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NOMENCLATURE

The following nomenclature is used in this paper:

$\left.\begin{matrix} a \\ b \end{matrix}\right\}$ dimensions of a rectangular plate

h Thickness of the plate

E Young's modulus

ν Poisson's ratio

q intensity of distributed load

D flexural rigidity of the plate $\left(\frac{Eh^3}{12(1-\nu^2)}\right)$

w deflection of the plate in general

∇^4 biharmonic operator in rectangular coordinates

GENERAL ANALYSIS

The problem of bending of a rectangular plate with clamped edges has attracted attention of authors for many years but exact solutions have never been obtained. The difficulty arises from the fact that there is no simple procedure for selecting a proper deflection function that will satisfy both the differential equation of bending and the specified boundary conditions. The only exception occurs when two opposite edges of a rectangular plate are simply supported. In such a case, a simple solution either in the form of a single series as proposed by Levy, or in the form of a double series as proposed by Navier (9) can be easily derived. However, these types of solutions cannot be applied to a plate with two adjacent edges clamped. Because of this difficulty, different approximate methods have been proposed by many authors since 1903. A rather complete list of references of the early works can be found in the discussion of Stiles paper by D. Young (2).

Recently, Weinstein (4) has proposed an approximate method based on variation principle which has been successfully applied to a plate with all edges clamped. Later, this method has been extended by Stiles (6) (7) to investigate bending of a plate with two adjacent edges clamped and the others simply supported. Huang and Conway (8) have also investigated the bending of a

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rectangular plate with two adjacent edges clamped and the others either simply supported or free subjected to uniformly distributed load only. In each case, solutions are obtained by the method of superposition. Since this method depends upon the symmetrical properties of the deflection surface to satisfy some of the boundary conditions, it is obvious that it can not be applied to other loading conditions.

It is the purpose of this paper to show that a systematic approach to this class of problem can be obtained by Fourier method (1) which is practically an extension of Timoshenko's method (3).

Based on conventional assumptions of thin plate theory for small deflections, the problem of bending of a laterally loaded rectangular plate is reduced to the integration of Lagrange equation:

$$\nabla^4 w = \frac{q}{D} \qquad (1)$$

We take deflection of the plate in the form:

$$w = w_1 + w_2 \qquad (2)$$

in which w_1 represents the deflection of a simply supported plate subjected to same type of loading as the considered plate, and w_2 represents the deflection of a correction state which is selected as two infinite series of particular solutions of the biharmonic equation

$$\nabla^4 w_2 = O \qquad (3)$$

Each of the two series of w_2 is so chosen as to give a Fourier series expansion for one component of displacement at the proper boundary. Then the coefficients of the series can be adjusted by Fourier analysis to satisfy not only the specified conditions at the boundary, but also to eliminate any undesirable quantities which may have been introduced by the other series. Because of this interrelated nature of the boundary conditions, the final solution is obtained by solving a system of an infinite number of linear simultaneous equations. Physically speaking, the correction state can be interpretated either as deflections due to distributed moments along the edges of the plate when it is a correction for slope or bending moment, or as deflections due to distributed forces along the edges when it is a correction for shear force or deflection.

It is seen that this method can be applied to a plate subjected to any kind of loading with any combination of even and odd order of boundary conditions. Except for minor adjustments, practically the same type of function can be used for the correction state for various problems. Therefore, different problems with different loading conditions and boundary conditions can be handled almost in a routine manner. The following two examples will serve to illustrate the procedures:

Example 1. Rectangular Plate with Two Adjacent Edges Clamped and the Others Simply Supported

We consider the case of a uniformly distributed load only. If the coordinate axes are chosen as shown in Fig. 1, the deflection

