1989

An analysis of computational errors in the use of the division algorithm in the fourth grade

Teri Sue Rokusek
University of Northern Iowa

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Abstract

Division i s considered by many teachers to be one of the most difficult skills in an elementary school mathematics curriculum (Holland, 1942). This study explored the first step of diagnostic teaching in division; identifying errors made by students. The study was specifically tailored to fourth grade students and their use of the division algorithm on the set of whole numbers. Research has shown that in the case of systematic errors, the child will continue to use the algorithm incorrectly if not corrected (Cox , 1974). Grossnickle (1936) concluded "Almost 60% of the total number of errors which will impede pupil progress in long division with a one-figure divisor were systemtic." (p. 368) Therefore, research on the detection and analysis of systematic errors is of educational importance. It is useful to distinguish a careless error pattern from a systematic error pattern because the remedial procedures will be quite different. Yhen dealing with systematic errors, no matter how many times the child works the problem, he/she will probably make the same mistake. Using a sample of 25 randomly selected test papers from a population of 57 fourth grade students, errors from a thirty-item test were classified as random , careless, or systematic. iv Systematic errors comprised 39% of the total errors and these were further categorized into six error groups. Faulty or incomplete procedures; 30%, and regrouping; 25%, were the most common systematic errors. These were followed by zero/identity concept; 18%, place value; 16%, and remainder; 11%. No basic fact errors were found in the tests analyzed in this study.

AN ANALYSIS OF COMPUTATIONAL ERRORS
IN THE USE OF THE DIVISION ALGORITHM
IN THE FOURTH GRADE

A Graduate Project

Submitted to the

Department of Curriculum and Instruction

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts in Education

UNIVERSITY OF NORTHERN IOWA

by

Teri Sue Rokusek

May 1989

This Research Paper by: Teri Sue Rokusek

Entitled: AN ANALYSIS OF COMPUTATIONAL ERRORS IN THE USE OF THE DIVISION ALGORITHM IN THE FOURTH GRADE

has been approved as meeting the research paper requirement for the Degree of Master of Arts in Education.

Feb 28, 1989	Greg P. Stefanich
Date Approved	Director of Research Paper
Feb 28, 1989	Greg P. Stefanich
Date Approved	Graduate Faculty Advisor
Feb 22, 1989	Marvin Heller
Date Approved	Graduate Faculty Reader
March 3, 1989	Roger A. Kueter
Date Approved	Head, Department of Curriculum and Instruction

ABSTRACT

Division is considered by many teachers to be one of the most difficult skills in an elementary school mathematics curriculum (Holland, 1942). This study explored the first step of diagnostic teaching in division; identifying errors made by students. The study was specifically tailored to fourth grade students and their use of the division algorithm on the set of whole numbers.

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It is useful to distinguish a careless error pattern from a systematic error pattern because the remedial procedures will be quite different. When dealing with systematic errors, no matter how many times the child works the problem, he/she will probably make the same mistake.

Using a sample of 25 randomly selected test papers from a population of 57 fourth grade students, errors from a thirty-item test were classified as random, careless, or systematic.

Systematic errors comprised 39% of the total errors and these were further categorized into six error groups. Faulty or incomplete procedures; 30%, and regrouping; 25%, were the most common systematic errors. These were followed by zero/identity concept; 18%, place value; 16%, and remainder; 11%. No basic fact errors were found in the tests analyzed in this study.

ACKNOWLEDGEMENTS

The author wishes to acknowledge people whose support and assistance were greatly appreciated throughout this research project.

A sincere thank you to Dr. P. J. Porter and Kathy Howell. P. J. "planted the seed" that gave me the desire to pursue my education. Kathy was the light at the end of the tunnel; a true inspirational model.

The guidance and suggestions from Dr. Greg Stefanich were a major factor in the development of this paper. He set specific guidelines, believed in me, and took the time to work one-to-one until we had a finished product. The author also wishes to thank her typist, Joyce Broell, whose patience and enthusiasm is greatly admired.

