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Writing in the middle school mathematics classroom

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A successful whole language reading and writing program aimed at students lacking in comprehension skills led to the development of a writing program in the mathematics classroom. Communication has been promoted as a key component in mathematics. The purpose of this action research was to research and implement whole language strategies for communication into the mathematics classroom. Based on reasoning, connecting, and communication, strategies for speaking and writing about mathematical concepts and problem solving were taught and analyzed within a sixth grade classroom. Writing was also examined as an assessment tool. Writing was found to have value in both the learning process and in the communication.

WRITING IN THE MIDDLE SCHOOL MATHEMATICS CLASSROOM

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A successful whole language reading and writing program aimed at students lacking in comprehension skills led to the development of a writing program in the mathematics classroom. Communication has been promoted as a key component in mathematics. The purpose of this action research was to research and implement whole language strategies for communication into the mathematics classroom. Based on reasoning, connecting, and communication, strategies for speaking and writing about mathematical concepts and problem solving were taught and analyzed within a sixth grade classroom. Writing was also examined as an assessment tool. Writing was found to have value in both the learning process and in the communication.

WRITING IN THE MIDDLE SCHOOL MATHEMATICS CLASSROOM

Introduction

The focus of this research arose from success. I had successfully implemented a whole language reading and writing program aimed at students lacking in comprehension skills for the previous three years when I began this study. The whole language program I developed for my students combined cooperative learning techniques with a reciprocal reading program providing opportunities for active participation and student-directed learning.

Specifically, the whole language program I developed taught directly five different comprehension strategies: webbing, summarizing, questioning, predicting, and using context clues to discover meaning of unknown vocabulary. Instruction of these strategies was deliberate and complete, usually requiring the first four months of each school year.

When students were independent in the use of these five comprehension strategies, cooperative groups were formed and students were then introduced to the reciprocal aspect of instruction. They became responsible for their own learning within each cooperative group. Using a journal page that included each of the five strategies taught (see Appendix A), students rotated responsibilities, each employing and writing about one strategy for a day. Students then shared their work with their group members, and used a set of rubrics (see Appendix B) developed to assess the quality of their work.

This program has been successful for several reasons.

1. The strategies taught are solid and teachable.
2. This program promotes independence and active participation.
3. Cooperative techniques are a developmental match at the upper elementary level.

4. Whole language concepts of reading, writing, speaking, listening are employed in a positive way in this program.
5. This program stresses metacognition through writing and questioning.
6. The act of reading and writing enhances the commitment of learning to long- term memory.
7. A deeper level of assessment of student's thinking was possible through examination of their writing than would be possible through quizzing for one word answers.

Following three successful years of using reciprocal teaching and whole language strategies, my next goal was to focus on mathematics using the information that had been gained through implementation of the reading program. From that goal, the problem addressed in this research project arose. The National Council of Teachers of Mathematics' (NCTM) Curriculum and Evaluation Standards for School Mathematics (1989), have promoted communication as a key component of mathematics. Therefore, I set out to determine how successful whole language strategies might be engaged in the mathematics classroom to enhance communication and learning.

Since I would be an active teacher/researcher in this endeavor, I found that action research as defined on page 3, best described the research methodology in which I would be engaged. During the process of the action research, the implementation of whole language strategies in the mathematics classroom and throughout the curriculum became the long-term goal. Writing in a mathematics classroom was more difficult and less readily accepted by the students and parents than was initially anticipated. I found that the ability to communicate in written and verbal modes was a prerequisite to the employment of whole language strategies for learning in mathematics. Therefore, the depth

of the action research for this project was limited to addressing the problem of the guiding of students' growth in their ability to communicate through written and verbal expression in the mathematics classroom.

Definitions

In this study, the following definitions will be assumed for each term.

action research - the collaborative efforts of teachers to identify an important problem and to develop a workable solution; on-site inquiry aimed at problem resolution rather than the production of research data and reports (Owens, 1993, p. 320)

active participation or active student learning - the ability to raise one's own questions, utilizing teachers and other resources to pursue self-defined goals (Good & Brophy, 1987, p. 499)

communication - involves the ability to read and write mathematics and to interpret meanings and ideas. Writing and talking about their thinking clarifies students' ideas and gives the teacher valuable information from which to make instructional decisions. Emphasizing communication in mathematics class helps shift the classroom from an environment in which students are totally dependent on the teacher to one in which students assume more responsibility for validating their own thinking. (NCTM, 1989, p. 95)

cooperative learning - a method of learning that includes four basic elements: positive interdependence (mutual goals, division of labor and materials, assigned roles, and joint rewards); face-to-face interaction; individual accountability; appropriate use of interpersonal and small-group skills (Johnson, Johnson, Holubec, & Roy, 1984, p. 8)

journals - booklets, notebooks, or folders where students keep personal reflections about their reading and writing [mathematics, or other content areas (Cooper, 1993, p. 352)

metacognition - thinking about one's thinking (Owens, 1993, p. 17)

reciprocal teaching - a process in which the goal is for students to take gradual ownership of their own learning and to be able to self-monitor that learning.

rubric - a mechanism for assessing (scoring, monitoring) student performance given the conditions for the problem and/or the task (Lockett, 1994, p. 3)

whole language - a professional theory, and a theory in practice, based upon the following beliefs: reading and writing are learned through really reading and writing; reading and writing must have a purpose that is valid to the child; process, product and content are all interrelated; respect for and trust between teachers and learners is necessary. (Edelsky, Altwerger, & Flores, 1991, p. 7-9)

The development of successful strategies for teaching language arts in a whole language classroom led to the goal of implementing whole language strategies into the process of communication in the mathematics classroom. Could the premises of the whole language theory be applied to mathematics instruction in a sixth grade classroom? Does writing affect learning in the mathematics classroom? These are the questions addressed in this action research.

Literature Review

This review of literature will demonstrate correlations and connections that can be made by information known to the whole language teacher and the process that mathematics teachers are experiencing in learning about communication in general, and specifically writing in the mathematics classroom. Both the language field and the mathematics field are in the process of aiming at a similar goal - better communication. Therefore, the first

question addressed in this review will involve the analogy between the pedagogical transition currently taking place in the fields of language, and in mathematics, specifically for communication and writing.

Second, this review will examine the specific purposes for writing in the mathematics classroom. The purposes for writing in the mathematics classroom seem to vary, but tend to be narrower than the purposes of writing in language arts.

Finally, research and my own teaching experience have yielded information about successful techniques for assisting student's writing in the mathematics classroom. These techniques will be discussed and analyzed through this review of literature.

Communication in Language and Mathematics Classrooms

Where are we, in education, headed in the development of curriculum and instruction? Changes in goals in both language instruction and in mathematics instruction reflect changes in society and the demands placed upon students who have completed their education. Language changes addressed in Literacy: Helping Children Construct Meaning include the following view:

> Educators and psychologists have noted the importance of comprehension as part of reading and have tried to understand what happens when a reader comprehends. ...the actual process of comprehension itself has not changed. ...But two things have changed: our understanding of how comprehension takes place and the literacy demands of society. In today's technological world, there is an escalating need for literate, critical thinkers who can fully participate in society. (Cooper, 1993, p. 4)

Similar goals are reiterated in Curriculum and Evaluation Standards for School Mathematics.