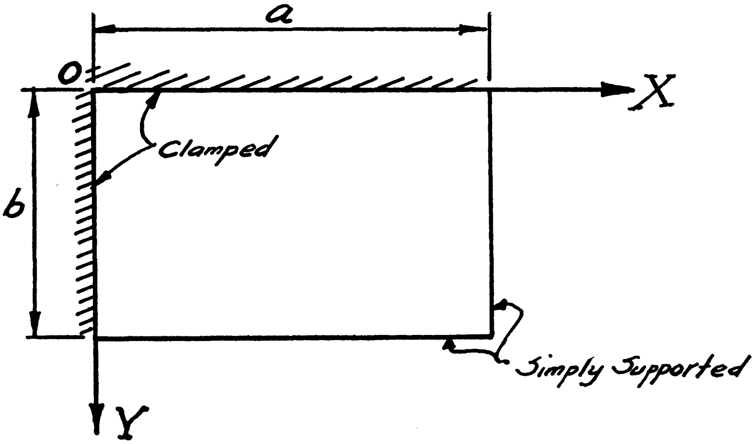

Fig. 1 Rectangular Plate with Two Adjacent Edges Clamped and the Others Simply Supported

of a simply supported plate subjected to a uniformly distributed load of intensity q is given by (9):

$$w_1 = \frac{4q_0 a^4}{\pi^5 D} \sum_{i=1,3,5}^{\infty} \frac{\sin \frac{i\pi x}{a}}{i^5} \left[1 - \frac{\alpha_i \tanh \alpha_i + 2}{2 \cosh \alpha_i} \cosh \frac{i\pi}{a}\left(y - \frac{b}{2}\right) + \frac{i\pi\left(y - \frac{b}{2}\right)}{2a \cosh \alpha_i} \sinh \frac{i\pi}{a}\left(y - \frac{b}{2}\right) \right]$$

$$\text{where} \quad \alpha_i = \frac{i\pi b}{2a} \quad \text{..............................} (4)$$

For the correction state, we take w_2 in the form:

$$w_2 = \frac{q_0 a^4}{D} \sum_{m=1,3,5}^{\infty} \left(A_m \cosh \frac{m\pi y}{a} + B_m \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} + C_m \sinh \frac{m\pi y}{a} + D_m \frac{m\pi y}{a} \cosh \frac{m\pi y}{a} \right) \sin \frac{m\pi x}{a}$$

$$+ \frac{q_0 b^4}{D} \sum_{n=1,3,5}^{\infty} \left(A_n \cosh \frac{n\pi x}{b} + B_n \frac{n\pi x}{b} \sinh \frac{n\pi x}{b} + C_n \sinh \frac{n\pi x}{b} + D_n \frac{n\pi x}{b} \cosh \frac{n\pi x}{b} \right) \sin \frac{n\pi y}{b}$$

$$\text{..............................} (5)$$

The boundary conditions are:

$$\left. \begin{array}{l} w = 0, \text{ when } x = 0, \text{ and when } x = a \\ \phantom{w = 0, \text{ when }} y = 0, \text{ and when } y = b \end{array} \right\} \text{..............................} (6)$$

$$\left.\begin{array}{l} \dfrac{\partial^2 w}{\partial x^2} + \nu \dfrac{\partial^2 w}{\partial y^2} = O, \text{ when } x = a \\ \dfrac{\partial^2 w}{\partial y^2} + \nu \dfrac{\partial^2 w}{\partial x^2} = O, \text{ when } y = b \end{array}\right\} \quad \text{(7)}$$

$$\left.\begin{array}{l} \dfrac{\partial w}{\partial x} = O, \text{ when } x = O \\ \dfrac{\partial w}{\partial y} = O, \text{ when } y = O \end{array}\right\} \quad \text{(8)}$$

