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Estimation: A principle component for the mathematics curriculum

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Abstract

In the age of advanced technology when a push of a button begins a series of connections, the importance of human reasoning power becomes more and more crucial. Only through thinking, scanning the limits of the brain, can the technology be of use to society and to oneself. Throughout this period of technological progress, the skills necessary to exist in this society are changing and in question. The goals and approaches of the elementary school mathematics curriculum are also being questioned. In mathematics education the major influence of technology is its potential for shifting the focus of instruction from an emphasis on manipulative skills to an emphasis on concept development and problem-solving skills. "Technology will change what we teach as much as how we teach it" (Campbell & Fry, 1988, p. 54).

ESTIMATION:

A PRINCIPLE COMPONENT FOR THE MATHEMATICS CURRICULUM

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Estimation:
A Principle Component for the Mathematics Curriculum

In the age of advanced technology when a push of a button begins a series of connections, the importance of human reasoning power becomes more and more crucial. Only through thinking, scanning the limits of the brain, can the technology be of use to society and to oneself.

Throughout this period of technological progress, the skills necessary to exist in this society are changing and in question. The goals and approaches of the elementary school mathematics curriculum are also being questioned. In mathematics education the major influence of technology is its potential for shifting the focus of instruction from an emphasis on manipulative skills to an emphasis on concept development and problem-solving skills. "Technology will change what we teach as much as how we teach it" (Campbell & Fry, 1988, p. 54).

The field of mathematics is in the process of revision. The National Council of Teachers of Mathematics is expecting to affect a change in mathematics education within the next decade. It is currently generating a set of Curriculum and Evaluation

Standards for School Mathematics (Standards). These recently released standards propose "a balanced curriculum, a curriculum that focuses on both mathematical ideas and processes, but one in which reasonable manipulative proficiency is also expected" (Thompson & Rathmell, 1988, p. 18). The expectation is for the standards to serve as a prototype for curriculum writing. It is expected that they will influence the content of the textbooks in addition to the content and emphasis of local, state, and national tests. Emphasis in the mathematics curriculum seems to be shifting from paper-pencil algorithms into the realms of problem solving, conceptual development, and mathematical reasoning based on the assumption that the impact of technology will be widely available.

Estimation along with mental computation plays an important role in this development. Estimation should be a principal, not parenthetical, component of the curriculum. It should not be limited to isolated, discrete exercises. Proponents of this skill feel strongly about its inclusion in the learning-teaching act, but there has been little research to support the position. Another obstacle to inclusion into the curriculum is the lack of adequate testing procedures.

Research teams have difficulty evaluating estimation strategies and understandings. Despite these limitations, it is through estimation that the thinking abilities are strengthened, and understanding of basic mathematical concepts increased. It has been found that students are better at finding exact answers through paper-pencil computation. They have had unlimited practice in this process but little experience with estimation and mental computation. Typically very little formal instruction has been devoted to these operations. It is relatively easy to test and measure the success in computing algorithms but much more challenging to check and measure the level of understanding and skill in estimation.

If this skill of estimation and related mental computation is so important, and proficiency with estimation has long been recognized as an essential outcome for the study of mathematics, then why has it been neglected in the mathematics curriculum? What does research suggest for the inclusion of this skill? What are the characteristics of a good estimator? What strategies are used when estimating, and how can they be used in the existing mathematics curriculum?

Support for Inclusion of Estimation in the Elementary School Curriculum

To develop a perspective about the proper role of estimation in the mathematics curriculum, an examination was made of the concept and the current utilization of estimation exercises in the schools. The benefits of estimation in everyday life were also examined. The information gained in this manner provides a background for making sound curricular decisions.

The Concept of Estimation--What Is It?

Estimation is "the process of producing an answer that is sufficiently close to allow a decision to be made" (B. J. Reys, 1986, p. 22). It is done mentally and with reasonable quickness.

One widely used type is computational estimation. Reys, Reys, Trafton, and Zawojewsky (1984) defined it as:

> the interjection and/or combination of mental computation, number concepts, technical arithmetic skills including rounding and place value, and less straightforward processes such as mental compensation, that rapidly and consistently produce answers that are reasonably close to a

> correctly computed result. This process is done internally without the external use of a calculating or recording tool. (p. 1)

Mental computation is "the process of producing an exact answer to a computational problem without any external computation aid" (B. J. Reys, 1986, p.22). This sets mental computation apart from estimation. In estimation many different but reasonable estimates can exist for a given problem. Mental computation, however, is the process necessary to arrive at an estimate that requires a numerical computation. Because of the close relationship between estimation and mental computation, an instructional program must include the teaching of both (Hope, 1986).

