cases of simply supported plates, the problem of thermal stresses due to unequal surface temperatures is equivalent to a membrane problem under normal surface pressure; and (2) to give some results for an isosceles right-triangular plate.

It has been shown by various writers (1) that the bending and twisting moments which exist in thin plates when there are constraints imposed by the edge conditions upon the free deformation which would exist under thermal action are

$$\begin{aligned} M_x &= -N[w_{xx} + \nu w_{yy} + T], \\ M_y &= -N[w_{yy} + \nu w_{xx} + T], \qquad (1) \\ M_{xy} &= -N(1-\nu)w_{xy}, \end{aligned}$$

where N and ν are plate constants, w is the deflection of the plate, and the temperature function is $T = \eta(1+\nu)(t_2 - t_1)/h$ with η the coefficient of thermal expansion, $(t_2 - t_1)$ the temperature difference of the surfaces and h the thickness of the plate. Equations (1) may be deduced from the fundamental relationships existing between the space changes of the displacement vector and the stress-strain relationships. They may also be interpreted for simply supported plates as the superposition of the two actions: (A) The determination of the stresses in a plate free from constraints produced by uniformly distributed couples; (B) The determination of the stresses in a simply supported plate submitted to couples uniformly distributed along the edges. In order to have translational and rotational equilibrium of all plate elements, Eqs. (1) are related by

$$-p(x,y) = \frac{\delta^2 M_x}{\delta x^2} + \frac{2\,\delta^2 M_{xy}}{\delta x \delta y} + \frac{\delta^2 M_y}{\delta y^2}\,. \qquad (2)$$

If no surface load p (x, y) is present, then one finds from Eqs. (1) and (2)

$$N\,\Delta^2(\Delta^2 w + T) = 0. \qquad (3)$$

In the case of polygon plates having straight line edges, the harmonic function of Eq. (3) vanishes at a rectilinear boundary and hence is known to vanish everywhere. The fourth order equation (3) is reduced to second order and is identical with the deflection equation of a membrane, that is

$$\Delta^2 w + T = 0. \qquad (4)$$

Hence the theorem: In simply supported thin polygonal plates, the deflection due to the thermal action from unequally heated surfaces, is that of a membrane over the same region with a normal pressure proportional to the temperature function T.

Consider the case of an isosceles right triangular plate with the edges $x = a$, $y = a$, and $x + y = 0$ simply supported. Let $T =$ constant, that is the temperature difference is the same everywhere and is not a function of x and y. The solution of (4) is

$$w(x, y) = \frac{T}{2} \left\{ - \frac{x^2 + y^2}{2} - xy + a(x + y) \right. \qquad (5)$$

$$\left. - 2 \left(\frac{2}{\pi}\right)^3 a^2 \sum_n \frac{(-1)^{\frac{n-1}{2}}}{n^3 \, \mathrm{Sh} \frac{n\pi}{2}} \left[\mathrm{Sh} \frac{\alpha\gamma}{2} \cos \frac{\alpha x}{2} + \mathrm{Sh} \frac{\alpha x}{2} \cos \frac{\alpha\gamma}{2} \right] \right\},$$

where $\alpha = n\pi/a$ and $n = 1, 2, 3, \ldots$

This deflection vanishes on all edges. From Eq. (1) the moments may be found. Of these the twisting moments M_{xy} are of interest. At the corners these twisting moments induce shearing forces which hinder the free curling which would be present under the unrestrained thermal expansion. At the corners with 45° angles these corner forces are

$$\frac{1}{24} E \, \eta \, h^2 \, (t_2 - t_1).$$

and are infinite at the right angled corner. It is known that at the corners of a rectangular plate (2) these forces are infinite and in the equilateral triangle they are

$$\frac{\sqrt{3}}{12} E \, \eta \, h^2 \, (t_2 - t_1).$$

One notes the increase of these corner shearing forces as the angle increases. In practical cases the forces, which become infinite by the mathematical solution, are redistributed due to the plastic action in the neighborhood of high stress concentrations.

For obtuse angles, the corners forces could be studied from the results of the equivalent torsion problem (case of $T =$ constant) for any regular polygon. (3)

1. TIMOSHENKO, S. "Strength of Materials," Vol. II, p. 487 (1930).
MARGUERRE, K. Thermo-Elastic Plate Equations, Zeit. f. Angew. Math. U. Mech., Bd. 15, S. 369. (1935).

OLSSON, G. Norske Ved. S. Skrifter, Nr. 2, (1933).
2. MAULBETSCH, J. Thermal Stresses in Plates, Applied Mech. Journal, V., 2, A-141, Dec., 1935.
3. SETH, B. R. Torsion of a beam whose cross-section is a regular polygon of n sides, Proc. Camb. Phil. Soc., 30, p. 139 (1934).

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