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Navigation Instruction of Naval Officer Candidates

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The title of this paper has appeared in two forms. First as "Mathematics of Navigation" and then as given above. We shall limit the presentation to a discussion of some topics in mathematics which the Naval Officer Candidate has need of, most of them wish they had been exposed to, or which are required for an understanding of navigation. These remarks are based on several year's experience with Midshipmen during World War II and four summers with the Navigation Department of Reserve Officer Candidate School beginning in the summer of 1949. Our thinking is directed largely to surface navigation but applies almost as well to aerial navigation.

In its general sense, "navigation is the science of location; i. e., the science by which one determines his position within a given coordinate system from observations made upon objects either within or without the system". Using the knowledge of his position, the navigator of a ship or plane may then set a course to any other object whose coordinates are known. All navigation by observations made upon objects within the coordinate frame of meridians and parallels is lumped under "geonavigation" whereas when the objects are extraterrestrial, the science is termed "celestial navigation". When it is necessary to resort to extrapolation, the method is termed "dead reckoning".

The following breakdown may be made:

1. Geonavigation (Pilotage
   (Sonic methods
   (Radio location
2. Celestial Navigation
3. Dead Reckoning

In piloting, one moves from origin to destination by noting visual ly the location of prominent landmarks along the route. Just as the pedestrian or motorist may proceed toward his destination by turning left at Schulz's delicatessen, so the ship in pilot waters may change course when abeam of some prominent building whose position is marked on its chart. Sonic types of navigation depend upon the reflection of compressional waves traveling either in air or in water. The time required for a whistled signal or other sound to travel from a vessel to a nearby cliff is measured, and from the known speed of the wave the distance from cliff to ship is quickly determined. Radiolocation is coming into widespread usage. Some of these systems are Gee, Shoran, Racon, Consol, Decca, Raydist, Loran, etc. One of these, loran, will be mentioned again later. The term celestial navigation is self-explanatory. Its practice consists of establishing the position of at least two heavenly bodies upon the celestial sphere from the observer's location. By a transformation of coodinates, the observer's position upon earth is determined. Dead reckoning consists of maintaining a record of the velocities and times which have elapsed since leaving the last positively identified position.

One requirement is basic to all navigational methods. This stems from geometry; one observation establishes a given constant coordinate, which the navigator terms a line of position (LOP). A second observation defines a second line of position. The intersection of two lines of position fixes the navigators position. Sometimes each line may be a great circle, but the second intersection does not cause much confusion as the navigator usually knows whether he is in the Atlantic or the Pacific. Position may be specified by latitude and longitude (x, y), by distance and bearing (r, θ), or in the case of dead reckoning, by velocity and time (v, t). Familiarity with various coordinate systems is therefore highly desirable.

Direction, distance, velocity are of course, fundamental to all navigation instruction. The use of signed numbers enters in compass correction, tide and current table. Sometimes the signs are replaced by E or W, or S or N, but the problem of combining them is the same. The frequent use of tables calls for a knowledge of interpolation. The need for a knowledge of logarithms is somewhat on the decrease. Trigonometric functions are used for the solution of both plane and spherical tri-angles, for map projection, bow and beam bearings and the like.

The word vector is somewhat frightening to the average officer candidate but the difficulties encountered when he meets such problems as determination of course to steer to compensate for current, determination of true wind, solving maneuvering board problems indicate strongly that knowledge of this subject is highly desirable.

Standard instructional procedure for the first class in navigation is for the teacher to introduce himself, outline the procedure for the course, and then hand each candidate a chart, or navigator's map, of the local coastal area. He is reminded of the fact that it is not possible to make a true representation of a spherical surface on a plane without distortion and a brief discussion of chart projection follows. The vast majority of the charts used for marine navigation are made on the Mercator projection. In this projection the meridians are straight vertical lines, evenly spaced, and the parallels are straight horizontal lines, the interval between them increasing with latitude. The expansion of the latitude and longitude scales approximate the secant of the latitude. This ratio may be used without appreciable error for charts covering a relatively small area but when great distances are to be involved, a more exacting method based on the calculus must be used. (Finding even a meager knowledge of the calculus in these students exceeds our wildest dreams.) The Mercator projection is conformal. On the surface of the earth a line making the same angle with all meridians is called a rhumb line (or loxodromic curve). Such a line on a Mercator chart is a straight line.