> Schools, as now organized, are a product of the industrial age. In

> most democratic countries, common schools were created to provide most youth the training needed to become workers in fields, factories, and shops. As a result of such schooling, students also were expected to become literate enough to be informed voters. Thus, minimum competency in reading, writing, and arithmetic were expected of all students...The educational system of the industrial age does not meet the economic needs of today. New social goals for education include (1) mathematically literate workers, (2) lifelong learning, (3) opportunity for all, and (4) an informed electorate. (NCTM, 1989, p. 3)

The basic principles in these two areas have changed and are highly interrelated for both language arts and mathematics. Learning and valuing learning, in both language and mathematics, encompass many aspects. Language experts hold the view that reading, writing, speaking, listening and thinking develop simultaneously as learners grow into literacy (Cooper, 1993). Math experts suggest that students should have numerous and varied experiences related to the cultural, historical, and scientific evolution of mathematics in order to learn to value mathematics (NCTM, 1989). They also view mathematics as the ability to solve problems, communicate and reason mathematically, and to make mathematical connections (NCTM, 1989). Within both mathematics and language, experiences that develop student understanding include multi-faceted aspects of each discipline. To develop literacy understanding, language experiences should include reading, writing, speaking, listening and thinking while mathematical understanding includes experiences related to the evolution of mathematics from a cultural, historical, and scientific perspective, using problem solving, communication, reasoning and connections.

Learning in both language and mathematics involves actively doing. Language experts profess that students learn to read and write by reading, writing, and responding to their reading and writing (Cooper, 1993).

Mathematical confidence develops as a result of studying mathematics, realizing that doing mathematics is a common human activity, and having numerous and varied experiences to encourage trust in their own mathematical thinking (NCTM, 1989). In other words, both the field of mathematics and language believe that the process of doing, students actively engaged in reading, writing or solving mathematical problems, is the best method of instruction.

Learning in both language and mathematics is based upon prior knowledge and connections within the student's background. Language experts note that prior knowledge and background are major elements in one's ability to construct meaning when reading, writing, or speaking (Cooper, 1993).

The Curriculum and Evaluation Standards state:

> As they [mathematics students] learn new ideas or solve new problems, students enrich their own thought processes and skills by drawing on previously developed ideas; this ability to integrate ideas and concepts fosters students' confidence in their own thinking as well as in their skills of communication" (NCTM, 1989, p. 85).

The following connections exist between the development of whole language in recent years and recent developments in the field of mathematics: (1) Connections are abundant between how students develop understanding of mathematics and how students develop language. (2) Society is demanding different skills and knowledge from students in both their ability to process mathematically and communicate (to read, write, speak, and listen). Society today demands that employees be able to think divergently when solving mathematics problems and to communicate their ideas clearly and effectively. Society is requiring adults that possess the ability to use language to communicate in varied forms. (3) Both mathematical development and language development are multi-faceted. Proficient language ability includes

reading, writing, speaking and listening. Proficient mathematical thinking includes problem solving, communicating, connecting and reasoning. (4) Language and mathematics are both learned best through a process of actively doing. To learn to read, one must read. To learn to write, one must write. To learn to speak, one must speak. To learn to listen, one must listen. To learn to solve problems, one must be presented with problems and be asked to solve them. To learn to communicate mathematically, students must be given opportunities to read, speak and write about mathematics. To learn to reason mathematically, students must be asked to question their thinking. To learn to make connections mathematically, students must experience mathematics and the progression of mathematics. (5) Language and mathematics development is based upon prior knowledge. As educators come to understand how students learn mathematics and how students learn reading, writing, speaking and listening connections between mathematical and language knowledge are being made.

The Goal of Communication in the Mathematics Classroom

What is the purpose, function, or goal of communication in the mathematics classroom? Perhaps to answer this question, a definition of mathematical communication must be established. Watson (1991) has described communication as "... not just through symbol manipulation in working towards the *right answer*, but [mathematical communication] includes listening to, reading about, writing about, speaking about, reflecting on, and demonstrating ideas" (p. 18). According to the NCTM Standards (1989), "Communication involves the ability to read and write mathematics and to interpret meanings and ideas" (p. 78). Shepherd (1990) defines communication as "...[the ability of students to] convey their ideas about

mathematics, deal with it in social contexts, make connections, and make known their thinking as they learn and become involved in mathematics" (p. 18). These definitions include both the ability to receive information and ideas, and the ability to express information and ideas.

Guidelines for implementing the NCTM Curriculum and Evaluation Standards assert the following function of communication about mathematics in the classroom:

1. *Communication helps students enhance their understanding of mathematics.* Both expressing ideas and listening to other's ideas deepens understanding through clarification of thought and the appreciation that people think in different ways.

2. *Communication helps establish shared understandings of mathematics.* By discussing and sharing ideas, students develop the need for a common language, appreciate the role of definitions, and eventually grasp the significance of discussing and clarifying assumptions.

3. *Communication can empower students as learners.* When we ask students to talk or write about their thinking, we express value in what they have to say, and students by presenting what they think, exercise power and control over their own learning.

4. *Communication promotes a comfortable environment for learning.* Talking and listening to others in small groups is an anxiety-free way to try out new ideas.

5. *Communication assists the teacher in gaining insight into the students thinking.* (Shepherd, 1990, p. 18-19)

Watson (1991) states two objectives for writing in the mathematics classroom. First, it allows students to practice communicating their understanding of mathematics through written expression. Second, it enhances the learning process in how students think about mathematics and the attitudes they have toward mathematics.

Again, according to Watson (1991), research results suggest the following three facts:

(1) writing in mathematics classes can produce a rich resource for enhancing teacher's ability to assess students' understanding of school mathematics;

(2) reading students' writing can have a direct influence in teachers' instructional practices, both short and long term;

(3) using writing in mathematics classes can positively influence student teacher interaction patterns.

According to Lappan and Schram (1989),

> Major goals of mathematical power are seeing connection and relationships, making sense of mathematical situations, reasoning and conjecturing, and having confidence in discussing one's ideas. [A] powerful means of communicating that is seldom used is writing about mathematics. Writing helps students clarify their ideas about a specific topic or problem and also helps them develop a better conception of mathematics. Having students write about how they tackled a problem and how the members of a group thought about aspects of the problem can help them realize that the teacher values their thinking and reasoning about a problem. (p. 16)

Writing in the mathematics classroom enhances metacognition or the ability to think about one's own thinking. Writing hones a student's ability to receive and express ideas. Both teacher to student and student to student interaction is positively influenced by writing in the classroom. And, teacher's insights into student's thinking increases, therefore improving the ability to assess student's understanding. Writing can be an empowering tool for both the student and the teacher.

Although writing in the mathematics classroom (a) has a broad base of support from the language discipline, (b) aligns with the NCTM Standards goals, and (c) is based upon sound purposes, it is probably the subject area in which students write least. Writing has never been seen as a natural part of

mathematics (Wilde, 1991). In addition, mathematics teachers must contend with trying to explain to colleagues, administrators, and parents that communication in the mathematics classroom is, in fact, important.

Successful Techniques

What techniques have teachers and researchers found to be successful when integrating the writing process into the mathematics classroom? Taylor found, that based upon her experiences as a first grade teacher, the following suggestions positively affect writing in the mathematics classroom:

1. Try to maintain an atmosphere that encourages pupils to feel free to express themselves.

2. When in doubt, remain silent and give pupils the chance to work through their mathematical difficulties.

3. Don't paraphrase for the students. Ask students to clarify their thoughts through their own expression rather than to comply with your thoughts.

4. Encourage constructive arguments.

5. Play dumb (without condescension) and let the students teach you as well as each other.

6. Don't focus on "right" answers. More than one solution frequently exists and alternative answers can create a positive tension that leads to discourse and interaction.

7. Allow time to work as individuals before sharing thoughts.

8. No best way exists to form small groups.

9. Take time to model expected etiquette for discussing in small groups.

10. Save the written work of students to evaluate and substantiate learning. (Taylor & Taylor, 1990, p. 36)

These techniques are as applicable in the first grade classroom as in the twelfth grade classroom.

