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The importance of teaching problem solving

Edward Dvorak
University of Northern Iowa

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Abstract

Each day mathematics teachers and their students go through the daily routine of solving basic textbook mathematical problems. The teacher discusses the basic concept that is being studied and then assigns a number of problems. The assignment is generally limited to performing algorithmic computations in a rote manner. Problem solving is limited or nonexistant.

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Ed Dvorak

University of Northern Iowa

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Co-Director of Paper
Dr. Glenn Nelson

Graduate Faculty Advisor
Dr. Len Froyen

Barry J. Wilson

Co-Director of Paper
Dr. Len Froyen

Department Head

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Examining Committee

Chairperson
Dr. Len Froyen

Member
Dr. Glenn Nelson

Member
Dr. Melissa Heston

Member

Transmitted by:

Barry J. Wilson

Department Head

Chapter I: Introduction and Purpose

Each day mathematics teachers and their students go through the daily routine of solving basic textbook mathematical problems. The teacher discusses the basic concept that is being studied and then assigns a number of problems. The assignment is generally limited to performing algorithmic computations in a rote manner. Problem solving is limited or nonexistant.

Many mathematic educators are not familiar with the distinction between story problems and problem solving. The Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics (NCTM, 1989, p. 75) defines problem solving as:

> . . . the process by which students experience the power and usefulness of mathematics in the world around them. It is also a method of inquiry and application, interwoven throughout the Standards to provide a consistent context for learning and applying mathematics. Problem situations can establish a 'need to know' and foster the motivation for the development concept.

Purpose

Due to growing concerns about problem solving I will do a research paper that builds a case for making problem solving a

necessary part of the classroom in the nineties. The teacher will be the key to how this challenge is handled.

There have been research studies done in problem solving. One study by Muth (1986) examined the lack of extraneous information in math problems. This study emphasized that math textbooks are not adequate in preparing students for real life problem solving. Another study was conducted by The Institute For Research On Teaching at Michigan State University (Staff, 1988). This study emphasized that there was a lack of time spent on problem solving in mathematics. Since educators tend to overlook the importance of problem solving in mathematics, the teacher will have to be convinced that problem solving has its place in the curriculum.

Math teachers have a particular responsibility to teach problem solving in their classroom. Knowledge and training are the keys to teachers adapting a problem solving curriculum. When teachers witness the excitement and experience the enthusiasm that problem solving research has revealed, they should become a major factor in endorsing this curriculum in the nineties.

Problem solving in mathematics involves knowledge of the basic facts and understanding what to do with them. Most

students enter high school with adequate skills in the four basic operations, addition, subtraction, multiplication, and division; but are weak in the area of problem solving. I have seen this in my own classroom. Many of the students are able to perform algorithmic computations. When it comes time to transform a word problem they are unable to decide what to do. When students are trying to solve a basic algebraic distance, time, and rate problem, they are capable of reading the problem in standard sentence form and solving it, but when given the equation and asked to make up a problem or write a question that could be answered using the data, the students are lost.

John S. Thaeler (1986, p.682) states the following about problem solving: (He uses the term word or story problem to mean problem solving which adds confusion to what the term problem solving means.)

> Many high school and college students dislike word or story problems because they have not been very successful in solving them. Teachers have had a difficult time going beyond teaching students specific approaches to certain types of problems. Students tend to memorize a certain way to work each type of problem and then are ill-equipped to deal with new situations. Another reason why

> students dislike word problems is that too many teachers avoid applications, either because they cannot do them well or they want to emphasize the "easier" material.

Renate Lippert (1987, p. 479) states:

> Problem solving has indisputably become the focus of curricular reform in mathematics in the eighties (National Council of Teachers of Mathematics, 1980). In science and mathematics, the ability to recall formulas and manipulate them algebraically does not by itself indicate complete understanding of a subject area. Students must be able to give qualitative explanations of principles and make direct inferences from them without referring to the results of numerical calculations. That is, emphasis should be on developing and using algorithms to solve problems, not on performing algorithmic computations in a rote manner.

In March of 1989, the NCTM established standards to be used in teaching mathematics. Standard 1 is Mathematics as Problem Solving. Much research has been done. I will describe, interpret, and discuss the findings from some of these studies. Hopefully this synthesis of the research and my own experience applying these ideas in my classroom can be used to convince

math teachers of the importance of consistently teaching problem solving in their classrooms.

Chapter II: Importance of Problem Solving in the Classroom

Many teachers feel that if they are teaching traditional textbook story problems then they are tapping the problem solving skills of their students. This is an error that needs to be addressed. A story problem gives the information and asks the student to find the answer. Many times the students do not even need to read the problem. They just take the numbers and perform one of the four basic arithmetical operations. As an example:

> Jon bought a $28.00 video game. He plays it 8 times a week. At the corner store it cost $ .25 each time he plays the same game. How many weeks will it take Jon to break even?

The students may look at the problem and immediately devote their entire attention to the numbers needed to calculate an answer. The teacher has told them to do the problem and to get the right answer. The student may take 8 x $ .25 = $2.00 and divide $2.00 into $28.00 and get an answer of 14 weeks. Very little thought has gone into the problem. It is a basic algorithmic computation.