Another projection of special interest to the navigator is the gnomonic which is the projection obtained by projecting the meridians and parallels from the center of the earth to a plane tangent to the earth at some point. All great circles on the earth project as straight lines. This projection is useful in navigation for planning long voyages because great-circle tracks are shortest.

Another conformal projection of interest is the stereographic which has found favor in star charts and for navigation in the earth's polar regions. Other projections of lesser use to the navigator could be mentioned and frequently are.

The use of the maneuvering board has been mentioned. This is nothing more or less than a convenient polar-coordinate diagram published by the Hydrographic Office. This diagram was developed particularly for problems of relative motion. This is where some knowledge of vectors would be of tremendous assistance. To sell the idea that the velocity of a moving point A relative to a moving point B is the vector difference $V_A - V_B$ of their velocities is a major instructional achievement. As these problems frequently require the finding of time to complete a maneuver, and the distance covered, a nomogram is placed at the bottom of the maneuvering board for this purpose. Time, distance, or speed, may also be determined by means of the logarithmic scale forming the top line of the nomogram. This is used as a slide rule by the aid of a divider.

Celestial navigation remains the most difficult part of navigation to learn. With more extensive use of various electronic aids to navigation, the role of this type of navigation has dimned somewhat but must still be considered of considerable importance. As late as 1942 this required the application of time-consuming formulas from spherical trigonometry. Many of us cut our navigational teeth on the cosine-haversine formula: hav (Co-H) = hav (L∼d) + cos L cos d hav t, where H represents altitude, L∼D the algebraic sum of latitude and declination, and t the meridian angle. Officers under instruction at a historically unique indoctrination school had their military posture impaired from carrying around the ponderous volume of Bowditch, Practical Navigator, which contained a few pages of natural haversines and logarithms of haversines. Basically, our problem is to solve a spherical triangle with vertices at the observed body, the elevated pole, and observer's zenith for the altitude and azimuth of the heavenly body. The usual procedure is to divide this triangle into two right spherical triangles and apply Napier's rules. These earlier formulas have been simplified, and as a still easier alternative, full tables and graphs of solutions have been computed, which require only accurate arithmetic for their application. These include such tables as H.O. 214, H.O. 218, and H. O. 249.

Since our original premise was that understanding rather than rote learning was desirable, and with a firm conviction that an understanding of "why" would frequently save time in the learning process and help avoid ridiculous numerical results, we shall set down the basic mathematical formulas used in Ageton's method, or

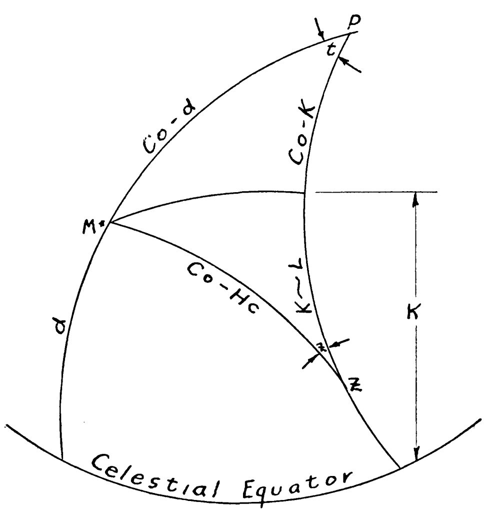

Fig. 1. The navigational triangle as divided in the H.O. 211 method.

by use of H.O. 211. By referring to Fig. 1 is can be seen that the resulting formulas are:

$$(1)\quad \text{Csc R} = \text{csc t sec d}$$

$$(2)\quad \text{Csc K} = \frac{\text{csc d}}{\text{sec R}}$$

$$(3)\quad \text{Csc H}_c = \text{sec R sec } (\text{K} \sim \text{L})$$

$$(4)\quad \text{Csc Z} = \frac{\text{csc R}}{\text{sec H}_c}$$

It will be noted that all four formulas are in terms of secants and tangents. The table in H.O. 211 is arranged in parallel A and B columns, the A functions being log cosecants multiplied by 100,000 and B functions being log secants multiplied by 100,000.

Upon the completion of the solution of the astronomical triangle, or the navigational triangle, as it is now called, the navigator puts down the line of position as a Sumner line which means a line tangent to the circle of the equal altitude. As this circle may have a radius of a thousand miles, and since the navigator knows moderately well where he is, this approximation introduces no appreciable error.