Writing in the mathematics classroom is implemented in various ways. Letters about what is being studied can be written in order for teachers to learn about students' needs. Letters can include the following points: what students understand and what they don't understand (with examples); and what they are wondering about. Learning logs can be used to maintain a regular record (daily, weekly, or at the end of a unit) of learning as it happens. Learning logs are designed to teach students to write in their own language. Requiring a student to ask a question is often more difficult than answering one. Students can demonstrate their understanding of mathematics concepts by writing word problems; this process requires prior modeling and the opportunity to discuss writing word problems. Problem solving that involves the use of specific strategies can be reported on by students; the reporting can include both an expression of students' feelings while solving problems and their attempts at solving the problem (Kennedy, 1985).

Another form of writing used extensively in the language area that can be easily applied to writing in mathematics is journaling. "The journal is a diary-like series of writing assignments" (Nahrgang & Petersen, 1986). Journals can be used as a means for students to monitor their own understanding of mathematical concepts. Especially effective are dialogue journals in which the student's writing is read by the teacher and a response by the teacher is written. These communiques then become letters between the teacher and student (Wilde, 1991).

Writing in the mathematics classroom can be as individual and fluid as each group of students tends to be. Although there are techniques that are effective, flexibility in the process and trust between student and teacher is imperative. Kennedy (1985) states that:

> Because ...writing is tentative, exploratory and personal, it's also a little scary. To open themselves that much, your students must first know they can trust you. They must know you won't make judgments about their personal worth on the basis of what they've written, and that you'll keep confidential any admissions of ignorance..If that trusting atmosphere exists, your students - and you - will find that "writing in mathematics" opens a door to awareness and understanding not just of mathematics, but of how they think and learn. (p. 61)

Summary

There is a specific purpose and function for writing in the mathematics classroom. The purpose and function of communication in the mathematics classroom is to give and receive information, enhance and establish shared understanding, empower students, encourage metacognition, and to create a relaxed atmosphere that encourages learning. Another function of writing in the mathematics classroom is to provide the teacher with insights into student thinking.

Techniques and methods have been and are in the process of being developed in mathematics classrooms. Techniques that cultivate writing ability in the mathematics classroom include (a) providing a positive, uninhibited atmosphere; (b) avoiding "teacher talk" and avoiding paraphrasing student thought; (c) encouraging constructive argument; (d) stressing that there is not one "right" answer in mathematics; (e) modeling communication etiquette. Methods of writing include (a) letter writing; (b) learning logs; (c) students writing word problems; (d) journaling. The method that a teacher employs to write in the mathematics classroom can be fluid and reflect the needs of the students.

Connections between whole language and mathematics would indicate that writing has a place in the mathematics classroom. Enhancing learning and communication appears to be the function and purpose for writing in the mathematics classroom. Methods and techniques that have been tried for

writing in the mathematics classroom encourage success.

Methodology

This study began as an attempt to implement the basic principles of the successful reciprocal teaching program that I had developed for reading instruction. As stated in the introduction, the whole language program I developed for my students combined cooperative learning techniques with a reciprocal reading program. The program directly taught five comprehension strategies: webbing, summarizing, questioning, predicting, and using context clues to discover meaning of unknown vocabulary. Instruction of these strategies was deliberate and complete, usually requiring the first four months of each school year. When students were independent in use of these five comprehension strategies, cooperative groups were formed and students were then introduced to the reciprocal aspect of instruction. They became responsible for their own learning within each cooperative group. Using a journal page that included each of the five strategies taught, students rotated responsibilities, each employing and writing about one strategy for a day. Students then shared their work with their group members, and used a rubric to assess the quality of their work.

After successfully employing this program for three years, I set out to develop a similar program for teaching mathematics using the information that had been gained through implementation of the reading program. This original goal proved too lofty. The first problem that needed to be addressed was considered during the spring of 1994. That being the difficulty of applying a limited number of teachable strategies to mathematics instruction that would be similar to the comprehension strategies (webbing, summarizing, questioning, predicting, and using context clues). Through discussion with other educators, I

decided to view mathematics instruction through a slightly different paradigm.

Rather than developing specific strategies for understanding mathematics, I implemented the premise of the NCTM Standards that progressive mathematics instruction includes problem solving, reasoning, connecting and communicating. From that premise, I developed a journal form (see Appendix C) to structure the reciprocal mathematics discussions. This form included six sections: (1) a statement of the objective; (2) a section for calling up previous knowledge of the objective in which students were asked what they already knew about the objective; (3) a section for communicating what was learned during the direct instruction lesson by recording their understanding of the objective; (4) a section for recording reasoning by asking students to draw a picture or in some way state how the objective made sense to them; (5) a section for making connections to what they knew or needed to know before the objective became understandable, for instance, to understand equivalent fractions students must understand the size of fractions, multiplication and division, etc. and; (6) a section for writing problems related to the objective. The original plan was to have students record their thinking on the journal form and then to assign each student a section of the journal as their responsibility to share during the cooperative discussion the following day.

In conjunction with the journal form, I had planned to use an assessment instrument (see Appendix D) that I had developed during the fall of 1993 that sought to assess students' number sense. The instrument was a compilation of questions taken from various sources and related to the objectives taught in each unit of study. The assessment for each unit included a limited number of questions (usually six) that required students to explain their thinking.

My intention was to model use of the journal and encourage student talk

about their thinking during whole class instruction of each objective and small group discussion to develop the ability to write about their thinking. Assessment of understanding was to be made through the daily journaling and through the assessment at the end of each unit of study.

The research design used was action research. According to the document, Research Interpretation Project, prepared by the National Council of Teachers of Mathematics, which was edited by Owens (1993),

> Action research is characterized by the collaborative efforts of teachers to identify an important problem and to develop a workable solution. It is on-site inquiry aimed at problem resolution rather than the production of research data and reports.
>
> Historically, much educational research evolved from a comparison of treatments with different groups of students. Findings are often reported statistically, with the goal of generalizing from the random sample to an entire population., Educational research of this type has produced a large quantity of data but little information about the way education operates in naturalistic settings. Consequently, it appears to have had little effect in changing classroom practice. Because it is conducted in isolation, university-generated research is often seen as impractical and inaccessible to classroom teachers.
>
> According to Wallace, action research is now a well-established strategy whereby teachers may improve their work in classrooms. Their role is that of researcher. Stenhouse uses the phrase *teachers as researchers* to describe teachers who are developing their art as practitioners through a reflective approach to and ongoing inquiry about the activities of their classroom.
>
> Inquiry is an interesting way to describe the research of teachers in their teaching-learning environments. It implies a sense of discovery, a curiosity, an openness to explore different phenomena observed in the classroom. Although they are not always doing formalized research, teachers constantly assess and modify their actions and behavior in order to make student learning more meaningful. The inquiry view of research is tremendously liberating because it validates the everyday work of classroom teachers; it emphasizes the importance of teacher and student interaction as a source of information on learning and teaching. (p. 320)

As stated earlier, this was a lofty goal. As I was developing the journal form, I used the form with students. It was readily apparent very early that using the form in a reciprocal teaching format was not a reasonable goal. The writing aspect in the mathematics classroom was enough to address initially. My formal action research took place in the fall of 1994 and focused on writing in the mathematics classroom.

What actually happened the fall of 1994 was (1) an implementation of the journal form for guiding writing in the mathematics class, (2) use of the assessment designed to determine number sense or conceptual understanding, (3) use of materials and curriculum that encouraged writing and conceptual understanding and, (4) rich reflection on my part of this interrelated process.

The subjects of this study included forty-two sixth grade students from an elementary school classroom in a building of approximately 300 students. Mathematics was taught for approximately 60 minutes per day. Student abilities varied within the normal range and no special education students were included in the study.