On the other hand, problem solving creates a thinking situation for the student. Lee Von Kuster, chairperson of

Teacher Education and Professor of Mathematics at the University of Montana, (1984, p. 61) states:

> Everyone is talking about it; some are even doing something about it. Problem solving comes under different titles and with various definitions, but generally it is something at which students are not very good. Just what constitutes problem solving will always be open to debate. On the most basic level, however, it means tackling problems that do not have obvious answers. As students mature mathematically, situations which were once problems become simple exercises. Thus teaching problem solving may seem like a problem in itself, but it is actually a relatively simple concept, involving a little extra time and imagination.

Patricia F. Campbell and Honi J. Bamberger, from the University of Maryland, define problem solving as envisioned in the National Council of Teachers of Mathematics Standards. (1990, p. 15) They state:

> It is not simply instruction for problem solving or about problem solving. It is students actively involved in constructing mathematics through problem solving; it is cooperation and questioning as students acquire, relate,

and apply new mathematical knowledge. Problem solving is a setting for communicating mathematical ideas, a context for investigating relationships, and a catalyst for connecting mathematical concepts and skills.

Here is an example of the same problem, but with a problem solving approach.

> Write three questions that can be answered using the data in this problem. Then find the answers. Jon's father purchased the video game Space Wizard for $28.00. Jon had been playing that game 8 times a week at the corner store, where it cost $ .25 to play one game.

This approach allows students to think of different questions that could be asked from the problem. It shows that there is more than one question that could be raised from the given information. It lets them see which data in the problem are necessary to find a solution and if there are unnecessary data.

Thus there are differences between story problems and problem solving. Story problems are relatively easy to solve and take very little thought. Typically, the students just have to pick out the numbers and perform one of the basic operations with them. Problem solving enhances the story problem. This method creates a situation where the students must analyze,

synthesize, and then evaluate the information given in the story problem.

There are advantages of the problem solving approach. Problem solving exposes students to problems of the kind that they will encounter in the real world. Problems in the real world may not have single answers or easy answers. Problem solving creates an awareness of the different strategies that can be used in dealing with real life situations. Problems can involve a lot of mathematics and some insight to consumer awareness. "Problems are all around us. It isn't necessary to seek out a book that has a vast supply of them. Local situations offer an abundance of ideas, with the added bonus that students will be more interested in situations which they know something about." (Von Kuster, 1984, p. 62)

Problem solving develops the mathematical knowledge that the students need in order to be successful in school and out. It prepares them to use the four basic operations in mathematics. These operations will be used in all areas. One of these areas is working with a checkbook. They must know how to add and subtract. They need some knowledge of how to balance their account every month. They may need to know how to double a recipe or to reduce it by a certain amount. If they

ever need to build something, they have to know how to measure a board and be able to change measurements from feet to inches. Budgeting is an important skill to learn where knowledge of percents is necessary. They have to know what interest is and how to look for the best interest for their economic situation. When taking a vacation, they will use distance, rate, and time problems. Robertta H. Barba (1990, p. 34) states:

> Successful problem solving involves fact, rules, skills, and strategies, called heuristics. Heuristics include selecting appropriate notation, studying the solution process, making a figure or drawing, checking the solution, identifying necessary information, generalizing, restating the problem, looking for patterns, writing mathematical statements, working backwards, drawing from known information, constructing a table of data and physical models, predicting, and breaking a complex problem into simpler components. If information (including facts, rules, and heuristics) is stored in memory in an organized fashion, then the student should become a good problem solver.

This statement sums up what a person needs to function in the real world. Many students will use some of the knowledge

that they receive in mathematics class. Others will use more. This will depend on the career that a person chooses. But in any profession, a good solid mathematical background is needed to handle any problem with confidence.

Students should be encouraged to devise their own problems. These problems will be more relevant to them, thus motivating them. There will be more willingness to accomplish the task. They can develop skills that apply to the entire school program. "Problem solving strategies are important parts of our school curriculum. Teaching strategies is an excellent way to help students attack mathematical, as well as other, problems.... This experience promotes the development of intuition and number sense in young students." (Kerekes, 1988, p. 432)

As students become better problem solvers, application of learned math skills should improve. They could make a sketch, drawing, or table of data to organize the information. Students will be able to take a complex problem and develop it into a simpler problem, make a guess, check the guess, and use appropriate labels. After reflecting on the problem and method that was used, the students may use this information for future reference.

Problem solving creates an atmosphere of thinking. It

makes the student understand how to do the problem and see what can be accomplished from this knowledge. "Students need to work on problems that may take hours, days, and even weeks to solve. Although some may be relatively simple exercises to be accomplished independently, others should involve small groups or an entire class working cooperatively." (NCTM Standards, 1989, p. 6)

In 1978, Larry L. Hatfield examined rationales for problem solving instruction. Then in 1989, Thomas L. Schroeder and Frank K. Lester, Jr. re-examined his work and related it to the Standards. "Both of these references define and distinguish among (1) teaching about problem solving, (2) teaching for problem solving, and (3) teaching via problem solving " (Schroeder and Lester 1989, p. 32).

Teaching about problem solving refers to teaching strategies that help the students solve problems. Students are taught to consider what the problem means before they decide how to solve the problem. This approach allows the students to reflect on and evaluate their solution method.