Once an estimate is computed or determined, this approximation must be judged, or evaluated, to be reasonable, or appropriate. Reasonableness of the derived estimate(s) is, therefore, an essential skill. This process requires higher levels of critical thinking. "Estimation in computation, after all, is no more than a form of less precise mental calculation" (Hope, 1986, p.49). The reasonableness of the estimate is the final judge of the accuracy of the estimation.

There is a link among the three skills: estimation, mental computation, and reasonableness of the resulting quantity. Each skill is used in conjunction with each of the other skills. The user through experience, becomes increasingly more precise with each skill. Since the use of high technology is increasing and society is emerging into the information assimulation age, mastery of these skills becomes more and more valuable.

Current State of Estimation--What Is It?

Textbooks are the main vehicle for instruction. A review of basal textbooks in mathematics shows that very little attention is devoted to the systematic development of computational estimation. An examination of textbooks indicates that "as much as 60-75 percent of lessons...center on computational skills, but less than five percent are devoted to computational estimation" (Reys & Reys, 1986, p. 4). Driscoll, in a study of three widely used mathematics textbook series cited by Reys et al. (1984) revealed that estimation appeared in less than three percent of the lessons. As for experiences with mental computation, it was not encouraged, or even suggested, in most basal text series.

When students are tested on mental computation and estimation skills, data show that student performance is low. From the limited research studies completed, consensus is that performance on estimation tests is disappointing. "The ability studies show, on the whole, across all age levels and for both computational and measurement estimation, a disappointing ability to estimate" (Benton, 1986, p. 24). "The data generally reflects a lack of instructional attention" (Reys et al., 1986, p. 4). "Research studies of computational estimation are rare, because such skills are difficult to test" (Reys, Rybolt, Bestgen, & Wyatt, 1982, p. 184).

Standardized achievement tests usually include a section on computation but do not cover mental computation and rarely include computational estimation. Assessment procedures must be developed to profile an individual's performance and to monitor progress over the years. This is a challenge, but possible. The microcomputer has the potential for this type of assessment. Alternatives to traditional testing need to be explored.

Beyond the classroom, the demand for the use of these skills is much different. Adult surveys show that estimation and mental computation are used in more than 80% of all real-world problem-solving situations (Reys et al., 1986). Adults utilize estimation more often than exactness. Claiming that adults solve most everyday calculative tasks by applying methods quite different from those taught in school mathematics classes, Maier (1977) dubbed these unconventional and often untaught procedures as "folk math."

> Some of the general differences between school math and folk math are clear. One is that school math is largely paper-and-pencil mathematics. Folk mathematicians rely more on mental computations and estimations and on algorithms that lend themselves to mental use. When computations become too difficult or complicated to perform mentally, more and more folk mathematicians are turning to calculators and computers. In folk math, paper and pencil are a last resort. Yet they are the mainstay of school math. (p. 86)

Benefits of Estimation--What Are They?

Students benefit from instruction and from their experiences with estimation. With maturity the students' conceptual base in mathematics broadens and deepens. They can view problems in a variety of ways. The calculator provides opportunities for exploring without the burden of pencil-and-paper calculations. The students' daily routine changes when they become owners of the ability to estimate.

One benefit that can be achieved through estimation and mental computation is a thorough understanding of mathematical concepts. Estimation offers an alternative way of developing concepts (R. E. Reys, 1986). The act of estimating in measurement, for example, develops better understanding of the measurement and familiarity with the size of the basic unit. The act of estimating also enhances the ability to deal with numbers sensibly. Without actually counting, very young children can estimate the answer to questions like, "Who has the most?" The beginning of conceptual understanding might begin with estimation activities. Estimate how many 12s there are in 144. The constant subtraction function of the hand-held calculator can be used to lay the groundwork for an understanding of the division concept. Activities where students must estimate how many out of 100 there are lay the groundwork for percent (O'Daffer, 1979). Barbara J. Reys (1986) observed "that the time spent developing basic concepts through a mental computation and estimation approach greatly enhances and gives meaning to later work with exact computation" (p. 33).