One of the instruments employed to collect data included comparisons of actual student work through the course of the year. Data were analyzed through the use of a rubric designed to assess the quality of writing done by mathematics students. The process was also analyzed through themes that arose during the implementation.

The results of this study are (1) to be of use in further investigation of writing in the mathematics classroom, (2) especially designed to be of use to the teacher/researcher (me) of the study and, (3) to be of use to other teachers in their journey of teaching and learning.

Results, Discussion, Summary

Metacognition is an important aspect of learning. As I began to reflect on the reflections I have been making through the course of the past semester, I was overwhelmed. At times, I fluctuated between being excited about what had happened and frustrated. This change was stressful, yet at the same time, moved in the direction that mathematics research is recommending.

There are several different aspects of the teaching of mathematics that I have focused on and which show evidence of change in my classroom. Most obvious is the curriculum itself and the approach that I am taking to the curriculum. Writing in the classroom relates to the curriculum and my approach. Another aspect is the reaction and interaction of parents in this process of change. A major hurdle that I have addressed and continue to address is assessment. I have also been conscious of the student's self-confidence in mathematics. The topic that I have chosen and will address in this action research is writing in mathematics class.

Many purposes justify the writing that takes place in my classroom: (a) using metacognition (asking students to think about how they are thinking), (b) recording ideas introduced in the classroom, (c) communicating with parents, (d) committing ideas to long term memory, (e) learning technical writing skills, (f) and more precise assessment of knowledge. These are, however, the most important. Writing in the mathematics class has passed through several phases during the fall of 1994, and as problems have occurred, various solutions have been attempted. These results and discussion will reflect the progression that writing has taken through the course of the fall of 1994.

Writing in mathematics class is not a popular activity to introduce to sixth grade students. I have, however, used writing in both types of concept lessons:

understanding of fractions or equivalent fractions, and problem solving. In the initial stages of using writing in mathematics, I found the students that have typically done well in a more traditional approach involving number crunching struggled with the writing. Students that typically struggle with mathematics see little hope that anything will help them understand mathematics and resist because it has to do with mathematics. However, there is hope. With (a) direction, (b) commitment of the teacher to read the writing, and (c) practice, students can learn how to express their mathematical thoughts in writing.

I started the year with a unit on data analysis in which students communicated verbally with little emphasis on unstructured writing. Writing during that unit involved specific tasks. The emphasis during the data analysis unit was on collaboration and verbal communication to accomplish a task that involved data and manipulating data. We did do some writing on the concept of "average". Through this limited writing, I was able to somewhat see the challenge for some and pleasure for others that writing about mathematics was going to be.

Our next unit involved number relationships. We studied place value and comparisons. During this unit, I introduced the journal sheet that I had designed during the spring of 1994 to guide our reciprocal teaching of mathematics (see Appendix C). The journal sheets were assigned daily, but I did not read them daily. The journal sheets included a place to record the objective or concept that we talked about, a section for quickly writing down what the student already knew, a section for (a) what they learned, (b) how it made sense to them, (c) how it connected to what they already knew, sample problems, and (d) how they would explain this to someone else. For many students, this task was extremely frustrating. That was apparent by the great number of journal pages

that were incomplete. For a few, this task was an opportunity to do something that was relatively easy for them (writing) in mathematics class.

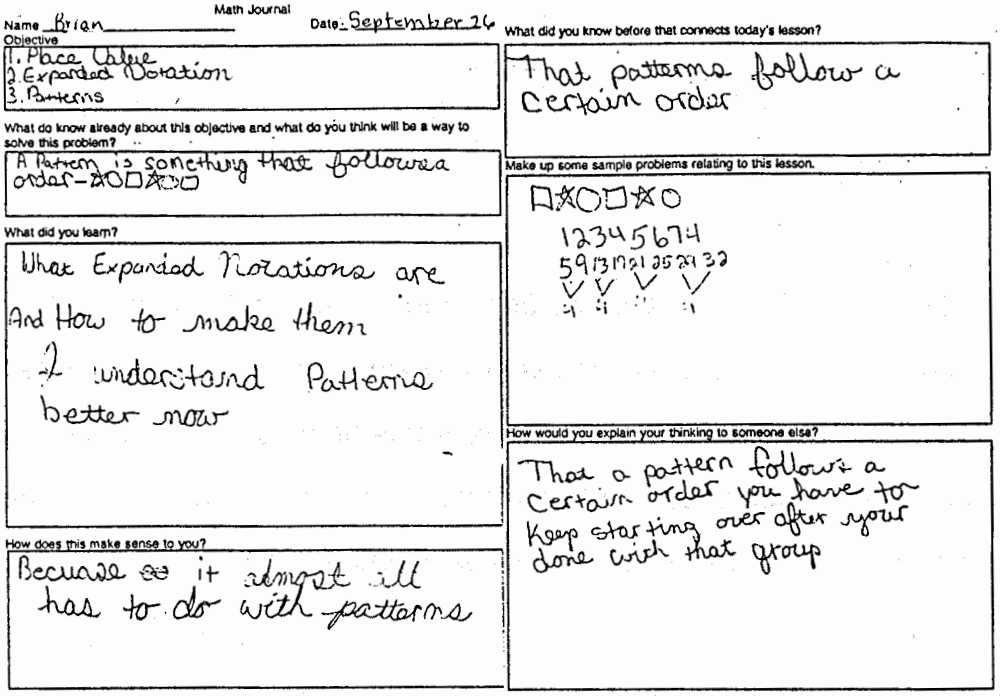
As stated earlier, one of the purposes of journaling was to record their present understanding of the concepts and ideas introduced. During our third unit on fractions, journaling became an even more important step in the learning process because new ideas were being introduced and the textbook was not being relied upon for information. This was one of the points of frustration for students and parents. Students were having a difficult time recording information in a new way, and parents (who were being asked to sign the journals each night after talking to their student) were not able to understand what their student was supposed to be learning when the journals were not clear.

This frustration led to the realization that I needed to be very clear about what response I expected from students in the journal and the need to model, and at times, write for them on the overhead, the way to keep a record of what we were studying. We worked through this problem by (a) me honing my skills at being clear and direct, (b) some lessons on the skill of technical writing, and (c) developing a rubric for directing the writing of the journals.

The rubric served two purposes. First, it gave students and parents the ability to judge the quality of the work in their journals and to hold them accountable for quality . Second, it provided a way for other people (my volunteer and student teacher) to help me read the journals and respond to them each day; it became evident that this was of great importance. Developing the rubric that we used was a thought-provoking process. I had to think through exactly what I felt represented quality journaling. The rubric is included in Appendix D of this paper.

Early examples of writing show an attempt to write that is very controlled and teacher directed. The students simply did not know where to start. At this phase, I was modeling on the overhead for the students so that they would have some idea of how to write. Although modeling the writing process seems contrary to exploring student thinking naturally, this was a necessary phase to help students begin to write on their own. Early examples shown in Figures 1 and 2, indicate the difficulty students had in expressing their own thoughts, especially when they were asked to make connections to other ideas or to make sense. All examples shown are labeled with pseudonyms for student names.

Figure 1. Early sample of journaling.