Teaching for problem solving involves applications. Real-life problems are used so the students can apply and practice concepts and skills that have been taught. In this manner the

students get to practice their basic operations in a variety of meaningful situations.

Teaching via problem solving goes beyond the other two approaches. New mathematical ideas may be learned and connected to existing skills. "Students come to learn concepts, connect ideas, and develop skills as they solve carefully constructed problems that embody essential aspects of the mathematical content being studied." (Campbell and Bamberger, 1990, p. 15)

Barba (1990, p. 35) states, "The work of mathematics researchers has revealed that problem solving is a multifaceted construct that is difficult to teach, and yet vital to the educational process of students in our classrooms." For students to become good problem solvers, they must practice the necessary problem solving skills. Barba lists the following implications: (1990, fig. 2, p.35)

1. Problem solving is not developed through simply doing exercises or worksheets. Problem solving skills require time to develop and are the result of solving many problems.

2. Good problem solvers need a balanced routine of facts, skills, and heuristics.

3. Problem solving is highly teachable, but the process

requires time and practice; there is no simple, easy method to teach problem solving.

4. Problem solving skills are not highly generalizable, students need exposure to many problems to become good problem solvers.

5. Students learn by solving problems, not by observing a teacher solve problems. In teaching problem solving, teachers need to remember that watching is not the same as learning. Students need to actively develop problem solving skills.

6. Problem solving is facilitated by calculators and microcomputers in the classroom. Computer programming, per se, does not teach all problem solving skills.

Problem solving allows the student to make decisions on what action or operation is needed to solve the problem. It makes the student aware that many problems can be solved in more than one way. It improves students' abilities to implement solution strategies accurately. Problem solving develops a situation where the students may think, reason, create, and seek solutions. It prepares the students to handle the real world problems that they will encounter after high school.

Chapter III: Problem Solving--Success in a 9th Grade Classroom

Problem solving in my classroom has become a regular part of the curriculum. I have, in the past two years, been very interested in this aspect of mathematics. I became aware of problem solving through several different circumstances. One was a class I took at the University of Iowa. It was called Problems Teaching High School Mathematics. This class examined the new standards developed by NCTM. These standards are to be used by math instructors to improve their curriculum. One of these standards deals with problem solving. Part of the class was devoted to a discussion about whether problem solving was being used and if there had been any success using it. Several class members had used problem solving and one person had done a lot with problem solving. She had not done any formal research, but she had observed that her students were enjoying mathematics more and they seemed to have more interest in doing their math assignments. Her students were asking for more problem solving activities. She noted that math scores for this class had improved. She felt that these positive results were due to using problem solving in her curriculum. After listening to this success story, I felt it was time that I tried problem solving in my classroom.

The math conferences that I have attended have also encouraged me to do more with problem solving. I always check the brochures from these sessions for any meetings dealing with problem solving. One session that was held recently got me interested in using a calculator approach to problem solving.

Much of the information I use comes from other teachers who have had success using the problem solving approach. I observe what they have done and I adapt their uses to fit my needs and classes.

In the past, prior to using problem solving, I basically followed the textbook. I introduced the topic, discussed it, looked at some examples, and assigned a certain number of problems. The students would work on their assignment. If they had any questions, I would assist them at that time. The next day we would correct the assignment in class, start a new topic, and repeat the same lesson routine. After listening to other teachers describe their success with problem solving, I decided to try problem solving in my Algebra I class. I began with my Algebra I class as these students were more proficient in mathematics than my other classes. I wanted to see how they reacted before I attempted it with my General Math students.

The first activity that I tried was Activity One. This activity

uses a cooperative learning approach. The students were very excited about it. There was much discussion among the group. Students were interacting with one another. They were asking each other questions and were able to answer with accuracy. When it came time to test over this unit I felt they had a better understanding. The test scores were higher than the previous year's class. The students asked when we would do problem solving again. As a teacher, it is a good feeling to see students excited about something. This gives an excitement to teaching and seems to make the job easier and more satisfying. When the students want to learn, a great obstacle to success has been overcome.

I also introduced a "Problem to Challenge You" board this year. On this board I place problem solving problems with different levels of difficulty. I divide the levels into "Tough", "Bewildering", and "Impuzzable". The students can answer these problems any time that they wish for extra credit. After the problem has been answered, we take five minutes to allow the student who gave the correct answer to tell the class how he or she approached this problem.

The "Problem to Challenge You" is my own idea. I wanted the students to be able to work on problems that challenged

them any time they had completed the regular assignment. I include a variety of problems at all levels to involve all of my students. Some of the problems include real life situations, puzzles, logic, patterns, and stories. I have students taking these problems home where they have involved their parents, older brothers, sisters, and friends. I have also had students who see problem solving problems in the newspaper, magazines, or other places and bring them in to challenge the other students. The Appendix includes sample items students have brought to class.

The "Challenge" board has been very successful. The success is apparent when I look out at my students toward the end of the period and I see several of them working at this board. They are discussing the different answers that are possible. I see them challenge each other and discuss why they feel their answer is correct and the other ones are wrong. I especially enjoy the look of pride when a student gets the correct answer and gets to put his/her name on that problem. This is a very good learning tool to use to motivate students. The students work very hard on these problems. They are challenged and they like the challenge because they have chosen it.