This paper was written during my teaching career at LaPorte City Elementary. The entire staff did everything in their power to help me see the project through. Little words of encouragement and interest made all the difference. A special thank you to my two team teachers, Ms. Kim Colpitts and Ms. Doris Winter. They sacrificed their schedules to accommodate my research more than once. Neil Mullen also put in a lot of proofreading time, offered constructive criticism, and gave moral support.

This section of my paper would not be complete without acknowledging my husband, Dennis Rokusek. He deserves a gold medal for all he has done and been patient with so that this paper could be completed. His caring attitude, enthusiasm for learning, and respect for my goals, is forever appreciated.

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CHAPTER I

INTRODUCTION

Introduction to the Problem

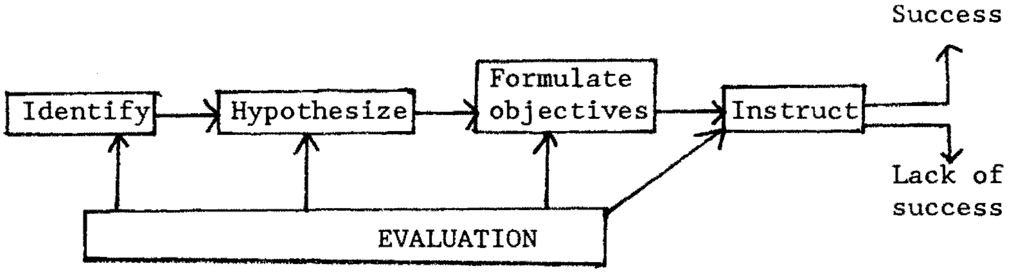
Division is considered by many teachers to be one of the most difficult skills in an elementary school mathematics curriculum (Holland, 1942). Students find it hard to learn because it requires the knowledge of place value, subtraction, and multiplication concepts.

When children make errors in computation, teachers should analyze whether the errors are careless errors, random errors, or systematic errors. It is very important to identify the students making systematic errors because this type of error is potentially remediable. Cox (1974) found that without instructional intervention, systematic errors will continue for long periods of time. She reported that 23% of the children making systematic errors were still making the same error, or another very similar error, almost one year later.

Children are taught to look for patterns when they work with numbers to help them discover the structure of our number system. Similarly, teachers must look for patterns in the data they collect from children who are experiencing problems in computational skills. Recognizing patterns in the errors a child is making is the initial step toward remediation of the error (Cox, 1974).

Diagnostic teaching enables the teacher to determine how he can help the student in the best way. Fredricka K. Reisman (1978) has formulated the following "diagnostic cycle."

Figure 1. The Diagnostic Teaching Cycle Model



The diagnostic teaching of elementary school mathematics involves five processes:

1. Identifying the child's weaknesses and strengths;

2. Hypothesizing possible reasons for these weaknesses and strengths;

3. Formulating behavioral objectives to serve as a structure for the remediation of weaknesses or the enrichment of strengths;

4. Creating and trying corrective remedial procedures;

5. Continuing evaluation of all phases of the diagnostic cycle to see if progress is being made.

This study concerned itself with the first of the above processes --identifying the child's weaknesses.

Statement of the Problem

This study analyzed errors made by students in grade four in their use of the division algorithm on the set of whole numbers. This involved identifying the thinking patterns (strategies) that led to the errors.

The study carefully identified guidelines for use in classifying all incorrect responses as one of the following: (a) systematic error, (b) random error, or (c) careless error. The systematic errors were further analyzed according to the thinking strategy employed.

Purpose of the Study

If one is to be effective in guiding children in the process of division, it is necessary to give consideration to the errors they are making and the thinking strategies that led to those errors. The analysis of the child's systematic errors can and should be used as a guide for follow-up remediation.

When the teacher is aware of the relationships that are basic to mathematical concepts and generalizations, he or she is better able to make decisions regarding selection of curriculum that is appropriate to the learner and offer effective instructional assistance to the learner.