Math Journal

Name Brian Date: September 26

Objective
1. Place Value
2. Expanded Notation
3. Patterns

What do know already about this objective and what do you think will be a way to solve this problem?
A Pattern is something that follows a order-☆○□☆○

What did you learn?
What Expanded Notations are
And How to make them
I understand Patterns better now

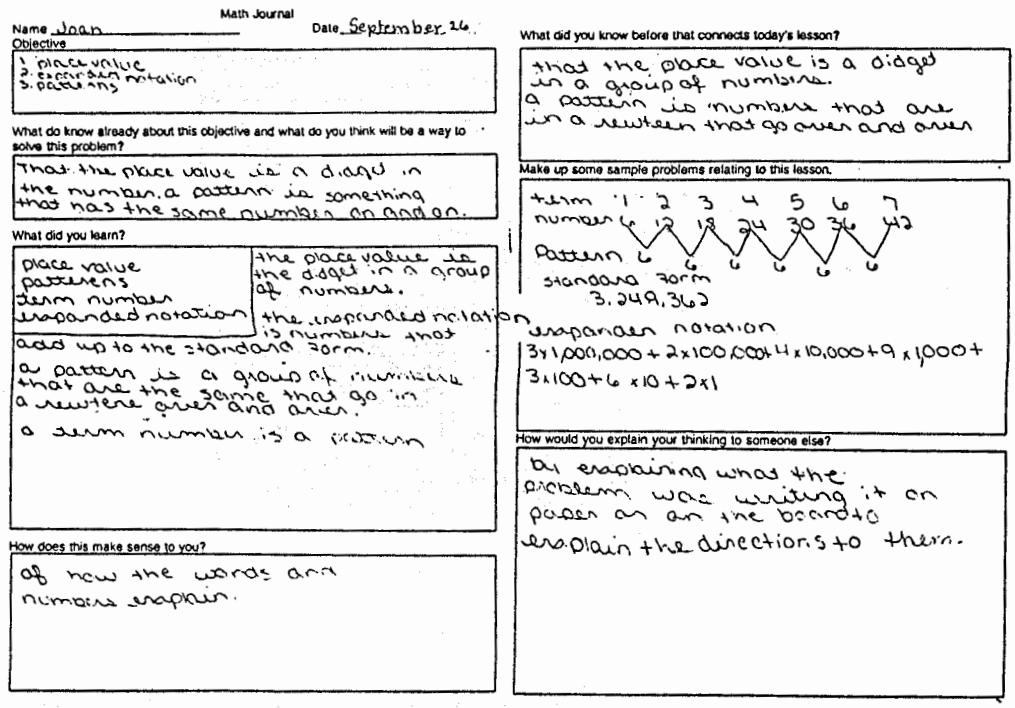
How does this make sense to you?
Because ∞ it almost all has to do with patterns

What did you know before that connects today's lesson?
That patterns follow a certain order

Make up some sample problems relating to this lesson.
□☆○□☆○
1 2 3 4 5 6 7 4
5 9 13 17 21 25 29 32
+4 +4 +4 +4

How would you explain your thinking to someone else?
That a pattern follows a certain order you have to keep starting over after your done with that group

Figure 2. Early sample of journaling.



Math Journal

Name Joan Date September 26

Objective

1. place value
2. expanded notation
3. patterns

What do know already about this objective and what do you think will be a way to solve this problem?

That the place value is a digit in the number. a pattern is something that has the same number on and on.

What did you learn?

place value
patterns
term number
expanded notation

the place value is the digit in a group of numbers.

the expanded notation is numbers that add up to the standard form.

a pattern is a group of numbers that are the same that go in a rewtern over and over.

a term number is a pattern

How does this make sense to you?

of how the words and numbers explain.

What did you know before that connects today's lesson?

that the place value is a digit in a group of numbers. a pattern is numbers that are in a rewtern that go over and over

Make up some sample problems relating to this lesson.

term 1 2 3 4 5 6 7
number 6 12 18 24 30 36 42
Pattern 6 6 6 6 6 6
standard form
3,249,362

expanded notation
3x1,000,000 + 2x100,000 + 4x10,000 + 9x1,000 + 3x100 + 6x10 + 2x1

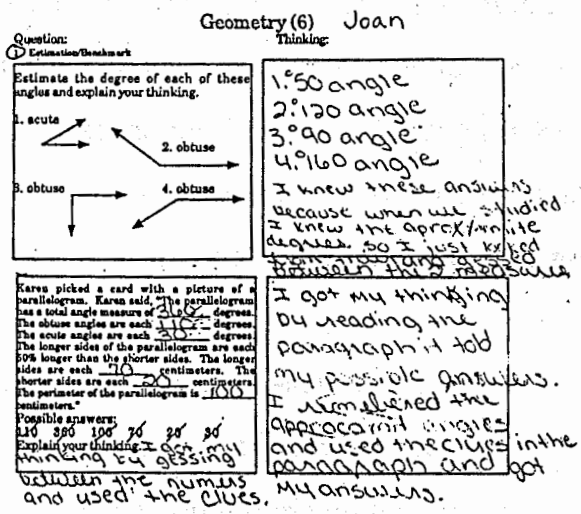
How would you explain your thinking to someone else?

by explaining what the problem was writing it on paper on the board to explain the directions to them.

We continued to use the journaling process during the next unit - geometry. By this time, students were becoming more proficient at journaling and I attempted to teach mathematical concepts much more clearly. Some students were responding, others were not. For most students, the quality of responses for the end of the unit assessment that asked students to record their thinking was improving.

The following is an example of an assessment activity used following our geometry unit. It demonstrates how this student improved in self confidence in ability to express herself. Her writing reflects more original thought and a greater ability to express her thinking.

Figure 3. Geometry assessment activity.

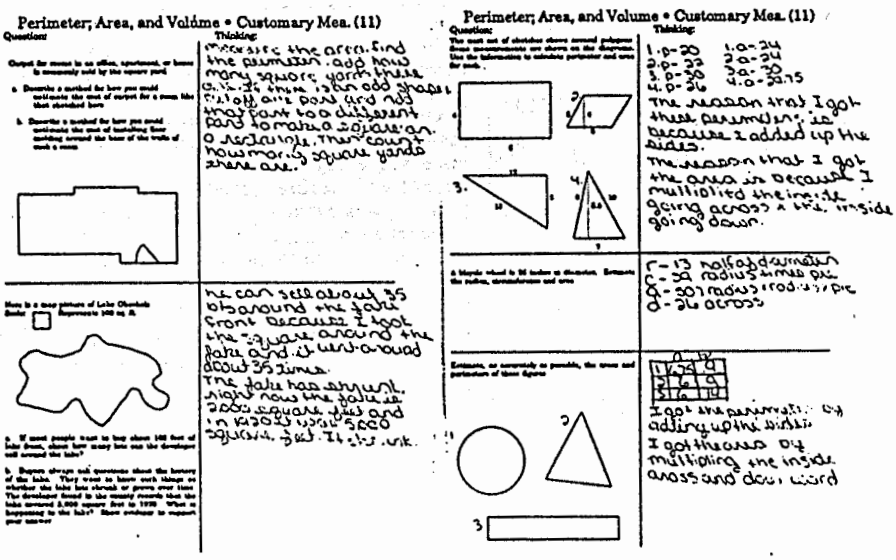


We followed the geometry unit with a unit on metric measurement. Students continued to write; some complained. At this point, we had just held parent-teacher conferences and I had explained the purpose of writing and the rubric. For the most part, parents responded in a positive way. A week after conferences, one student misunderstood the directions that had been given and became very frustrated at home with the writing. Her parents called and expressed their frustration. About the same time, another parent came to school expressing concern because her daughter had been forging her name on the journal sheets each night and because she was very concerned with the quality of her daughter's work (I was, too). That parent's second concern was that there had been an inconsistency in the scoring of the problem solving efforts between the student teacher and myself. A third parent came to school confused after the student teacher began teaching because her son was not grasping the

concepts of probability. I addressed the concerns of parents through parent - teacher communications. Most of these concerns were alleviated. However, a few parents remain skeptical of this "writing in mathematics" stuff. I also called a parent meeting to discuss mathematics today and to view a Marilyn Burns' video "What are You Teaching My Child?". Through talking with parents and further explanation of what we were attempting to accomplish by writing in the mathematics classroom, I have encountered both support and further skepticism.

