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A Comparison of Methods of Interpolation:

by Don Barry—

The original project consisted of refining a new method for square root determination so that a simple, accurate, and single-step method would exist. Naturally, it was hoped that such an examination of the method would lead to other discoveries concerning square roots. Merely for the sake of challenge, it was decided to use as simple mathematics as possible. Eventually, the author was able to develop an equation, not merely a method, for square root determination. However, a proof of this equation has not been discovered.

The original method determined the positive $x$-intercept of the parabolic equation: $x^2 - a = y$ by defining $\{x_1, x_1 < x < (x_1 + 1)\}$, $x_1 \in \text{integers}$ and interpolating using the points $((x_1, x_1^2 - a)$ and $(x_1 + 1, \lceil x_1 + 1 \rceil^2 - a)$ on the curve.

Here, $\sqrt{a} = x_1 + \frac{a - x_1^2}{2x_1 + 1}$

For no logical reason, 1 was added to the numerator and to the denominator of the fraction, resulting in

$$\sqrt{a} = x_1 + \frac{a - x_1^2 + 1}{2x_1 + 2}$$

It was then proved that:

$$\left[ x_1 + \frac{a - x_1^2 + 1}{2x_1 + 2} \right]^2 = a + \frac{(x_1 + 1)^2 - a}{2x_1 + 2}$$

This indicated that the error diminished as $a$ approached $(x_1 + 1)^2$. It seemed that a number considerably less than 1 should be added, in the above manner, to an approximation of $\sqrt{a}$ for $a$ approaching $x_1^2$. Also, for $a$ approaching $((x_1 + 1)^2$, a number close to 1 must be used. Logically, this problem could be resolved by adding the fractional part of the interpolation to the interpolation in this manner:

$$\sqrt{a} = x_1 + \frac{a - x_1^2 + \frac{a - x_1^2}{2x_1 + 1}}{2x_1 + 1 + \frac{a - x_1^2}{2x_1 + 1}}$$

This was found to be more accurate than the previous methods, and it was proved that the resulting number was less than $\sqrt{a}$. It seemed that this method could be extended, and each extension was found to be more accurate. Therefore the suggested solution seemed to be:

$$\sqrt{a} = x_1 + \frac{a - x_1^2 + \frac{a - x_1^2}{2x_1 + 1} + \frac{a - x_1^2}{2x_1 + 1} + \cdots}{2x_1 + 1 + \frac{a - x_1^2}{2x_1 + 1} + \frac{a - x_1^2}{2x_1 + 1} + \cdots}$$

However, as has been stated, this is still conjecture.

Donald Barry is currently enrolled in Carleton College. He has been a participant in a special National Science Foundation Institute in mathematics, and was a National Merit Scholarship finalist. He hopes to continue working with number theory.