This study will look at the systematic errors in the division of whole numbers. Research has shown that in the case of systematic errors, the child will continue to use the algorithm incorrectly if not corrected (Cox, 1974). Therefore, research on the detection and analysis of systematic errors is of educational importance.

Assumptions

1. It was assumed in this study that the division errors made by students in grade four could be classified as systematic, random, or careless according to the operational definitions used.

2. Tests used in this study were assumed to measure adequately six levels of the division algorithm with one-digit divisors.

3. The students used as a sample population for this study were assumed to be representative of a typical public school classroom in Iowa.

4. The study took place over a short period of time so maturation did not interfere with the results.

5. No pretest was used, eliminating pretest sensitization.

6. The materials were designed for a minimum of teacher input; therefore, teacher attitudes and methods did not influence the results.

Limitations of the Study

The study was limited to fourth grade students in one Iowa community. The community represents a rural population of varying socio-economic backgrounds. Caution should be used in generalizing to other geographic areas and other grade levels.

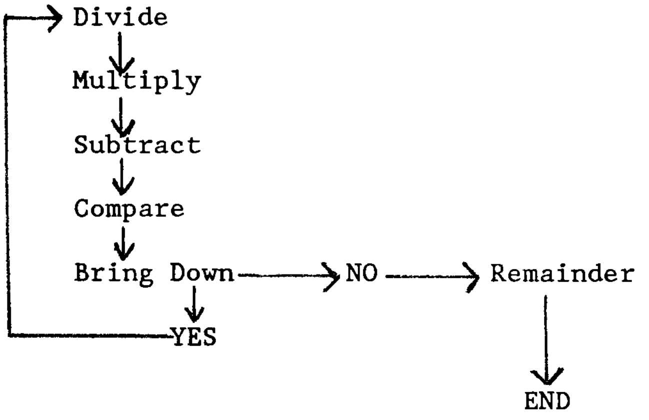
Definition of Terms

Basic fact--is any two, one-digit numbers along with their sum (for addition) or missing addend (for subtraction). There are 100 basic addition, subtraction, and multiplication facts, but since division by zero is ruled out, there are only 90 basic division facts (Burns, 1965).

Careless errors--occur in one or two out of five problems. The child basically knows how to perform the correct computation, but due to distractions, boredom, or a lapse of attention he/she makes careless errors (Copeland, 1974).

Computational skills--the ability to use the basic math facts with the appropriate basic algorithm (Brueckner & Elwell, 1932).

Division algorithm--is the step by step process necessary to divide a number in arithmetic. The division process (Ashlock, 1972) is:



Division error groups--consist of errors which stem from a common root (Cox, 1975). In division, there are six major groups of errors. They are:

1. Basic fact errors occur when a child gives the same incorrect response for a particular basic fact.

2. Faulty or incomplete procedure errors occur when a student fails to complete the division algorithm or completes it in the wrong order.

3. Place value errors involving improper place value and occur when a child writes the correct quotient with incorrect place value or writes extra 0's in middle or right side of quotient.

4. Regrouping errors involve incorrect regrouping and occur when a child uses the digits in the divisor separately, doesn't regroup when subtracting the partial dividends, or adds the part remainders together for the final remainder.

5. Remainder errors occur when a student leaves the final remainder too large or does not write the remainder.

6. Zero or identity concept errors occur when a child does not write 0 in the quotient when necessary, ignores 0's in the dividend and/or divisor.

Random errors--occur in at least three out of five problems but contain no discernible pattern. They are hard to remediate because of the fact there is no discernible pattern to the error (Kalin, 1983).

Systematic errors--are those consistent computational errors that occur in at least three out of five problems for a specific algorithmic computation. They show a pattern of incorrect responses. The child will likely make the same error when encountering similar computational problems (Dutton, 1964).

CHAPTER II

REVIEW OF LITERATURE

Introduction to the Literature Review

There is a need for diagnostic teaching in mathematics education. The basis for this need lies in the projected positive effects of remediation as a result of specific error identification.