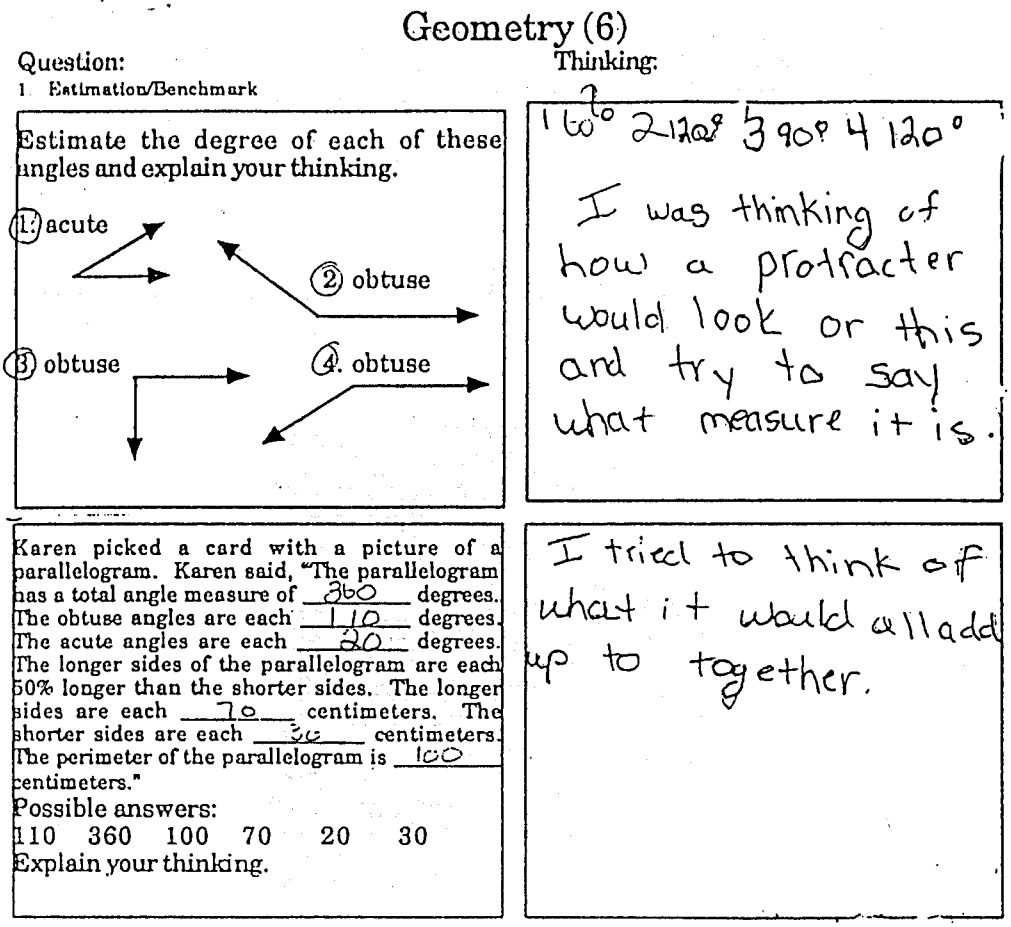
Journaling during the unit on measurement again suggested improvement in both the student's confidence in writing and their ability to express ideas and the assessment on the measurement unit shown in Figure 4 indicated more metacognition on the students' part. Especially noticeable is the improvement in the student's effort to explain her thinking. Answers are becoming more complete and reasonable. The examples in Figure 4 show this continued improvement.

Figure 4. Measurement assessment.



The following examples shown in Figures 5, 6, and 7, were written by the same student, Emily, and show the general progression of improvement of expression that was, for the most part, evident with all the sixth grade students. These examples were created over a period of about two and one-half months. My responses, as recorded on the geometry assessment, were helpful in guiding students expression.

Figure 5. Early assessment sample (September).



Geometry (6)

Question:

1 Estimation/Benchmark

Estimate the degree of each of these angles and explain your thinking.

1. acute

2. obtuse

3. obtuse

4. obtuse

Thinking:

1 60° 2 120° 3 90° 4 120°

I was thinking of how a protracter would look or this and try to say what measure it is.

Karen picked a card with a picture of a parallelogram. Karen said, "The parallelogram has a total angle measure of 360 degrees. The obtuse angles are each 110 degrees. The acute angles are each 20 degrees. The longer sides of the parallelogram are each 50% longer than the shorter sides. The longer sides are each 70 centimeters. The shorter sides are each 30 centimeters. The perimeter of the parallelogram is 100 centimeters."

Possible answers:

110 360 100 70 20 30

Explain your thinking.

I tried to think of what it would alladd up to together.

Figure 6. Later fraction assessment sample (October).

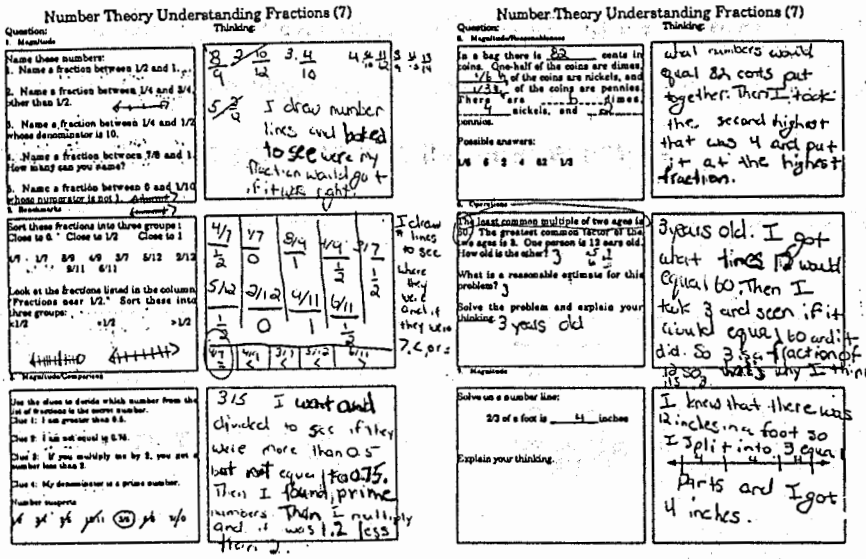
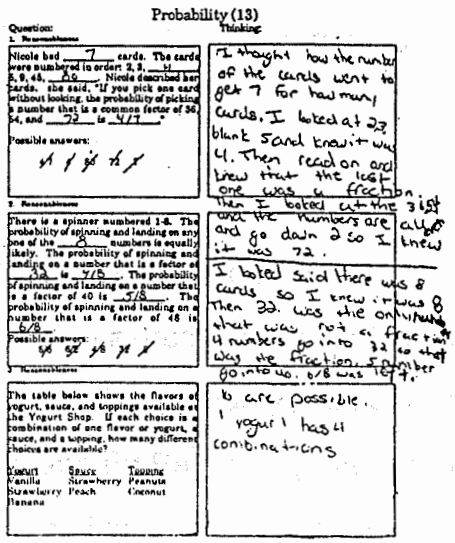


Figure 7. Probability assessment sample (November).



Writing was also a key aspect of problem solving. Students used a recording sheet and rubric (see Appendix E and F) to guide their writing. As is obvious by the following two examples shown in Figures 8 and 9, one selected from September, one from November, clarity of thought and completeness of thought were much more prominent in the later example.

Figure 8. September sample writing.