I did a unit in my Algebra I class over the first four chapters. I developed some activities that I incorporated into these chapters. Four of these problem-solving activities will be described as a way to illustrate the approach and the benefits of using it.

CHAPTER ONE--ACTIVITY ONE:

SKILL: Given a problem, the students will identify the operation(s) needed to find a solution.

PROCEDURE: We begin by talking about each person's role in the group. We go through an example and use one of the groups as a role model. This way each person has an idea of what is expected of him/her. The students are then divided up into groups of three. One will be the recorder, another will be the gopher, and the third will be the spokesperson. They will be given a collection of 10 problems that each group will try to solve. The recorder will write down all answers that the group comes up with. The gopher will be the person who may leave the desk to get material or ask questions. The spokesperson will give the answers when the group is called upon at the end of the period. Each person in the group is responsible for work on the problem. During the year I rotate the jobs so that everyone has a chance to do all three jobs.

BENEFITS: The students soon learn that if they interact, they solve the problem sooner and with less work. By putting in less effort, the students seem to enjoy the activity more. They are more willing to dig into the problem. They lose the bad taste of dull and repetitious work. They start to see a meaning to all the work they have put into it. A lot more work gets done without testing student perseverence. This activity encourages students to ask questions among themselves that they would not ordinarily be inspired to ask. Students in a small group will ask questions such as: "Is the number even or odd? What does 'sum' mean? Does this answer sound right to you? Could we write the numbers as two digit or maybe three digit numbers?"

All of these types of questions prompt the other students in the group to think of new ideas that are being suggested. As students interact with each other and learn from one another a certain unity develops. This is less likely to happen in a classroom where more traditional methods are used.

Small groups allow the quiet student to become actively involved. A quiet student may be more apt to speak out in a small group as the fear of someone making fun of his/her answers or questions is diminished. Students feel a certain safety when they are a valued member of a group. It may give

them a feeling of belonging and closeness that does not occur in a traditional setting.

The students "brainstorm" their ideas which makes them more aware of other problem solving strategies. The collaboration among students encourages intellectual risk-taking and deemphasizes right and wrong answers. Special attention is given to process as students identify their errors and focus on specific skills that enhance understanding. Students use the trial and error method and often see the plausibility of more than one answer.

The students learn from each other and gain self-confidence to solve problems on their own. Students must learn that mathematics is not a subject in which they either catch on immediately or they do not catch on at all. They learn by watching how some students approach a problem and start to see how to attack a problem. They start to have success in the group which builds up their ability to persevere when they encounter more difficult problems. They learn to ask sound questions that lead them on the right path to success.

CHAPTER ONE - ACTIVITY TWO:

SKILL: Given a number sentence, the students will write a story problem that would be solved using that number

sentence.

PROCEDURE: This is an individual activity. Each student will be given a group of 10 algebraic problems where a variable is defined and an equation is given. The students will be assigned to do at least 5 of the problems. They are encouraged to do more of the problems if they have time. They must make up a story problem that fits the algebraic equation. They will be required to read their problems to the class on the next day. After we have talked about the problems the next day I will take problems from their papers and copy them exactly. I will then have them try and write equations for these problems. We will also discuss why some of the problems were easier and why some were very difficult or impossible.

Following is an example of a dialogue between a teacher and a group of students using this activity.

Given: X = Number of books

$2X + 4 = 16$

Jane wrote the following story problem: John has 4 books more than twice the number of books. He has 16 books altogether. Find how many books did John start with?

Teacher: Is this problem easy to solve?

Mary: No, it does not make sense.

Teacher: What would you do to correct this problem?

Jim: The first sentence does not make sense.

Teacher: What part of it does not make sense?

Jane: It should say twice the number of books than he started with. It was missing this part.

Teacher: If you change that part does it make the problem easier to solve?

Mary: Yes

Jane: You could change it to say: John has read 4 books more than twice the number of books he read last year. He has read 16 books this year. How many books did he read last year?

Teacher: How many people feel that this problem is easier to understand and easier to solve than the original problem? What makes it easier?

Jim: When I read the problem I could not understand what they were asking for and I could not see how the equation would solve for it.

Teacher: Is there anything else that helped?

Jane: No, I can't think of anything.

Teacher: So in order to solve problems we must be able to read them and have an understanding of what we are

looking for and how we can approach the problem. Let's try the next one.

This is similar to how I use this activity and how it unfolds in my classroom.

BENEFITS: This activity encourages students to develop their own problems. When students develop problems by themselves they select information that is relevant to them. If the data are relevant to them they are more willing to engage in the problem solving process. When they view the information as being useful, they are more likely to utilize it to explore new concepts.

The dialogue encourages students to use strategies that they normally do not use in a regular classroom situation. Arithmetic and algebraic concepts are reviewed. Skills in working backward and using logical reasoning are developed in this activity. The dialogue improves students' abilities to monitor and evaluate their thinking while solving problems.

Students experience a feeling of pride when they develop a "good" problem. Students learn that there is more than one way to construct a problem. They have been told by teachers for years that they must organize a solution procedure in a specific manner. Some students can find the answer by using other

methods for setting up the problem. It is very important that the students realize that one problem can be approached from different angles and that the final results can be the same.