The importance and levels of diagnostic teaching are discussed in part one of the review. Part two explores the variety of error patterns emphasizing those specific to division.

Literature Review

Diagnostic teaching looks at the errors children make and subsequently structures the learning experiences so the errors will be eliminated. There is hardly a skill in the teacher's repertoire that is more important than the ability to identify pupil errors and prescribe appropriate remedial procedures (West, 1971).

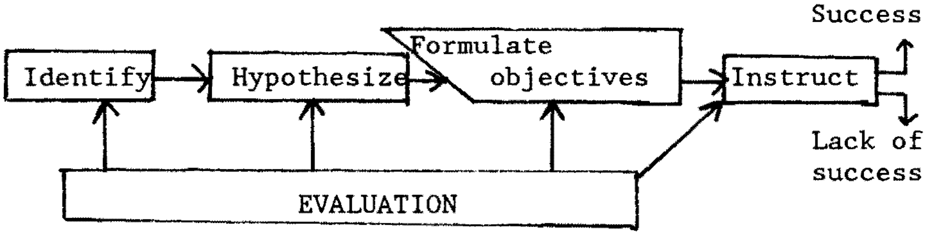
Fredricka Reisman (1977) conducted considerable research on the diagnostic teaching of arithmetic. She states, "All children, whether they are slow learners, average, or very bright, proceed optimally with and profit from diagnostic teaching" (p. 9).

The diagnostic strategy is based on a simple model, which involves the intermingling of three processes: (a) the identification of a child's level of achievement, (b) the analysis of the content to be taught, and (c) the implementation of appropriate instructional procedures. In order to base mathematics instruction on this model, the elementary school teacher must have a knowledge of mathematics and understand how children learn mathematics. To sustain the continuity necessary for good mathematics teaching, the primary teacher should know where children are headed along the mathematical path. This continuity must continue in the upper grades when teachers are aware of the arithmetic experiences that the pupils have already encountered (Reisman, 1977).

To examine where a child is conceptually, it is important to identify the pupil's strengths and weaknesses by using certain diagnostic procedures. These include observation, interview, and diagnostic testing of mathematics concepts (Copeland, 1974).

This identification of strengths and weaknesses is the initial step in Reisman's Diagnostic Teaching Model (1978).

Figure 2. The Diagnostic Teaching Cycle Model



The identification process involves analyzing such behavior products as achievement test scores, scores on teacher-made tests, responses to mathematical problems during an interview between teacher and student, or any other appropriate sample of the student's behavior. The identification component of diagnosis depends upon the teacher's skills in using a diagnostic strategy. These skills involve the ability to construct a diagnostic instrument, to interpret the results of tests and interviews, and to utilize techniques for investigating the child's thinking processes (Cox, 1975).

Ashlock (1972) agrees that the diagnosis of errors in arithmetic is an essential part of evaluation in the mathematics program, and any such diagnosis must be accompanied by remedial or corrective instruction. Ashlock feels strongly that if the written work of a child is to provide useful information for diagnosis, that work must not only be scored, it must be analyzed as well.

Frequently, usual scoring techniques do not distinguish situations in which the child uses an incorrect procedure and situations in which the child does not know how to proceed. Clearly, it is better for teachers to spend their professional time analyzing the written work of children and planning corrective instruction rather than using what time they have for scoring (Myers, 1924).

Diagnosis is not something reserved for special times and places nor is it an activity reserved for specialists. Effective teachers make diagnoses many times every day as they teach children (West, 1971).

There are different levels of diagnosis (Roseman, 1985). There are times when you merely seek enough information to decide what kind of help each child needs. You may be forming instructional groups or you may be deciding which children need the help of a resource teacher. Additional diagnosis is required before you can plan lessons.

At an even deeper level, you try to find a child's areas of strength and weakness. It is helpful to know which concept and skill categories are well developed and which require special attention.

Within the area of division, specific concept and skill categories exist which involve regrouping, place value, basic facts, and the like. For each child having difficulty, one must know which of these categories are strengths and which are weaknesses.