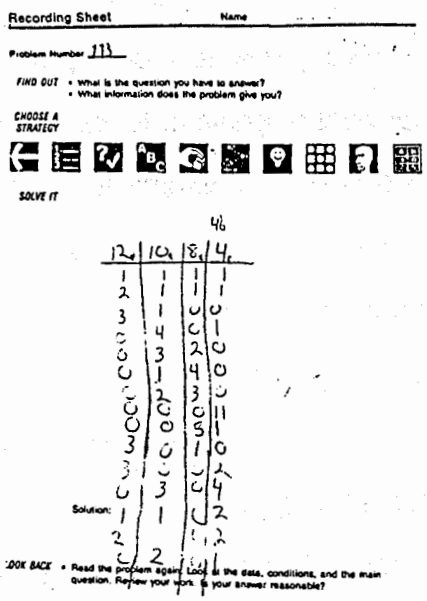
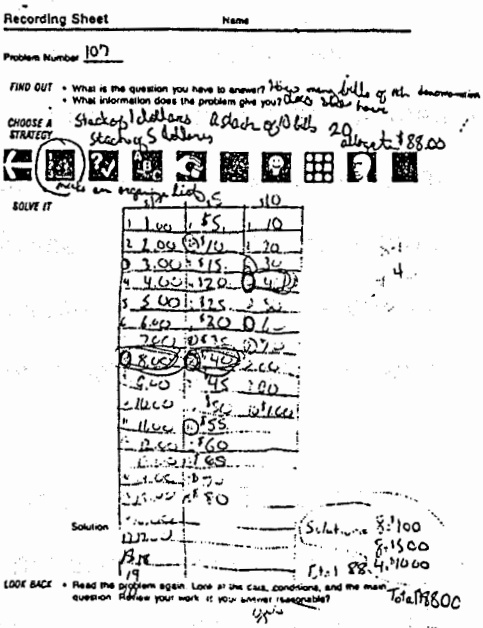


Figure 9. November sample writing.



Students continued to have mixed feelings about journaling at the end of the semester. Some were doing extremely well and others would prefer to be number crunching. I discussed this with Gary Nanke, Area Education Agency #7 Mathematics Consultant, and Ed Rathmell, professor of mathematics at the University of Northern Iowa, and they suggested some ways to make journaling a more positive experience. One suggestion was to limit journaling to once a week rather than every day. At that point, I was still feeling that journaling was

important on a daily basis. I did abbreviate the journaling form (see Appendix G) so that less writing was necessary. I was comfortable with that because students were still addressing the what did you learn, how does it make sense and how does it connect (which I feel is the heart of learning mathematics), but in a more succinct way and the three were integrated into one section.

Conclusions and Recommendations

Implementing writing in my mathematics classroom has been a challenge, but I do see growth in the students' ability to express their mathematical thinking in their journals and in the assessments that ask them to record their thinking. I believe that the journaling is actually the most beneficial for students that have typically not viewed themselves as strong mathematics students. I believe that traditional ways of assessment are not sufficient for assessing students' thought. I believe that some parents are resisting writing and some are supportive.

What can I suggest for others starting the process of writing in the mathematics classroom? First, as with most projects of value, writing requires time and commitment. The first experiences with writing for students who have not made the connection between communication and mathematical concepts may not be positive. Feedback to the students about their writing is essential. Giving quality, consistent feedback may require help for the teacher to read and respond in student journals. Seek help. Commitment to writing will bring rewards over time. For some students that have difficulty understanding mathematical concepts, learning will be enhanced through writing and their understanding of concepts will improve. For students that easily understand mathematical concepts, learning to express their understanding will enhance their ability to communicate ideas.

My recommendations for further study would include seeking answers to questions involving student thought processes. Which students benefit through acquisition of mathematical knowledge from writing and why? Which students tend to learn best by other methods and why? How can we effectively teach students to use communication as a tool of learning and as a tool of expression?

I would also recommend more dialogue between experts in the field of language development and experts in the field of mathematical concept development. What does one group of experts already know that can be acknowledged and used by the other group?

For my personal practice, where do I go from here? What would I do differently next year? Has the value I placed on writing changed? How can assessment more closely align with thinking about thinking? How can we make writing a more positive experience for students and parents?

We will continue to write. I have learned several things about writing from this investigation. Writing in my classroom will be fluid and continue to change.

I feel that writing does have value, especially for qualitative thinkers who struggle with quantitative thought. The value that I place on writing, may be less all - encompassing, however. For the student that responds and learns well through writing, it is an excellent tool that has been long overlooked. For the student that does not respond well to writing in the classroom, but processes mathematics concepts easily another way, writing may be necessary only in learning to communicate ideas. That student needs writing as a tool in a much different way. Writing has importance as a function of learning for some students, and as a means of communicating ideas for other students.

Next year, I may abbreviate the journaling assignments even further, shortening the process so students become better at being succinct. Another possibility I might consider for making writing in mathematics class more doable is to make connections with university students who want to journal with the students in my classroom. This would be plausible for me in this setting because I could pick up the journals and deliver them daily. This may provide students with more motivation to write personal and careful responses.

Assessment improves when students write. When students write, it is much easier to "see" when students are understanding and when they are not understanding a concept. It is also easier to correct a misconception. I will continue to ask students to record their thinking as they process problems.

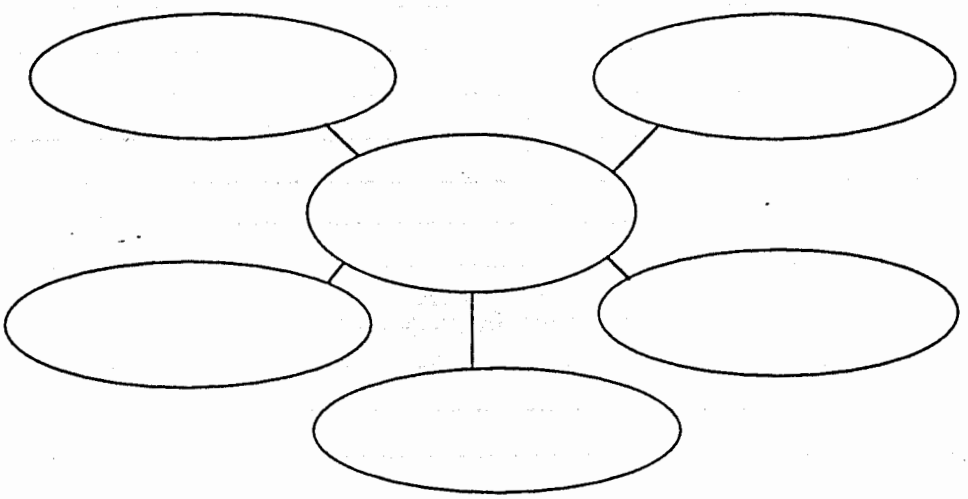
I probably will try to keep parents more informed about our goals and the method we plan to use to achieve them. Possibly, next year we will have a parent meeting earlier in the year.

My students and I share in the process of learning. I expect that I will understand more in March than I do in December, just as I understand more in December than I did in September. The journey is where learning takes place.

Appendix A

DAY/DATE TITLE NAME

WEB



QUESTIONS

1.

2.

3.

4.

5.

SUMMARY

VOCABULARY

WORD	PAGE #	MEANING FROM CONTEXT

PREDICTION

Appendix B

Webbing Rubric

0.....10.....20

Captures main ideas of the reading	Incomplete. Main ideas not represented.	Some main ideas and some less important ideas.	All main ideas represented. Clear and succinct.
Sequenced	Incomplete. Order and organization non-existent.	Partially complete. Order apparent, but unclear and less organized.	Sequence directionally as around the clock. Events follow the story line and show an understanding of the order intended by the author.
Shows knowledge of story structure.	Incomplete. Webs do not reveal the flow of story structure.	Webs show some sense of structure and address components of a story partially.	Webs indicate the overall flow of the book. Webs written at the beginning of the book address setting, characters, and events. Webs written through the middle parts of the book address the problems and solutions that become apparent. Webs written at the end of the book reveal resolution.
Clear and succinct	Incomplete. Student attempt to communicate ideas ineffective.	Communication fairly clear, but stated in an inefficient way.	Web ideas are easily understood and written with a minimum of words that accurately describe the ideas communicated.
Mechanics, grammar, neatness	Incomplete. Students use words improperly, no system of organization to the writing.	Use of mechanics and grammar are appropriate not neat.	Mechanics and grammar appropriate for the writing, indicating organization of ideas. Web is neatly written and legible.

