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Volume 52 | Annual Issue Article 24

1945

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Recommended Citation

Yamakawa, K. A. and Oelke, W. C. (1945) "The Determination of Surface Tension by Standing Waves," *Proceedings of the Iowa Academy of Science, 52(1),* 199-203.
Available at: https://scholarworks.uni.edu/pias/vol52/iss1/24

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THE DETERMINATION OF SURFACE TENSION BY STANDING WAVES

K. A. YAMAKAWA AND W. C. OELKE

The velocity with which a wave travels over the surface of a liquid depends upon the surface tension of the liquid as well as on a gravity force due to the displacement of the liquid from its undisturbed position. For waves of large wavelength the effect of surface tension on the velocity of the wave is small; however for small wavelengths the gravity force is very small and the velocity of the wave is determined almost entirely by the surface tension. We shall therefore consider waves of very small wavelength.

J. H. Poynting and J. J. Thomson in their "Text Book of Physics" derive an expression showing this relation for the case of a wave whose amplitude is small compared to its wavelength neglecting the effect of the depth of the liquid.

Their expression is:

eq 1 $$T = \rho\left(\frac{\lambda^3 n^2}{2\pi} - \frac{g\lambda^2}{4\pi^2}\right)$$ where T - surface tension, λ - wave length, ρ - density, g - acceleration of gravity, n - frequency, h_{av} - average height of wave above normal

Lord Kelvin deduced an equation for the case of an infinitely small wave and included the effect of the depth of the liquid. This expression agrees with the above for depths of the order of a centimeter and wavelengths of the order of a half centimeter.

Lord Rayleigh was the first (Phil. Mag. xxx page 386) to successfully apply this method to the determination of the surface tension and his method was used by Dorsey (Phil. Mag. xliv page 369) to determine the surface tension of a large number of solutions. An electrically driven tuning fork generated the waves and care was taken to keep the amplitude small compared to the wavelength (to the extent that they were invisible to the unaided eye).

In the present case standing waves were produced in a glass tube containing the liquid by vibrating the tube. In this case the amplitude of the waves was no longer small compared to the wavelength. Furthermore the wave shape changed and peaked waves were produced as shown by the photograph in Figure 1. An expression for the form of the wave in such a case is derived in A. G. Webster's "The Dynamics of Particles and of Rigid, Elastic, and Fluid Bodies"; however it is very general and appears difficult to handle. An attempt is made in this paper to derive and experimentally check a relation for this type of wave so that the surface tension of liquids can be determined in this way. It can be seen from figure 1 that the shape of the wave approximates an inverted cycloid. This was assumed and an expression derived by following the general method of Poynting and Thomson in their "Text Book of Physics."

The force of gravity causing the wave to travel is due to the vertical pressure produced by the weight of the liquid above the undis-

Proceedings of the Iowa Academy of Science, Vol. 52 [1945], No. 1, Art. 24

turbed level of the liquid. The surface tension will give rise to an additional vertical force. The addition of the surface tension force produces an effect equivalent to that of increasing the normal gravity force. The velocity of a gravity wave, neglecting the effect of the depth of the liquid, is the velocity of a body would acquire under gravity by falling vertically through a distance $\frac{1}{4} \pi \lambda$ where λ is the wavelength. An expression for this equivalent "g" is obtained and in-

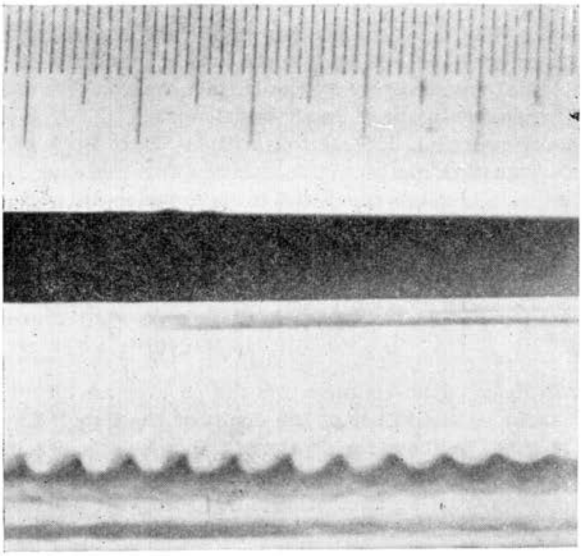


Figure 1

serted in the expression $v = \sqrt{2gh}$ relating velocity and distance in free fall. This results in the expression as given on the first page for the case of a wave whose amplitude is small compared to the wavelength. The wave was generated by a point fixed close to the center of a circle rolling in a straight line. In the following derivation the point is fixed at a point on the circumference of the circle resulting in a cycloid.