The most fruitful diagnosis is one that looks for patterns of errors rather than careless errors. Identifying systematic errors is made easier by the use of error pattern groups. Error groups consist of errors which stem from a common root (Shaw & Pelosi, 1983). Teaching is more effective if a class of errors can be eliminated by a single remedial procedure (West, 1971).

It is useful to distinguish a careless error pattern from a systematic error pattern because the remedial procedures will be quite different. The careless error will probably be caught if the child reworks the problem. If the child is given a procedure for checking his/her work or for estimating the reasonableness of his/her solutions, then careless errors may be reduced. The systematic error is quite different. No matter how many times the child works the problem, he/she will probably make the same mistake. We detect systematic errors by seeing patterns of errors in a child's paper. The child may consistently make the error, which means that he/she has an incorrect understanding of the procedure; or he/she may make the error only sporadically, or only under certain circumstances, indicating that he/she has some grasp of the procedure but not a complete understanding (Ashlock, 1972).

Cox (1974) pointed out that every teacher can expect to encounter at least one child each year who will make systematic errors in division. In her study more children made systematic errors than careless errors.

This is important to note because action can be taken to correct systematic errors. Once a systematic error is identified, a remedial instructional program can be developed and implemented.

The error patterns used in this study were taken from Grossnickle's studies. F. E. Grossnickle (1936) revealed many errors in long division with a one figure divisor. He studied errors made by 453 students in grades five to eight. Before making a detailed analysis of each paper, Grossnickle constructed a tentative list of errors and faulty procedures from the studies reported by Breckner and Elwell (1932), and Lazar (1928). Grossnickle found types of errors in his study not listed in the previous studies. He compiled a list of fifty-seven errors involving division with a one-digit divisor. The errors were grouped under six classifications as follows: (a) place value, (b) use of remainders, (c) zero errors, (d) faulty procedures, (e) regrouping, and (f) basic facts.

Although Grossnickle (1936) cited fifty-seven errors in long division, he noted in his summary that many of these errors were infrequent. Errors that fell into the six categories constituted 59.4% of the total frequencies of errors noted in his study. Thus Grossnickle concluded:

> If a teacher is able to give special consideration to the six types of errors enumerated, provision has been made for almost 60% of the total number of errors which will impede pupil progress in long division with a one-figure divisor. (p. 368)

Several approaches are helpful in dealing with error patterns: (a) develop and examine the sequence of steps leading up to the algorithm in which the error pattern occurs, (b) provide physical models for manipulative activity, (c) develop a diagram approach to the algorithm, and (d) use an alternative algorithm (Suydam & Reys, 1978).

Each algorithm has a prior sequence of learning steps (Brueckner, 1932). Brueckner's skill levels listed in Appendix A are organized by the number of digits in the divisor and dividend, the inclusion or exclusion of zeros in the dividend and quotient, and the existence or absence of remainders. Analyzing such a sequence will allow you to discover the point at which the child has gotten into trouble.

West (1971) believes teachers who develop the practice of looking at children's papers critically to detect such patterns of errors are in a better position to provide for remediation of learning difficulties. They will also develop better teaching procedures for avoiding the very appearance of such errors in children's work.

<u>Summary</u>

A basis for the need of diagnostic teaching in elementary mathematics education lies in the projected positive effects of remediation as a result of error identification. The diagnostic strategy is based on the intermingling of three processes: (a) the identification of a child's level of achievement, (b) the analysis of the content to be taught, and (c) the implementation of appropriate instructional procedures (Reisman, 1977).

The analysis of childrens' written work, accompanied by remediation and corrective instruction has shown to significantly improve student performance in mathematics. The most fruitful diagnosis is one that looks for patterns of errors (West, 1971).

Research indicates that more children make systematic errors than careless errors. Once systematic errors are identified, a remedial instructional program can be developed and implemented (Roberts, 1968).