The second benefit of estimation is the opportunity it offers for mathematical thinking. Mathematical thinking and problem solving are inseparable. Estimation provides a natural context not only to develop but also to practice many important thinking skills (R. E. Reys, 1985). In problem solving, the issue is examined, and a decision is made on what type of answer is needed. The students, using mental flexibility, examine multiple solutions. Strategies and solutions are accepted or aborted using key thinking processes. As the solutions are checked for reasonableness of results the following is considered:

> If students are encouraged to estimate the answer to a problem before becoming involved in a systematic solution, the estimation process frees students to think about the problem in a nonmechanical way. If children are encouraged to estimate the answer to a problem after they have solved it, they will see whether their specific answer makes sense and will have gained a valuable

technique for checking solutions to word problems. (O'Daffer, 1979, p. 47)

When looking ahead in a problem, the first decision is whether an estimated answer will suffice or is an exact answer required? Attendance at a ball game requires only an estimate for the size of the crowd. However, the cost of electrical consumption needs to be calculated to an exact amount.

Mental flexibility, viewing numbers in a variety of ways, is probably the most difficult skill to accomplish. The mind-set for exactness in all mathematical problems seems to overshadow estimation which is the direct opposite of exactness, or preciseness. Using the strategies of rounding (the predominant strategy), front-end estimate, averaging, and compatible numbers increases students' performances (R. E. Reys, 1985). "As a repertoire of estimation strategies increases through instruction and practice, students become more aware of the choices that can be made" (R. E. Reys, 1985, p. 39). Students become more analytical and exercise greater judgment in the decision making process. Tunnel vision and the one-right-answer syndrome will decrease. Several strategies might be applied to the same problem. If one strategy becomes messy or laborious, another is tried. Willingness to try another approach helps build self-confidence--an essential part of mathematical thinking.

Once an estimate is formed the estimate is evaluated for reasonableness. Generally two types of criteria are used. One is related to the context of the problem. A check is made to see whether the answers are realistic for the given situation. The second criterion involves estimation and is related to number manipulation. When multiplying 2.4 times 7.06, the resulting answer will be in the teens. Or when adding 134 and 689 the resulting answer will be only three digits. This second criterion requires the kind of number sense that comes from the knowledge of numbers and skill in manipulating them. Encouraging regular, but not perfunctory, checks should be the central focus of mathematical instruction.

The third benefit of estimation is the support the skill gives to questioning and checking the reasonableness of answers. The widespread use of the hand-held calculator has added importance to the ability to estimate and recognize reasonable answers.

This was supported in an interesting study by Reys, Bestgen, Rybolt, and Wyatt (cited by Schoen, 1987).

> A calculator was wired to give incorrect answers. Students and adults who were good estimators were told to use the defective calculator to check their estimates. Even those good estimators, when confronted with the answer on the calculator, usually accepted it as correct and assumed their estimate was in error. This research suggests that students often place too much trust in a calculator's answer. (p. 29)

The ability of the hand-held calculator and the computer to free-up the student from cumbersome calculations has also created a definite need for the skill of recognizing the reasonableness of an answer. The use of these tools makes a strong case for the inclusion of estimation into the curriculum as a tool to judge reasonableness.

The fourth benefit is that estimation experiences may be helpful for developing a positive attitude toward mathematics--supporting the affective domain and the self concept. Estimation activities can be enjoyable. They also provide a new kind of freedom for students by challenging them to devise their own procedures. Children also are not competing against other children for the "right answer". The competition is with themselves to improve the accuracy of their own estimates (O'Daffer, 1979).

Instructional Implementation

Characteristics of Good Estimators--What Are They?

Few researchers have examined strategies or processes used by estimators. To investigate this, Reys interviewed 59 good estimators from a sample population of 1200. He found that all good estimators had quick and accurate recall of basic facts and an understanding of place value. They used their knowledge of the properties of numbers and order of operations, preformed mental computations quickly, and had a tolerance for error. They had the ability to change data to a mentally manageable form (reformulation), had the ability to restructure a problem to a more mentally manageable form (translation), and had the ability to adjust an initial estimate (compensation) (Suydam, 1984).