The pressure (p) due to surface tension (T) is given by the relation p=T/R where 'R' is the radius of curvature at that point. The pressure due to surface tension can therefore be obtained from the general expression for the radius of curvature of any curve:

$$\text{eq 2} \qquad \frac{1}{R} = \frac{d^2y/dx^2}{\left[1 + \left(dy/dx\right)^2\right]^{3/2}} \qquad \text{where } R = \text{radius of curvature}$$

by substituting the equations of the cycloid:

$$\text{eq 3} \qquad x = a(\theta - \sin\theta) \quad , \quad y = a(1 - \cos\theta)$$

It can be shown that this results in the following expression for the radius of curvature:

$$\text{eq } 4 \qquad \frac{1}{R} = \frac{1}{4a} \csc \frac{\theta}{2}$$

The pressure due to surface tension acting normal to the surface of the liquid, is therefore:

$$\text{eq } 5 \qquad P = \frac{T}{R} = \frac{T \csc \frac{\theta}{2}}{4a}$$

It can be seen that the vertical component of this force is active in propagating the wave and that the horizontal component cancels. Therefore the effective pressure is:

$$\text{eq } 6 \qquad P_v = P \cos \phi \quad \text{where } \phi = \arctan {}^{dy}/_{dx}$$

The relation between θ and ϕ can be obtained by obtaining an expression for dy/dx from the parametric equations of the cycloid and equating it to that obtained in terms of ϕ.

$$\text{eq } 7 \quad \text{From equations of a cycloid} \quad {}^{dy}/_{dx} = \tan \phi = \frac{\sin \theta}{1 - \cos \theta}$$

$$\text{From which it can be shown that} \quad \cos \phi = \sin \frac{\theta}{2}$$

Therefore:

$$\text{eq } 8 \qquad P_v = \frac{T}{4a}$$

It is interesting to note that the vertical component of the pressure is independent of θ.

The pressure due to gravity is $P_{\substack{\text{pressure}\\ \text{due to gravity}}} = g h_{av} \rho$ that is the average pressure)

The total average pressure is therefore:

$$\text{eq } 9 \qquad P_{v_{Total}} = \left(\rho g h_{av} + \frac{T}{4a}\right)$$

This can be changed to the form:

$$\text{eq } 10 \qquad P_{v_{Total}} = \left(g + \frac{T}{4ah_{av}\rho}\right)(\rho h_{av}) \qquad \text{this results in an equivalent "g"}$$

This results in an equivalent "g".

$$\text{eq } 11 \qquad g_{equiv.} = \left(g + \frac{T}{4ah_{av}\rho}\right)$$

The average height h_{av} is taken as the average height of the curve above the normal undisturbed level of the water. The normal undisturbed level of the water is taken as the line shown as mn in Figure 2 drawn so that A is equal to B. Upon integration and averaging over the portion of the curve above the normal undisturbed level there results in 0.53a for the average height. The velocity of a gravity