CHAPTER III

DESIGN OF THE STUDY

Procedure Used

The subjects were fourth grade students from LaPorte City, Iowa. The students are from a rural community and have various socio-economic backgrounds.

A table of random numbers was used to select twenty-five fourth grade test papers from a population of 57 students who took the test. The test was administered on Thursday, February 2, 1989.

The diagnostic instrument was administered by three regular classroom teachers. They made no reference to the fact that the test was to be used in a research study. Students were asked to show all work including any crutches they normally used. There was no time limit. Students were asked to leave their work unchecked.

Description of the Instrument

The instrument consisted of a thirty-item test. The items were taken from Diagnostic Test IV - Division of Whole Numbers of the 1965 SRS Computational Skills Development Kit by Charles Proctor and Patricia Johnson. The instrument included five items per skill level. The procedure is similar to the one employed by Brueckner (Cox, 1974) as an acceptable minimum number for reliable diagnosis. The skills levels and test items are listed in Appendix A. Appendix B contains the thirty-item test.

Use of the Instrument

The sample of twenty-five tests was then selected by a table of random numbers. The criterion for the three types of errors was applied as defined in the Definition of Terms section. The errors that were found to be systematic errors were further analyzed to determine the child's thinking strategies that lead to the incorrect response.

A description of some of the systematic errors found are regrouping errors, place value errors, zero or identity concept errors, and remainder errors.

The researcher did the error pattern analysis. Training to criteria was obtained in the Diagnosis and Remediation of Mathematics class taught by Dr. Glenn Nelson at the University of Northern Iowa during the fall semester, 1983. After initial analysis by the researcher, the results were confirmed by Dr. Edward Rathmell, a professor in the University of Northern Iowa Mathematics Department.

CHAPTER IV

RESULTS OF THE STUDY

Introduction

This chapter reports and discusses test data concerning systematic, random, and careless errors in the use of the division algorithm. Twenty-five tests were randomly analyzed from a population of fifty-seven fourth grade students.

Classification of Errors

There were a total of 143 errors on the 25 division tests. Table 1 lists the breakdown of careless, random, and systematic errors. There is also a category for the 4 problems the students omitted.

Table 1

Classification of Errors

Error	Number Errors	% Total Errors	% Total Test Items
Careless	68	47.55%	9.06%
Random	15	10.49%	2.00%
Systematic	56	39.17%	7.46%
Omit	4	2.79%	.53%
Total	143	100.00%	19.05%

Careless errors occur in one or two out of five problems. The child basically knows how to perform the correct computation, but due to distractions, boredom, or a lapse of attention, he/she makes careless errors. Sixty-eight errors of this type were noted, which accounted for 47% of the total errors made. This was the largest error group.

Random errors occur in at least three out of five problems, but contain no discernible pattern. There were only 15 of this errors type found, resulting in 10% of the total errors.

Systematic errors are consistent computational errors that occur in at least three out of five problems for a specific algorithmic computation. They show a pattern of incorrect responses. The child will likely make the same error when encountering similar computational problems. The tests revealed 56 of these errors, which total 39% of the errors. This was the second highest error group.

Systematic Errors in Division Algorithm

The 56 systematic errors were broken down into six categories: (a) regrouping, (b) place value, (c) zero/identity concept, (d) basic fact, (e) faulty or incomplete procedure, and (f) remainder. Appendix C defines each of these categories and gives student examples for each.

Classification of Systematic Errors

The systematic error statistics were compiled to determine the percentage of errors in each category. Table 2 shows this breakdown.

The faulty or incomplete procedure category of error pattern occurred most often with 17 errors; 30% of the total systematic errors. Regrouping errors were the second most common type of systematic error. These comprised 25% of the total.