These three key processes used by estimators were first identified by Reys and his colleagues (1982). Reformulation is the process of altering numerical data to produce a more mentally manageable form. This is accomplished by rounding, using front-end or left-end numbers, or substituting compatible numbers which are relatively close to the original numbers but which are easier to manipulate. Translation is the process of changing the mathematical structure of the problem to a more mentally manageable form. Compensation is the adjustment made to reflect numerical variation that comes about as a result of reformulation or translation. That is, the actual answer may be less or greater than the estimate because numbers were, for instance, rounded.

Schoen and his colleagues (1981) taught fourth, fifth, and sixth graders to use front-end estimation and rounding with addition and multiplication. Their study showed estimation strategies can be taught in a short period of time. The pupils became better estimators by adopting a valid estimation strategy. Replacing computational drill by instruction in estimation did not have an adverse effect on computational skills. Good estimators were usually able to judge the magnitude of the relationship between their estimate and the exact answer by noting whether they had rounded to higher or lower numbers.

Progress is being made in understanding the skills that a good estimator employs. "We know that systematic instruction on computational estimation results in significant improvement in students' performance...and reinforces and stimulates good problem-solving processes" (Reys et al., 1986, p. 5).

Characteristics of Instruction--How Is It Taught?

Usiskin (1986) states four reasons to use estimation. In many situations there is no choice but to estimate. Estimates are used to increase clarity or ease of understanding. Estimates are used for facility in simplifying later work or making the answers more reasonable and realistic. At times estimates give consistency to the information being compared.

The reasons for teaching estimation are not the same as the reasons for using estimation. Instruction in estimation is often misleading or misunderstood. In some situations the only alternative or the preferred alternative is estimation. Estimation procedures can lead to new insights about exact procedures. Procedures which are inappropriate for obtaining exact answers may, in fact, be useful procedures for estimating. Estimation ideas affect student grading and placement. Teachers also need to be more aware of how estimation applies to standardized test results. To ignore instruction in estimating gives students a distorted view of mathematics (Usiskin, 1986).

Estimation, much like problem solving, requires a variety of skills and is developed and improved over a long period of time; moreover, it involves an attitude as well as a set of skills. Like problem solving, estimation is not a topic that can be isolated within a single unit of instruction. To be truly effective, a careful integration of estimation must occur. It must permeate many areas of the existing curriculum, and to be developed effectively, it must be nurtured and encouraged throughout the study of mathematics. A comprehensive estimation curriculum must address several areas: development of an awareness for, and an appreciation of, estimation; development of number sense; development of number concepts; and development of estimation strategies (B. J. Reys, 1986).

Unlike specific strategies which can be taught in individual lessons, these aspects of estimation can be viewed as threads that need to be woven regularly into the instruction of current topics of mathematics.

Developing an Awareness

The process of developing computational estimation must begin with an awareness of what estimation is so that students develop a tolerance for error. Obtaining an answer that seems reasonably close to the exact answer seems strange to students because most mathematics instruction emphasizes finding the exact answer. The importance of error tolerance in estimation is supported by Reys and his colleagues (1982) who found all good estimators to have a tolerance for error.

The first hurdle to overcome is the mind set for exactness. Students need to begin to view estimation as a valid and useful tool. This "grows out of an understanding of what estimation is and why it is done as well as knowing how to estimate quickly and easily" (Trafton, 1986, p. 17). The initial effort to overcome this hurdle should include many examples of where estimated amounts are useful. This helps students realize approximations are part of daily life.

Situations where only an estimate is required and obtained should be emphasized. The teacher should attempt to replace or overshadow the common textbook exercise that requires first an estimate that is to be used as a check on the following exact computation. This kind of exercise leads students to a wrong impression regarding the role of estimation in mathematics. Many students have difficulty in seeing the purpose of estimating to check the reasonableness of the exact computation. The value of estimating for this purpose eludes them because they have usually inappropriately computed the problem first and then merely rounded the answer to get an estimate. Teachers need to guard against this inappropriate practice.

To make estimation part of the students' everyday experience, use real-world situations and settings. When students recognize how valuable estimation can be in this context, they will be more inclined to estimate than to compute exact answers. Use easy and obvious examples in the initial stages with appropriate language like close to, just about, between six and seven but probably closer to seven. Accept a range of acceptable estimates. Students need to be convinced that estimation is easy and worthwhile. A wide variety of responses adds richness to the lesson as students share how they deduced their estimate. Others gain new insights by listening. Having students respond orally instead of recording their work on paper also promotes mental estimation. In this way they are shown that estimating is quick and easy because it can be done without the written computation. If paper and pencil were required, students would feel the need to process thoroughly and produce an exact answer; they would mistakenly overlook the benefits of knowing how to estimate.