Students find out that some problems do not have a single answer. Problems that students will face in the real world do not always have a simple and specific answer. Students suspect this already, but when they get to math class the teacher tells them that they must get a specific answer. This "one rule for the classroom and one rule for the real world" contradiction is hard for students to comprehend or accept.

This type of activity also encourages them to focus on the characteristics of the problem not just the answer. Students learn that a significant part of the intrigue involves figuring out ways to translate the problem into a mathematical sentence. The answer is the exclamation point to the problem. It gives it the final touch to one's fascination with the process.

Developing these problems allows the students to decide what information is useful and what information is not. Students will deal with this process throughout their entire lives. They must learn how to sort out the relevant information and act upon it.

CHAPTER 3 and 4 - ACTIVITY THREE:

DREAM HOUSE:

SKILL: Given a challenge to build something, students will make a scale drawing of it.

PROCEDURE: "Have your parents ever remodeled or built a house? Do you plan to buy a house?" These are some of the questions with which this activity will deal. If you plan to remodel or build something, a picture or drawing will give you an idea if it will fit and how it will look. In order to do this you have to make a scale drawing of it. This activity will look at how proportions help do this.

Each student will be given a worksheet of a scale drawing. They will answer the questions on their own and will discuss them the following day.

The next day after discussing this worksheet, the students will be assigned to draw a scale drawing of their "Dream House". They will be asked to include as much detail as they wish. They will have one week to complete their "Dream House". Graph paper will be provided to help students prepare their drawing.

BENEFITS: Students will see that this is an activity to which they can relate. It is very important for students to be able to see where and how they can use a skill. If they see the usefulness of an activity, they have a tendency to dig deeper into

it and pursue it with more enthusiasm.

Skills learned from this activity could be used in the future. It is a visual activity using measuring instruments. It gives students a chance to use mathematical manipulatives. It is a "hands on" activity that helps develop spatial relationship skills. Students can visualize what is taking place as they produce a tangible project.

The students will compare their drawings and note differences. This activity shows students that there is more than one way to approach a problem and that there is more than one solution. The students develop the feeling of doing their own project and develop a sense of pride in their accomplishments. The students look at each other's drawings and ask questions. These questions and the answers that are given help develop the students' critical thinking skills. They also see that the more detail they put into the drawings, the better they convey the pictures that are in their minds.

CHAPTER TWO - ACTIVITY FOUR:

SKILL: The students will improve their abilities to select an appropriate solution to a problem from among several solutions that are generated during a group discussion.

PROCEDURE: Each group will be given a list of 15

problems, 1 bonus, and 5 YOU DO problems. The YOU DO problems are ones they come up with to stump the other groups. The 5 problems that they make up help develop their ability to think of similar problems.

The class will be divided into groups of 3 again as in activity one. A very competitive game is to take the YOU DO problems and have the other groups try to solve them. If the other groups are unable to solve them, the developers are declared the winner and receive a prize.

Here is the work sheet that I use.

1. 7 D of the W _____
2. 12 M of the Y _____
3. 100 L on a C _____
4. 4 Q in a D _____
5. 8 L on a S _____
6. 3 S on a T _____
7. 26 L in the A _____
8. 3 B M _____
9. 24 H in a D _____
10. 50 S in A _____
11. 365 D in a Y _____
12. 12 I in a F _____

13. 4 S in a S _____

14. 18 H in a G C _____

15. 11 P on a F T _____

Bonus:

99 C in I _____

List five ideas that your group can think of?

1. _____

2. _____

3. _____

4. _____

5. _____

BENEFITS: This activity puts the students' minds together to come up with ideas that will stump the class. It develops a strong bonding within the group. A sense of pride is developed. All members are able to provide input during the creating phase of the activity. As the discussion takes place in the group, ideas flow. From this flow, skills and enthusiasm build and students apply content learned earlier in the course.

This process encourages students to be careful not to rely on wild guessing, but to utilize wrong answers as a way for learning. Students learn that making mistakes is acceptable if information is gained from these mistakes. Students learn that

when a mistake is made, subsequent questions can be more productive. Life is full of mistakes; learning from them develops better problem solvers.

Probing questions may be used to guide the students to discover the solutions. Students learn very quickly which questions are useful and which ones may be eliminated. Some of the questions they may ask are: "What field does it deal with, math, science, history, or geography? Can you see it? Can you touch it? Is it something we have studied?" These questions may be helpful in some situations, while not very helpful in others. Learning to be a discriminating questioner can be an outgrowth of this process.

Expanding knowledge of the real world and not just knowledge in the classroom is encouraged in the groups. Students learn that problem solving is not limited to the math classroom. Problem solving is used all the time, everyday throughout a lifetime. Using problem solving in small groups not only expands knowledge by exposing the students to multiple viewpoints, but helps students appreciate a variety of ways of viewing and solving problems.

A sense of pride is developed among the group members. When a group stumps the class or a group gets the answer a

feeling of power and success is developed in the group. Some students have never felt this success. It gives them a sense of pride, enthusiasm, and importance. They feel good about themselves and their accomplishments.

Students learn the difference between general and specific knowledge. They learn that certain problems that seem difficult to solve may be solved very quickly. Problems that seem easy to their group may stump the other groups for a long time. Some problems call for narrow meanings and specific skills; others call for broad interpretations and the application of principles.