Table 2

Classification of Systematic Errors

Error	Number Errors	% Total Systematic Errors	% Total Errors	% Total Test Items
Regrouping	14	25.00%	9.79%	1.87%
Place Value	9	16.07%	6.29%	1.20%
Zero/Identity Concept	10	17.85%	6.99%	1.33%
Basic Fact	0	0.00%	0.00%	0.00%
Faulty or Incomplete Procedure	17	30.36%	11.90%	2.26%
Remainder	6	10.72%	4.20%	.80%
Total Systematic Errors	56	100.00%	39.17%	7.46%

The next three categories fell close together and weren't as substantial as the two previously listed. There were 10 zero/identity concept errors representing 17% of the total systematic errors. Sixteen percent of the total systematic errors were place value errors. Ten percent of the total systematic errors were remainder errors. No basic fact errors were found in the tests analyzed in this study.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

This study analyzed fourth grade student errors in their use of the division algorithm. Their errors were classified as systematic, random, or careless. The systematic errors were analyzed further to determine each student's thinking strategy.

The objectives of this study were to: (a) compile a literature review of diagnostic teaching and its role in division, (b) identify each child's strengths and weaknesses in the use of the division algorithm, (c) determine the percentage of systematic errors in contrast to random and careless errors, (d) provide a sound base for remediation and curriculum decisions, and (e) improve teaching effectiveness in the use of the division algorithm.

The literature review showed that the basis for diagnostic teaching in elementary mathematics education lies in the projected positive effects of remediation as a result of error identification. The analysis of children's written work, accompanied by remediation and corrective instruction has shown to significantly improve student performance in mathematics. The most fruitful diagnosis is one that looks for patterns of errors (West, 1971).

Research indicates that more children make systematic errors than careless errors. Once systematic errors are identified, a remedial instructional program can be developed and implemented (Roberts, 1968).

This study administered one thirty-item test to 57 fourth grade students in LaPorte City, Iowa. A sample of 25 tests were analyzed to determine systematic, careless, and random errors. The systematic errors were then classified into six error groups.

Careless errors accounted for 47% of the division errors. Systematic errors were next with 39%. Of the six systematic division error categories, faulty or incomplete procedure errors were the most common. They accounted for 30% of the total systematic errors.

Conclusions

Through error analysis and a review of literature, the researcher concluded:

1. Error pattern analysis of fourth grade students in the use of the division algorithm using whole numbers indicated 47% of student errors are careless errors.

2. The error pattern analysis resulted in 39% of the errors being systematic.

3. The most common types of systematic errors were faulty or incomplete procedure errors and regrouping errors.

4. Diagnostic teaching, using error pattern analysis, is a viable procedure for improving instructional effectiveness in the use of the division algorithm.

Recommendations

The following recommendations are based on the literature review, testing, and follow-up work:

1. An extension of this problem is remediation of the errors detected. It is not within the scope of this paper to present ideas regarding remedial instructional techniques. This is a problem for

future research. The results of analyzing the systematic errors is of little value if methods of remediation are not known.

2. The long-term impact of remediation needs to be investigated. A longitudinal study of student error patterns should be conducted to determine if these patterns reappear later.

3. Error patterns involving other types of mathematical algorithms need to be researched.

4. Error pattern analysis is only one means of identifying systematic student errors in mathematics. Other procedures, such as interviews, need to be explored.

5. Application of similar strategies to other subject areas need to be examined.

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APPENDIX A

Test Items Listed by Levels of Skill in Division*

Level 1: One-digit divisor; two-digit dividend; no remainders.

4)48 3)96 2)78 5)85 6)90

Level 2: One-digit divisor; two-digit dividend; with remainders.

5)48 5)79 7)86 2)33 4)95

Level 3: One-digit divisor; three-digit dividends; no remainders.

5)455 8)232 6)342 4)292 7)497

Level 4: One-digit divisor; three-digit dividends; with remainders.

5)346 4)470 7)961 8)255 3)391

Level 5: One-digit divisor; three-digit dividends with zeros; with and without remainders.

5)608 3)702 4)507 7)905 6)804

Level 6: One-digit divisor; three-digit dividends that produce zeros in tens column in quotient; with and without remainders.