As the teacher guides students through activities built upon the previous ideas, their awareness and recognition of estimation as a valuable tool develops. For this to occur, experiences with estimation must be planned regularly and integrated into daily assignments and into lessons dealing with computation and problem solving.

Development of Number Sense

Flexibility in thinking and decision making continually increases as one acquires greater experience with numbers, and the relationships and factors that affect them. Estimation experiences can provide opportunities to strengthen number sense and develop sensitivity to factors that affect the processing of numbers.

Experiences involving a range of estimates need to be provided. Sometimes a rough, or ball-park estimate, is all that is necessary. Other times a more precise estimate is essential. The situation usually determines the choice of estimate. Practicing different estimation approaches for the same situation is also valuable. Follow the practice with a "how do you think" session where different methods are described and evaluated by students. In this way students begin making decisions by matching the appropriate strategy(s) to the situation.

Fine tuning the estimation process by developing a sense of the relationship between an estimate and the exact answer, is a developmental process. Recognizing whether an estimate is an overestimate or an underestimate is useful. This requires insight into number concepts and a good sense of number relationships; these develop with practice. In the early developmental stages, refined estimates are not required; it is not critical that the initial estimates be close. If students are not given instruction and not given frequent opportunities to apply this new skill, ball-park estimates will have to suffice.

The greatest concern about the failure to teach estimating skills is, however, the "students' failure to recognize when answers are not sensible" (Trafton, 1986, p. 28). This is the major reason for teaching estimation. It gives students a means of checking to see if an answer is reasonable. Further study is needed to discover the factors involved in recognizing unreasonable answers (Trafton, 1986).

Providing noncomputational situations encourages students to supply a reasonable answer. Having them identify the number of digits the answer should contain is a quick useful check for reasonableness. Giving computation examples that have been solved and asking students to quickly check to find unreasonable answers is another way to refine estimation skills.

Development of Number Concepts

Developing the concept of number size and developing the skill of estimation are complimentary processes. As students internalize the size of a number, it is possible for this to help them develop the ability to estimate size. When this skill is refined it serves to reinforce the original concept of a number.

Rational number concepts compared with whole number concepts are more difficult for students to conceptualize. This is supported by the research of Behr, Post and Wachsmuth (1986) which indicates that "learning of rational number concepts is more complex than originally thought and is in fact very difficult for many children" (p. 111). Whole number dominance seems to interfere with this concept. Fractions need to be regarded as numbers which are not only parts of a whole but as ratios or as the quotient of two integers. "It seems reasonable to assert that it is not possible to estimate satisfactorily the sum, product, difference, or quotient of two rational numbers unless one has the ability to determine the relative size of two or more rational numbers" (Behr et al., 1986, p. 105). Determining the size of a rational number requires an understanding of the relationship between the numerator and the denominator. Estimation is an excellent vehicle for developing this concept.

Development of Estimation Strategies

"The mathematics curriculum of the 1970s and early 1980s included rounding as the core estimation strategy or, more usually, the only strategy" (B. J. Reys, 1986, p. 34). Although rounding is a strategy that aids a student in estimating quickly by making numbers more manageable, it has not been regularly used this way by the students. It is also not the only strategy that they should use. A good estimator is able to select and apply a strategy that fits the problem.

Front-end strategy

The front-end strategy is particularly appropriate for young students. It can be modified for the four main operations in whole numbers, mixed fractions, and decimals. The most common application is for addition. The "front-end," or left-most digits, is the focus. It is a two-step process.

1. Total the front-end amounts. This requires only a minimal amount of mental computation.

2. Adjust or group the other digits. This step is a powerful tool; it can be suited to the particular mathematical skill of the user (B. J. Reys, 1986).

This strategy can be introduced first by using money as shown here.

\$2.58	1. Total the front-end (dollar) amounts.
\$1.87	2 + 1 + 0 + 4 = \$7
\$ .26	
\$4.41	2. Adjust...Group the cents amounts to form dollar amounts.
	58 and 41 make around \$1
	87 and 26 make around \$1

So, \$7 (initial estimate) + \$2 (adjustment) = \$9

This same process works well with whole numbers as well as fractions and decimals. This strategy can be utilized with subtraction, multiplication, and division.