Radiolocation methods generally are unique in that they are for the most part independent of the weather. Sonic systems are also independent of weather, but their use is confined to very small distances of the order of miles. The basis of all so-called pulse navigation systems is the time required for a pulse of radio-frequency energy to traverse a given distance. This principle is utilized for navigation and position-finding in three general methods.

In the first method, which is called the circular method, a train of pulses of radio energy is radiated by a transmitter at the unknown position. Some of this energy is returned from a reflecting object or from a radar transponder at a known position, and the time for the round trip is measured. The navigator then knows he is located somewhere on a "circular" line of position.

In the seocnd method, which makes use of a conventional radar, the pulses are radiated in a narrow beam by means of a directive antenna, and the unknown position relative to the reflecting object is determined from the bearing of the beam and the indicated distance. The first method also may be carried out with a conventional radar, although it may be separately instrumented. During World War II radar was considered super-secret and military personnel were not permitted to mention the word; —not even if they spelled it backwards.

The third method of determining position by radio pulses is termed the "hyperbolic method". In it pulses are radiated simultaneously from transmitters at two known ground positions and are received by equipment at the unknown position. When the unknown position is closer to one transmitter than to the other, the pulses from the nearer transmitter are, of course, received before those from the more distant one. A comparison of the two received pulse trains gives the time difference of arrival, which is proportional to the differences to the two transmitters. Geometrically, therefore, the lines of position form hyperbolic curves having two known transmitters as foci. Strictly speaking, of course, a line of position in a hyperbolic navigational system lies on the intersection of a hyperboloid of two nappers with the geoid.

As a good example of such a system we shall briefly describe the loran system. The name is coined from the initial letters of LOng RAnge Navigation. The basic proposal was devised in 1940, and in 1942 the first fixed stations were placed in operation along various coastal regions. Unlike radar, which uses very high frequencies of millions of kilocycles, loran operates on a frequency of about 2000 kilocycles. This is the region of the radio spectrum just above the commercial broadcast band. The long waves of loran, therefore, travel not only over the surface of the earth but travel skyward and encounter the electrically ionized region of the upper atmosphere. (the ionosphere), and may be reflected back hundreds of miles from the sending antenna. With present techniques, the limit of distance is about 700 miles by day and 1400 by night.

Operation of loran involves measuring to a micro-second, the time interval between the reception of pulses. It does this by means of an electronic system which utilizes a cathode ray tube. One transmitter of a loran pair emits a number of uniformly-spaced pulses each second; this station is known as the "master station". Several hundred miles away a second transmitter at the "slave station" emits a corresponding series of pulses which are kept accurately synchronized with those from the master station. The time difference between the reception of a master pulse and the corresponding slave pulse establishes the loran line of position. To eliminate any ambiguity in identifying the pulses, the master and slave pulses are not transmitted simultaneously. Each slave pulse is delayed by a carefully controlled amount so that the corresponding master pulse is always received first.

The lines of constant time difference for each pair of stations are all precomputed taking into account curvature and eccentricity of the earth, and other factors, and are made available to the navigator in the form of loran tables or charts.

Loran shore transmitting stations are usually arranged so that a pair are separated by 200 to 400 miles, but under unfavorable geographical situations the separation may be as little as 100 miles and as much as 700 miles.

Loran stations are located so that signals from two or more pairs of stations may be recieved in certain areas and a loran fix obtained by crossing two or more lines of position.

In order to economize on station installations, one station is often made common to two pairs. Usually the master station is common to both pairs and is double-pulsed. Double-pulsed stations, however, send out two entirely distinct set of pulses, one set paired with pulses from each adjacent station. Therefore, from an operating viewpoint, a double-pulsed station can be considered as two separate stations at the same location.

It may be pointed out at this time that an inherent accuracy factor existing in each of the various radio location or other types of navigational systems is determined by the angle at which two lines of position intersect. Obviously, any system which could provide an orthogonal relationship between the lines of position determined by the navigator would contain the greatest accuracy factor in this respect. The hyperbolic type would require a family of orthogonal ellipses—a requirement difficult of fulfillment.

This by no means exhausts the application of mathematics to navigation for the officer candidate. As aerology becomes more and more an integral part of the navigation course there arises other demands. No mention has been made of sailings, and no mention of the mathematics hidden in the equipment (or gear) used by the navigator.

The mathematical needs we have outlined, it would seem, are quite modest. But dozens of my fellow officer instructors would be eternally grateful if the officer candidate could do arithmetic with accuracy and understanding.

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