4)436 2)814 5)533 5)508 4)407

*These levels are not necessarily in order of increasing difficulty. They are organized by the number of digits in the divisor and dividend, the inclusion or exclusion of zeros, in the dividend and quotient, and the existence or absence of remainders (Brueckner, 1932).

APPENDIX B

Division Test

Solve the following division problems. Show all your work. Do <u>not</u> check.

1. $4\overline{)48}$	2. $3\overline{)96}$	3. $2\overline{)78}$	4. $5\overline{)85}$
5. $6\overline{)90}$	6. $5\overline{)48}$	7. $5\overline{)79}$	8. $7\overline{)86}$
9. $2\overline{)33}$	10. $4\overline{)95}$	11. $5\overline{)455}$	12. $8\overline{)232}$
13. $6\overline{)342}$	14. $4\overline{)292}$	15. $7\overline{)497}$	16. $5\overline{)346}$
17. $4\overline{)470}$	18. $7\overline{)961}$	19. $8\overline{)255}$	20. $3\overline{)391}$
21. $5\overline{)608}$	22. $3\overline{)702}$	23. $4\overline{)507}$	24. $7\overline{)905}$
25. $6\overline{)804}$	26. $4\overline{)436}$	27. $2\overline{)814}$	28. $5\overline{)533}$
29. $5\overline{)508}$	30. $4\overline{)407}$		

APPENDIX C

Systematic Errors in the Division Algorithm

I. Regrouping errors involve incorrect regrouping. These errors occurred in several ways. Some are:

a.
```
    101 r 2
 4 |436
    4
    3
     6
     4
     2
```
The student did not regroup the partial dividends.

b.
```
     30 r 6
 8 |255
    24
     1
      5
      0
      5
```
The student did not regroup the partial dividends and added the part remainders together for the final remainder.

c.
```
    420 r 1
 2 |814
    8
     1+4 = 5
           4
           1
```
The student added the partial remainders to the next dividend digit.

II. Place value errors involve improper place value. They occur when a child writes the incorrect place value or writes extra 0's in the middle or right side of quotient. Sample place value errors are:

a.
```
     110 r 75
 5  |85
      5
     80
      5
     75
```

b.
```
    150 r 4
 5 | 79
     5
     29
     25
      4
      0
      4
```

c.
```
    50 r 0
 5 |508
    5
     0
```

III. Zero or identity concept errors occur when a child does not write 0 in the quotient when necessary, ignores 0's in the dividend and/or divisor. Examples:

a.
```
     110 r 3
  5 ⟌508
     5
     08
      5
      3
```

b.
```
     11 r 3
  4 ⟌407
     4
       7
       4
       3
```

c.
```
     160 r 3
  5 ⟌533
     5
     33
     30
      3
```

IV. Basic fact errors occur when a child gives the same incorrect response for a particular basic fact. There were no systematic basic fact errors found in this study.

V. Faulty or incomplete procedure errors occur when a student fails to complete the division algorithm or completes it in the wrong order. These errors occurred in several ways. Some are:

a.
```
      1
  6 ⟌ 90
      60
```
Student stopped dividing at the first partial quotient and did not write the remainder.

b.
```
     4 r 1
  2 ⟌814
     8
     1
```
Student stopped dividing at the first partial quotient and wrote a remainder.

c.
```
     100
  3 ⟌391
     30
```
Student stopped dividing at the first partial quotient and wrote 0 in the rest of the quotient place values.

d.
```
     170
  4 ⟌470
     40
```
Student stopped dividing at the first partial quotient and wrote the rest of the dividend digits in the quotient.

e.
```
     21
  4 ⟌48
     4
      8
      8
```
Student wrote the digits of the answer in reverse order.

f.

```
     424
 8 | 232
     2
     2
```

Student divided the smaller digit into the larger digit even when the smaller digit was in the dividend.

VI. Remainder errors occur when a student leaves the final remainder too large or does not write the remainder. Examples:

a.

```
      32 r 9
 8 | 255
     24
      15
      16
       9
```

b.

```
     121
 5 | 608
     5
     10
     10
      8
      5
```