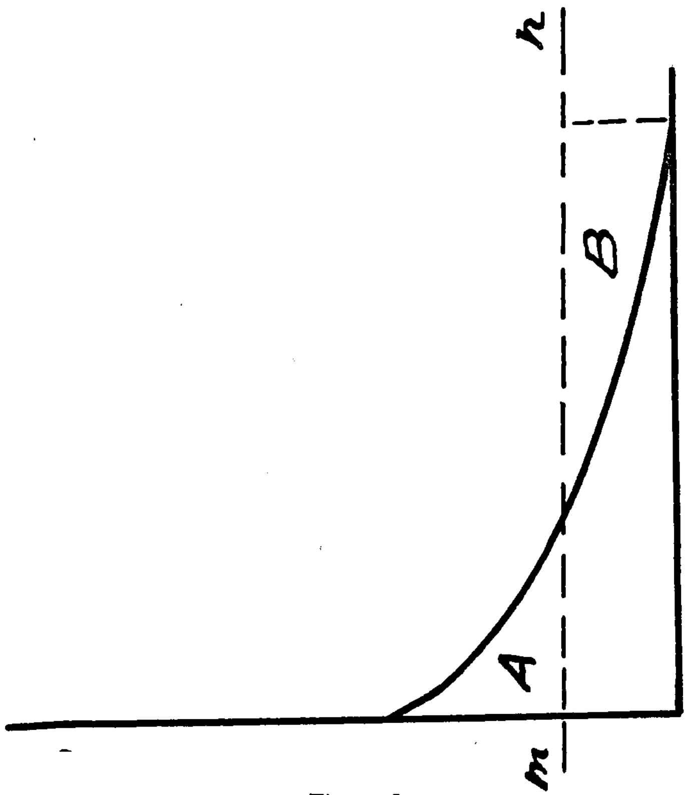


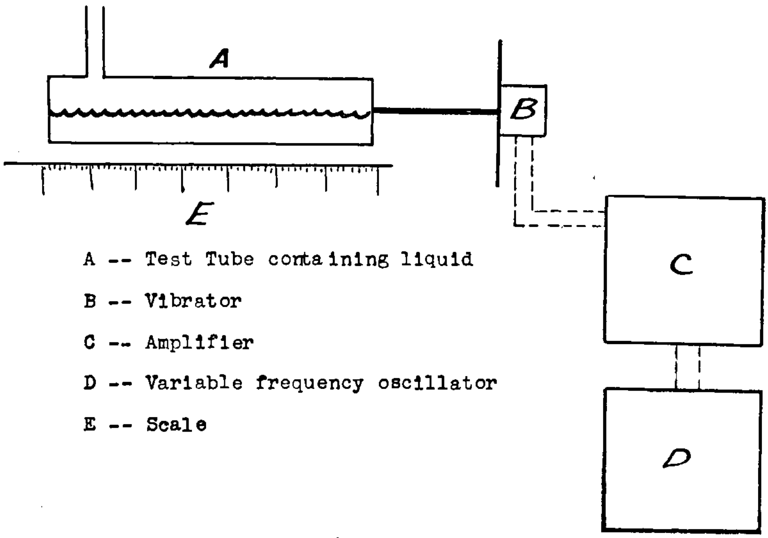
Figure 2

wave is that acquired by a freely falling body falling through a distance equal to $\frac{1}{4}\pi$ of the wavelength, and is therefore given by:

$$\text{eq } 12 \qquad V = \sqrt{\left(g + \frac{1.86\pi^2 T}{\lambda^2 \rho}\right)\frac{\lambda}{2\pi}} \qquad \text{since } \lambda = 2\pi a$$

$$\text{but } V_{\substack{\text{velocity}\\ \text{of a gravity}\\ \text{wave}}} = \sqrt{\frac{\lambda}{2\pi}\, g_{\text{equiv.}}}$$

Rigure 3 shows a diagram of the apparatus. The surface tension of water at 28°C was determined. Six determinations were made. The wavelength was determined by photographing the tube containing the liquid along with a scale mounted along the tube.



A -- Test Tube containing liquid

B -- Vibrator

C -- Amplifier

D -- Variable frequency oscillator

E -- Scale

Figure 3

EXPERIMENTAL RESULTS

Frequency cy/sec	Wavelength cms.	Surface Tension dynes/cm. Calculated from Eq. 1	Surface Tension dynes/cm. Calculated from Eq. 2
230	0.1613	34.0	72.2
163	0.202	34.2	74.6
194	0.181	34.5	74.9
84	0.198	8.6	57.4
194	0.181	39.3	84.9
163	0.213	40.0	85.6

CONCLUSION

The paper is not complete in that more experimental data is necessary. Also, we realize that many assumptions were made in the derivation of the equation. In particular, the assumption that the velocity of the wave is that of free fall through $\frac{1}{4}\pi$ of the wavelength may not be a valid one. However the method is very interesting and suggests many possibilities. We hope to continue the work.

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