4319	Think:
1533	4 + 1 = 5 or about 5000 plus about
976	2000 more equals about 7000.

1 4/5 + 2 1/8 + 3 1/2 Think:

1 + 2 + 3 is 6 plus 2 more equals about 8.

1.4 + 12.9 + 5.81 Think:

1 + 12 + 5 equals 18 plus one more is 19.

Clustering strategy

The clustering strategy is used when a group of numbers converge around a common value. It eliminates the mental calculation of a long list of front-end or rounded digits. This strategy is limited to a certain type of problem. It is one many students discover and use on their own (B. J. Reys, 1986).

Attendance at the Fair

Monday	72 378	1. Estimate an "average". These four numbers cluster, or "average", about 70 000.
Tuesday	63 915	
Wednesday	68 490	
Thursday	74 918	
		2. Multiply the"average" by the number of values. 70 000 x 4 is 280 000.

Rounding strategy

The rounding strategy involves rounding the numbers first then computing with the rounded numbers. The third step is adjustment. This strategy is a powerful and efficient one for estimating products of two multidigit factors (B. J. Reys, 1986).

	round up	
28 x 56	30 x 60	1800 is an overestimate. So, I'll adjust down: 1800 -

	round down	
62 x 23	60 x 20	1200 is an underestimate . So, I'll adjust up: 1200 +

	down up	
62 x 79	60 x 80	You can't really tell. So, I'll say 4800.
36 x 75	40 x 70	Since both numbers were close to the middle, I'll round one up and one down: 2800.

Compatible number strategy

Compatible numbers are those that are easy to manipulate, or easily fit together. The user takes a global look at all the numbers involved and changes or rounds each so it can be paired usefully with another number. This strategy is particularly effective when estimating division problems (B. J. Reys, 1986).

	These are compatible sets:	These are not compatible sets:
7) 3388	7) 3500	7) 3000
	8) 3200	7) 3300

This strategy can also be used for addition problems.

Look for hundreds!

27	27 and 81 is around 1 hundred.
49	49 and 56 is around 1 hundred.
38	38 and 65 is around 1 hundred.
65	
56	The sum is around 3 hundreds.
81	300 is my estimate!

Special number strategy

This strategy overlaps several others previously discussed. Special values include powers of 10 and common fractions and decimals. This strategy is best taught along with the development of fraction, decimal, and percent concepts. This and the previous strategy illustrate what estimation really is--"the process of taking an existing problem and changing it..." (B. J. Reys, 1986, p. 42).

	Think:	Estimate:
7/8 + 12/13	Each near 1	1 + 1 = 2
23/45 of 720	23/45 near 1/2	1/2 of 720 is 360

Integrating Estimation into the Curriculum At the National and State Level

Many educational organizations agree that estimation should become an integral part of the mathematics curriculum. On the national level, the National Council of Supervisors of Mathematics (1977) noted in a position paper, Basic Mathematical Skills - Ten Basic Skills Areas:

> Students should be able to carry out rapid approximate calculations by first rounding off numbers. They should acquire some simple techniques for estimating quantity, length, distance, weight, etc. It is also necessary to decide when a particular result is precise enough for the purpose at hand. (Curriculum Coordinating Committee Report, 1987, p. 11)

In addition, the NCTM's list of 10 basic skills includes a call for students to become alert to the reasonableness of their computational results.

> Due to arithmetic errors or other mistakes, results of mathematical work are sometimes wrong. Students should learn to inspect all results and to check for reasonableness in terms of the original problem. With the increase in the use of calculating devices in society, this skill is essential. (Curriculum Coordinating Committee Report, 1987, p. 11)

This trend is also reflected in An Agenda for Action (NCTM, 1980) which includes among the recommendations for school mathematics in the 1980s that teachers "incorporate estimation activities with all areas of the program on a regular and sustaining basis, in particular encouraging the use of estimating skills to pose and select alternatives and to assess what a reasonable answer may be" (p. 7 - 8). More recently the NCTM released a set of Curriculum and Evaluation Standards for School Mathematics (Standards) The Commission on Standards of the NCTM (1987) proposes the following standards for grades K - 4. Diminished emphasis on computational proficiency will leave more time for the development of other mathematical content. A greater emphasis is to be placed on problem solving and mathematical reasoning, measurement, geometry, and estimation. From the Standards for grades 5 - 8, the proposed changes concern computational proficiency which includes de-emphasis on paper-and-pencil computation with fractions and decimals; instead it is recommended that children be taught to become more adept at estimation with familiar fractions and their decimal representations and with mental computation involving familiar fractions and decimals. Using a calculator should be emphasized as a legitimate method of computation. Children should be expected to choose an appropriate method of computation according to the conditions presented by the problems. Legitimate computational methods include paper and pencil, mental arithmetic, estimation, and calculators or computers (Thompson et al., 1988).