I use these four activities as supplementary activities. I have achieved considerable success with these activities. I have used them as a review of the chapter or to emphasize a certain topic. They have contributed to improvements in students' ultimate command of the material. I believe that the students can answer questions like: "How did I get that answer?, Does my answer seem reasonable?," and, "Where can I use this process?."

All of these activities allow the students to ask questions among themselves that they might not ask in a large group. The fear of asking a dumb question or an inappropriate question is reduced. This approach gives special attention to the process of

doing mathematics. Students can capitalize on their knowledge, while learning to identify their errors and can focus on specific areas needing improvement. These activities help develop estimating skills and guide students to restructure their thought processes.

Students are encouraged to identify and examine a variety of methods and strategies rather than concentrating on finding the "one right answer". They learn that there is often more than one method to use to find the solution to a problem. Most of all, these activities help students develop a desire to learn mathematics for themselves, not because it is a requirement. Students become more comfortable with math when they realize that everyone makes mistakes, and that they can often learn as much from their errors as from their successes.

I plan to increase my knowledge in problem solving. I plan to gather more activities that I can incorporate into my regular lesson plans. I would like to have three to four activities per chapter. I am planning on ordering new textbooks for next year. When previewing these texts I will be looking for an emphasis on problem solving.

I plan on writing a unit using the calculator for my General Math class. The use of the calculator in the classroom is another

recommendation by NCTM. Calculators are a beneficial tool for problem solving. Many students who have difficulty in mathematics, can use a calculator to compensate for computational weaknesses. It allows students to solve more difficult problems. It takes away some of the math phobia and gives students a new look at mathematics. This look can be a positive one and may generate an interest in both remediation and the expansion of their knowledge.

I have seen how problem solving can affect students' performance. I have seen how students respond positively toward it. Students view it as something new and it encourages them to learn. They see it as something valuable that they will use after high school. Since it is useful to them, more interest is taken in it. Textbook companies are including problem solving and supplemental materials dealing with problem solving for use in the traditional curriculum.

I plan to use more problem solving next year in both my Algebra I classes and my General Math class. I have purchased several different books with problem solving activities for use in these classes. I plan to attend the problem solving sessions at the Fall Math Conference. While there, I can exchange ideas with other mathematics teachers.

I will not limit my teaching to problem solving. I will use it where it will do the most for the students. I intend to keep student interest high and not let problem solving become another boring aspect of math class. It would be difficult not to include problem solving in my curriculum when I see the success and positive results.

Chapter 4: What is Next?

The Situation

The NCTM Book (1989) lists Standard 1: Mathematics as problem solving. It states:

> Problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics instruction and an integral part of mathematics activity. Problem solving is not a distinct topic but a process that should permeate the entire program and provide the context in which concepts and skills can be learned.

Problem solving is discussed in the NCTM Standards at three levels. The K-4 Level deals with problems that arise from school and other everyday experiences. These problems will have meaning and are related to the students' environment. As children progress through the grades, they should encounter more difficult and complex types of problems that arise from both a real world and mathematical context. When problem solving becomes a part of the classroom program students will gain confidence in doing mathematics and develop a solid mathematical mind.

Classrooms with problem solving approaches encourage

the students to ask questions, speculate, investigate, and explore. The teacher's primary goal is to promote a problem solving approach to the learning of all mathematics content.

At the 5 - 8 Grade Level, the standards recommend that the curriculum should take advantage of the expanding mathematical capabilities of the middle school student to include more complex problem situations involving topics such as probability, statistics, geometry, and rational numbers. Situations and approaches should build on and extend the mathematical language students are acquiring and help them develop a variety of problem solving strategies and approaches. A balance should take place between real world problems and problems that arise from investigation of mathematical ideas. Students should also be allowed to engage in group projects that require students to use available technology and engage in cooperative problem solving and discussion. The most important way to motivate students at this level is to concentrate on problems in which they have interest.

Computers and calculators are powerful problem solving tools. The ability to compute rapidly, graph instantly, and to change one variable and observe what happens to other related variables can help students become independent doers of

mathematics.

At the 9 - 12 Level the problem solving strategies learned in earlier grades should become increasingly internalized and integrated to form a broad basis for the student's approach to doing mathematics, regardless of the topic at hand. Problem solving is much more than applying specific techniques to the solution of classes of word problems. It is a process by which mathematics is constructed and reinforced.

Problems and applications should be used to introduce new mathematical content, to help students develop both understanding of concepts and procedures, and to apply and review processes they have already learned.

At this higher level students should have some experience formulating their own problems. This can be done as an individual or group project. Developing their own problems broadens their awareness of "messy problems". These are problems that have too much or not enough information or that have multiple solutions. These are the kind of problems they will encounter in their daily lives.

The curriculum must give students opportunities to solve problems that require them to work cooperatively, to use technology, to address relevant and interesting mathematical

ideas, and to experience the power and usefulness of mathematics.