Vocabulary Rubric

	0	12	25
Words Well Chosen	Incomplete. Inappropriate words (too easy)	Some words well chosen .	Five words well chosen (either new and difficult or descriptive).
Definitions from context	Incomplete. Definitions do not make sense.	Words defined, some use of context clues.	Words defined using context clues have reasonable definitions.
Classification of words.	Incomplete. Inaccurate.	Attempt to classify words made. All words classified, but some inaccurately.	All words classified as noun, verb, adjective, adverb, etc. Classifications reflect reason.
Recognizes synonyms, antonyms, multiple meanings, prefixes, suffixes, root words.	Incomplete. Inaccurate.	At least one example included of a word chosen that has a synonym, antonym, multiple meaning, prefix, suffix, or root.	2-3 examples of words that have synonyms, antonyms, multiple meaning, prefixes, suffixes, or roots.

Summary Rubric

	0.....10.....20		
Captures main ideas of the reading	Incomplete. Main ideas not represented.	Some main ideas and some less important ideas.	All main ideas represented. Clear and succinct.
Sequenced	Incomplete. Order and organization non-existent.	Partially complete. Order apparent, but unclear and less organized.	Sequenced. Events follow the story line and show an understanding of the order intended by the author.
Shows knowledge of story structure.	Incomplete. Summary doesn't reveal the flow of story structure.	Summary show some sense of structure and address components of a story partially.	Summary indicates the overall flow of the book. Beginning summary of the book addresses setting, characters, and events. Middle summary of the book addresses the problems and solutions that become apparent. End summary of the book reveal resolution.
Clear and succinct	Incomplete. Student attempt to communicate ideas ineffective.	Communication fairly clear, but stated in an inefficient way.	Summary ideas are easily understood and written with a minimum of words that accurately describe the ideas communicated.
Mechanics, grammar, neatness	Incomplete. Students use words improperly, no system of organization to the writing.	Use of mechanics and grammar are appropriate not neat.	Mechanics and grammar appropriate for the writing, indicating organization of ideas. Summary is neatly written and legible.

Questions Rubric

0.....10.....20

Types of questions included	Incomplete. Question types are unidentifiable.	Student has attempted to include different types of questions	Student has included one Right There (Fact) question, two Think and Search (Inferential, put ideas together) questions, one Author and You (Inferential, requires previous knowledge to be combined with author's ideas), and one On Your Own (Personal, opinion)
Right There	Incomplete. Not relating accurately to story.	Question relates to story but to a n insignificant detail of idea.	Question is related to a main ideas of the reading and has general significance to the story. Written in an understandable format. One word response acceptable, yes, no response less desirable. Answer is in the text and easy to find. The words used to write the question and words used to answer the question are in the same sentence.
Think and Search	Incomplete. Not identified correctly.	Question requires student to combine information, but the subject of the questions is not well chosen.	Question requires student to combine information. Question represents a main idea or important thought of the author. Question stated in clear, understandable language and commonly understood by students. The answer is in the story, but you need to put together different story parts to find it.
Author and You	Incomplete. Not identified correctly.	Question is not related to the reading but requires the application of prior knowledge.	Question asks students to answer the question using prior experience and knowledge combined with information from the author and reading for the day. The answer is not in the story.
On Your Own	Incomplete. Not identified correctly.	Question not related to story or not asking students to express their own opinion.	Question is a personal question asking students to use their own experience to answer. You can answer the question without reading the story.

Prediction Rubric

	0	12	25
Addresses main points	Incomplete. Not related To reading.	Addresses some main points. Makes sense and is reason-able.	Addresses all main points of the reading.
Probability indicates thought and reason.	Incomplete. Prediction indicates little or no thought.	Prediction is possible. Some thought is evident.	Prediction is probable and the student's reason for the prediction is written and expressed in clear ideas.
Substantiated by text.	Incomplete. Prediction indicates student did not read.	Prediction relates to text, but student did not express how.	Student uses examples of text to support each prediction made.
Mechanics, grammar, neatness. (Punctuation, capitalization, use of right words)	Incomplete. No organization or effort to be neat.	Some mechanical and grammar errors. Somewhat neat.	No errors in mechanics or grammar. Legible and neat.

Appendix C

Math Journal

Name _____Date_____

Objective

What do know already about this objective and what do you think will be a way to solve this problem?

What did you learn?

How does this make sense to you?

What did you know before that connects today's lesson?

Make up some sample problems relating to this lesson.

How would you explain your thinking to someone else?

Appendix D

Journal Rubric

Objective	0..... Not written	.....5..... Partially written	.....10 Complete and legible
What do you already know about...	Not written	Response reflects little thought, unclear statements	Response reflects thought and is stated in mathematical terminology

	0.....	.....10.....	20
What did you learn?	Not written. Inaccurate.	Some ideas recorded with a degree of understanding.	All concepts of the objective taught are recorded in mathematical terminology that reflects complete understanding.
In what way does this make the most sense to you?	Not written. Inaccurate.	Response reflects some thought but ideas are not communicated clearly or understood clearly.	Response reflects complete thought and mathematical sense through manipulatives, drawings, estimation, real world connections, or in other ways that relate to the objective.
What math did you know that connects to today's objective?	Not written. Inaccurate.	Response reflects connections that are unclear or partially accurate.	Response connects to previous mathematical knowledge gained both this year and in previous years and is stated in understandable mathematical terminology.
Sample problems	Not written. Inaccurate.	Some sample problems included. Reflect some understanding of the concepts taught. Answers correct.	A minimum of 5 sample problems included that reflect complex thought about each objective. One or more word problems that relate to each objective. Clear problems with accurate answers that would be appropriate to share with the class.

Appendix E

Recording Sheet

Name _____

Problem Number _____

FIND OUT

- What is the question you have to answer?
- What information does the problem give you?

CHOOSE A STRATEGY

SOLVE IT

Solution:

LOOK BACK

- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

Appendix F

Problem Solver Rubric

	0	10	20
Find Out What is the question, what information is given?	No response. Inaccurate.	One part of the task complete. Response is accurate. Partial understanding reflected.	Both questions are addressed and response reflects understanding of the problem and information given.
Choose a Strategy	No response. Inaccurate.	Strategy chosen useful but inefficient.	Strategy chosen with thought. Circled and identified by writing name of strategy beneath icon.
Solve It	No response. Shows no thought for accuracy.	Effort reflects thoughtful use of strategy but is incomplete or inaccurate.	Accurate complete use of strategy. Clearly shown work that reflects thought and accuracy.
Solution	No response. Inaccurate. Response reflects no rational.	Reflects thought. Solution partially accurate or reasonable.	Solution that reflects reason and accuracy.
Look Back	No response. Obviously inaccurate. Problem incomplete.	Response indicates student reflection of part of the problem. Inaccurate solution, obvious errors, partial responses.	Student response reflective and obvious thought given with reasonable strategy and solution, complete understanding of the problem.

Appendix G

Math Journal

Name _____Date_____

Objective

What did you learn? Does this seem logical to you? How? How does this connect to math you already knew?

Make up five sample problems.

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