On the state level, the Curriculum Coordinating Committee's report A Guide to Curriculum Development in Mathematics (1987) issued by the Iowa Department of Education describes a curriculum for a changing society. "Problem solving (the most important), mental computation, estimation, and reasonableness of results are the overriding themes identified...for the entire curriculum model" (p.10).

At the Local Level

Students involved in estimation activities at all grade levels will not be reluctant to estimate. They will develop new techniques, improve the accuracy of their estimates, and develop confidence in their mathematical ability.

Students need to know that there are no specific rules to follow in making many types of estimates. The focus of estimation is not on right answers, but on improving the accuracy of their estimates. Activities should be kept simple and sequenced. Varying the experiences through the use of puzzles, games, contests, and calculators satisfies the students' need for variety (O'Daffer, 1979).

Controlling some variables of the estimation experience is necessary to force estimation instead of exact computation. The mental nature of the experience is stressed by recording only the estimate on paper or chalkboard. The experience is short and timed, using numbers that encourage and reward estimation. If the experience is open-ended, the ensuing discussion furnishes insight into the students' conceptual understanding. Narrowing the focus of estimation concentrates the experience on the operations being developed with paper-and-pencil computation. For example, teachers should have students estimate the product of two decimals before introducing them to the algorithms which produce the exact answers (R. E. Reys, 1988).

Concluding Remarks

The desired outcome of experiences with estimation is to apply the skill by actually performing it. The student needs to know what estimation is, when and how to use it, but most importantly, to actually experience estimation--to do it. The teacher, likewise, must provide experiences and guide them in estimation; this is how concepts are built. If the process stops here, however, there is little learning value to the student; the ideas must be applied to real situations. Much discussion on how to estimate is necessary. Process building experiences such as this are necessary if the student is going to expand the concept of estimation. The process of learning to estimate cannot end with the structured activities in the classroom. It must become so natural that it will become automatic when needed in real situations. "What", "when", and "do" are the three key words to guide learning and teaching estimation skills.

Experiences with estimation are especially appropriate for middle level students. They are moving from concrete manipulative experiences to abstract representations of experiences. They are also progressing from the concrete operational stage of development into the formal stage of development. Estimation skills fit this developmental stage especially well.

After reviewing the literature on estimation, it is evident that students exhibit a lack of skill though adults utilize the process regularly. With the escalated use of computers and calculators in the information age, it is imperative to improve estimation skills. There is also a clear and pressing need for experimental research on estimation and how it can be taught most effectively.

For instruction in estimation skills to be offered to all students, it must be included as an integral part of the mathematics curriculum. In reality textbooks drive the curriculum in most school districts, even though it appears on the surface that the curriculum determines the textbook. The textbook companies are not, however, the forerunners of change today. Why? Because too much money is invested by book companies to have a publication that might not sell even though it is designed around current research. The economic risk is too great. School districts continue to purchase the textbooks that are available even though they may fail to meet current needs of students. Educators are beginning to become more and more aware of the limitations of most textbooks. Local curriculum groups are becoming better informed and are, therefore, becoming more active in creating local curriculum. This takes a great deal of time and personal dedication. If textbooks are indeed determining the curriculum, they should be developed based upon sound research. Educators need to demand these changes and support the companies that meet the needs of the students of today.

Mathematics curriculum has changed very little in the past twenty years. Emphasis is still on calculating the algorithm. This understanding and manipulation of numbers is necessary, but not the only end-product. The process is merely a means to gaining further understanding of mathematical concepts. Problem solving and estimation must become interwoven into the core curriculum of mathematics. Both are valuable skills used throughout life.

Why are these skills so poorly taught in the classroom? Educators need guidance and help with putting research findings into practice in the classroom. A wide gap exists between what is practiced in today's classroom and what should be included in today's classroom instruction. The ideas presented in this paper have been studied to narrow that gap in my classroom.

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