In order to use problem solving in the curriculum, a teacher must feel comfortable with it. Many teachers who present problem solving do so in a highly structured, carefully, programmed way. They are afraid that the students will not accomplish the goals that the teachers have set. They start the lesson by saying "We are going to solve problems today by looking for patterns, You should use the guess and check method on these problems." Or, "You can make these easier by constructing a table". Instead, they should ask, "What is there about the problem that caused you to start there? What did the problem say to you? Have we studied problems similar to these, and if so, how or what method did we use? What did you assume that allowed you to do what you did?" These questions are essential if students are to develop problem solving skills. The students' last concern should be "What is the answer?" or "Is the answer correct?" After the students have gained success in developing the problem-solving procedures, there will be plenty of time to concentrate on answers.

The Challenges

There are three main factors that affect the classroom

climate and the willingness of students to take a chance on learning. The first is the appropriateness of the content (not too difficult and not too easy). The teacher must set the tone for the class by using problems that all the students will be able to solve. If the teacher starts out with problems that only a few can solve, other class members will lose interest. If students start out being able to succeed with a reasonable amount of effort, they will be more willing to dig into more difficult problems. Teachers must try to pose a variety of problem solving situations. Start with problems that the majority can solve and have them work into more challenging problems. It is also good to gradually introduce problems that do not have solutions. Students must realize that all problems do not have solutions in the real world.

The second factor is the evaluation practices used by the teacher. If a teacher wants students to take chances or play hunches they must use a variety of evaluation procedures. If students realize that a teacher only gives rewards for the "correct answer", they will work to produce right answers. When the students realize they can gain recognition, praise, and positive evaluations by being creative and taking risks, they will go for it. The teacher has the potential for controlling student

goal-directed behavior and needs to use this power wisely.

The third climate-oriented factor is the teacher's attitudes and actions related to problem solving. This is the most important factor in determining student receptiveness to problem solving. A teacher can make problem solving an exciting adventure by displaying enthusiasm and excitement in the classroom. If students see these qualities in the teacher, they are more likely to join the adventure. Teachers need to inspire students to dig into the problem and to look in other places for solutions. The students will begin to acquire and appreciate the problem solving skills that enable them to find solutions to a variety of problems.

Here are some things I am trying to do to create a positive climate and promote positive attitudes toward problem solving:

1. Be enthusiastic about problem solving.
2. Have students bring in problems from their personal experiences.
3. Personalize problems; whenever possible use the students' names within the problems and/or label the solutions to problems.
4. Recognize and reinforce effort and perseverance.
5. Use tangible and activity rewards.

6. Encourage students to play hunches.

7. Accept unusual solutions.

8. Praise students for getting correct solutions, but during problem solving emphasize the selection and use of problem solving strategies.

9. Emphasize persistence rather speed.

10. Emphasize that in some cases no solution is possible and that not all problems have a single answer.

There are a number of reasons for cultivating these qualities in my teaching. These reasons warrant consideration for all teachers of mathematics; particularly those who value problem solving.

A teacher must be enthusiastic in the classroom. Active engagement is a key to this enthusiasm. Do not sit behind the desk or stand at the board looking bored. Students pick up on non-verbal behavior very quickly and follow the mood. Jump around, be excited, and change your voice. The teacher is an actor who must entertain and keep the students' attention, while still meeting the content objectives of the lesson.

Students can relate to problems that they bring in from their personal experiences. These problems seem more realistic and have more meaning. When students see meaning

they are much more willing to dig into a problem. It becomes useful to them.

When students see their name in a problem or have a solution named for them, they feel special and thought about. If the teacher shows them that he/she cares, then the students will respect that teacher more. Students who respect their teacher will put more effort into their work and even try to please the teacher.

When students show perseverance and willingness to try a problem their noteworthy behavior should be praised in front of the entire class. Let the other members know that these characteristics are desirable. Other students will want to be identified as possessing similar praiseworthy qualities. This generates enthusiasm and improves morale.

Tangible rewards can be as simple as a sticker with "Great Job" or "Excellent Idea" written on them. A piece of candy can also an attractive reward. If the whole class works hard on a class project or the whole class does well on a big assignment, a class party or a class movie could be used.

Students should be encouraged to play hunches. Hunches lead the students into new and interesting directions. When students come up with hunches encourage them. Try to lead

them into the productive directions. This way they have a positive feeling about the situation and are willing to follow through with the problem.

Unusual solutions can be used in a positive sense. When the answer is correct, ask the student to share it and the procedure they used with the class. If the unusual solution is completely off track ask the student how they came up with that answer. Listen to them and as they proceed, guide them away from mistaken notions. Guide in such a way as to make them feel that they came up with the new solution.

Praise students for the right answer, but stress the procedure they used. Ask them to share the process with the rest of the class. Have them name the problem solving strategies that they used. This will help them practice the strategies and will serve to reinforce strategies that can be used in the future.

To emphasize persistence rather than speed, use the story of the tortoise and the hare. The tortoise won the race because he kept at it and did not give up. A person who is thorough, accurate, and persistent will ultimately solve problems at a better rate than the person who rushes through them. Speed is not one of the essential conditions associated with problem

solving.

Students must be aware that the world has many problems that do not have solutions. Students should learn that some problems may have more than one solution. Teachers must put problems like this in their examples and assign them to the students. The students will begin to look for several solutions to a problem. This attitude and practice will help them in later life.

Teachers should not be so single-minded about finishing the textbook each year. This disposition may be coming from the administration. The teacher and administrators should meet and discuss the idea of problem solving and how it works. The teacher should argue that problem solving can enhance mathematics skills that are more likely to transfer to subsequent mathematics classes. The teacher should also promote the benefits of problem solving skills because the students are able to use these skills in the real world. This is a goal that all mathematics instructors should be trying to achieve.

Problem solving also reduces the amount of time devoted to a more traditional style of teaching. It is sometimes more important to a teacher to cover material, relying on the lecture, than it is for the students to develop a thorough understanding

of the subject. Students with a solid foundation in problem solving will be able to use these skills as entry points in other mathematics courses and in real life. Teachers need to balance the process with the product. This balance will motivate the students to do their best and keep their interest.

Teachers and students have traditionally been so "answer-oriented" that it is often difficult to break this habit. But, there are many, many problems in today's world for which there are no easy answers, and in certain cases no answers at all. Students must become more conscious of this. Teachers need to be more realistic about what can actually be taught rather than getting through the book. Lee Von Kuster (1984, p. 63) states,

> . . . teachers often allow memorization to take precedence over discovery or the synthesis of information. If we allow the student to develop an understanding for problem solving we are giving them a true mathematical skill which will be far more useful and beneficial to them in the real world.

The Goal and Approach

The ultimate goal of any problem solving class and my goal is to improve student's performance at solving problems. In Problem-Solving Experiences in Mathematics, (1985, p. V)

Randall I. Charles, Robert P. Mason, Ethel Moffatt, and Catherine A. White have broken this goal into the following specific goals.

1. Improve students' willingness to try problems and improve their performance when solving problems.

2. Improve students' self-concepts with respect to their abilities to solve problems.

3. Make students aware of problem solving strategies.

4. Make students aware of the value of approaching problems in a systematic manner.

5. Make students aware that many problems can be solved in more than one way.

6. Improve students' abilities to select appropriate solution strategies.

7. Improve students' abilities to use solution strategies accurately.

8. Improve students' abilities to evaluate their thinking while solving problems.

9. Make students aware that some problems do not have a solution.

These goals are attainable by using the following activities:

1. Given a story, write a question that could be answered using the data in the story.

2. Given a problem with unneeded data, identify the data needed to find a solution.

3. Given a problem with missing data, make up appropriate data for solving the problem.

4. Given a problem, identify the operation(s) needed to find a solution.

5. Given a problem without numbers, identify the operation(s) needed to find a solution.

6. Given a table or organized list, write a story problem that would be solved by making and using that table or list.

7. Given a picture, write a story problem that would be solved using the picture.

8. Given a number sentence, write a story problem that would be solved using that number sentence.

9. Given the numerical part of the answer to a problem, write the answer in a complete sentence.

10. Given a problem and its answer written in a complete sentence, determine the reasonableness of the answer. (Charles, et al., p. V)

These activities are good to incorporate into a curriculum, but it is important to remember that problem solving is not developed through simply doing exercises or worksheets.

Problem solving skills require practice to develop and are the cumulative effects of solving many problems. Good problem solvers need to balance facts and skills. Problem solving is highly teachable, but the process requires patience and persistence. Researchers have revealed that problem solving is a vital educational experience that we as teachers can and need to provide students.

I have seen positive results in my classroom. I have seen how excited the students become when we do problem solving activities. I have students ask when we will do more of these types of activities. I do not hear that from my students when they are doing algorithmic computation. With all of the positive results described and illustrated in this paper I can hope that as other math teachers become familiar with problem solving they will share my enthusiasm for it.

Appendix

1. I spent 2/3 of my money in store A. I then spent 1/3 of what remained in store B. When I left store B, I had $4.00. How much money did I have when I entered store A?

2. The radius of a tire of Lyle's bike is 13 inches. How many complete turns does the wheel make in traveling 100 ft.?

3. Given the digits 1,1,2,2,3,3,4,4, write an eight-digit number using these eight digits such that the:

   1's are separated by 1 digit

   2's are separated by 2 digits

   3's are separated by 3 digits

   4's are separated by 4 digits

4. Nancy had $ .69 in coins. Jill asked her for change for a half dollar. Nancy tried to make change but found that she didn't have the correct coins to do so. What coins did she have, if each coin was less than a half dollar?

   _____ quarters

   _____ dimes

   _____ nickels

   _____ pennies

5. Cross out seven letters in the statement below so that what remains will still express a sum of 18.

FIVE PLUS SIX PLUS SEVEN

6. There are five numbers that each have five digits. Added together, the numbers total 145,677. No digit can be repeated within a number, although numbers can be repeated. All of the numbers must use the same five digits. What are the numbers?

7. I am a proper fraction. The sum of my numerator and denominator is 60, and their difference is 10. What is my simplest name?

8. A bag of 15 silver dollars is known to contain 1 counterfeit coin, which is lighter than the other 14. What is the minimum number of comparisons that may be necessary to identify the counterfeit coin using only a two-pan balance?

9. Six men were walking down the road. Each man had six sacks. Each sack had six cats. Each cat had six kittens. How many legs are there on all the creatures combined?

10. Roosevelt wrote to 12 of his friends from music camp at a cost of $3.08 for postage. If postage for letters is $ .29 and for postcards, $ .19, how many of each did he send?

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