To satisfy boundary conditions (6), we have:

$$A_m = A_n = 0 \quad \text{(9)}$$

$$\left.\begin{array}{l} B_m = -\left(\dfrac{C_m}{2\alpha_m} + D_m \coth 2\alpha_m\right) \\ B_n = -\left(\dfrac{C_n}{2\alpha_n} + D_n \coth 2\alpha_n\right) \end{array}\right\} \quad \text{(10)}$$

$$\text{Where } \alpha_m = \frac{m\pi b}{2a}, \ \alpha_n = \frac{n\pi a}{2b}$$

To satisfy boundary conditions (7), we obtain:

$$B_m = \frac{-2(\sinh 2\alpha_n + \alpha_m \cosh 2\alpha_m) D_m - (\sinh 2\alpha_m) C_m}{2(\cosh 2\alpha_m + \alpha_m \sinh 2\alpha_m)} \quad \text{(11)}$$

$$B_n = \frac{-2(\sinh 2\alpha_n + \alpha_n \cosh 2\alpha_n) D_n - (\sinh 2\alpha_n) C_n}{2(\cosh 2\alpha_n + \alpha_n \sinh 2\alpha_n)} \quad \text{(12)}$$

Substituting the relations between the coefficients (9), (10), (11) and (12) to equation (5), w_2 takes the following form:

$$w_2 = \frac{q_0 a^4}{D} \sum_{m=1,3,5}^{\infty} D_m \left[-\frac{m\pi}{a} (\tanh 2\alpha_m) y \sinh \frac{m\pi y}{a} - \frac{4\alpha_m}{\sinh 4\alpha_m} \sinh \frac{m\pi y}{a} + \frac{m\pi y}{a} \cosh \frac{m\pi y}{a} \right] \sin \frac{m\pi x}{a}$$

$$+ \frac{q_0 b^4}{D} \sum_{n=1,3,5}^{\infty} D_n \left[-\frac{n\pi}{b} (\tanh 2\alpha_n) x \sinh \frac{n\pi x}{b} - \frac{4\alpha_n}{\sinh 4\alpha_n} \sinh \frac{n\pi x}{b} + \frac{n\pi x}{b} \cosh \frac{n\pi x}{b} \right] \sin \frac{n\pi y}{b} \quad \text{(13)}$$

The coefficients D_m and D_n are determined by the boundary conditions (8). To satisfy the first of boundary conditions (8), it requires:

$$\frac{2}{\pi^4} \sum_{i=1,3,5}^{\infty} \frac{1}{i^4} \left[1 - \left(\frac{\alpha_i \tanh \alpha_i + 2}{2\cosh \alpha_i} \right) \cosh \left(\frac{i\pi y}{a} - \alpha_i \right) + \frac{\left(\frac{i\pi y}{a} - \alpha_i \right)}{2\cosh \alpha_i} \sinh \left(\frac{i\pi y}{a} - \alpha_i \right) \right]$$

$$+ \frac{2a}{b} \sum_{m=1,3,5}^{\infty} \alpha_m^2 D_m \left[y \cosh \frac{m\pi y}{a} - \tanh 2\alpha_m \, y \sinh \frac{m\pi y}{a} - \frac{2b \sinh \frac{m\pi y}{a}}{\sinh 4\alpha_m} \right]$$

$$+ \left(\frac{b}{a}\right)^4 \sum_{n=1,3,5}^{\infty} (\alpha_n) D_n \left[1 - \frac{4\alpha_n}{\sinh 4\alpha_n} \right] \sin \frac{n\pi y}{b} = 0 \quad \text{(14)}$$

Multiply by Sin $\frac{n\pi y}{b}$, and integrate from O to b. We obtain:

$$D_n = \frac{\frac{16\left(\frac{b}{a}\right)}{n^6\pi^6}\sum_{i=1,3,5}^{\infty}\frac{1}{\left[1+\left(\frac{ib}{na}\right)^2\right]^2} + 4\left(\frac{a}{b}\right)^5\sum_{m=1,3,5}^{\infty}\frac{D_m \tanh 2\alpha_m}{m\pi\left[1+\left(\frac{na}{mb}\right)^2\right]^2}}{\left(\frac{4\alpha_n}{\sinh 4\alpha_n}-1\right)} \qquad (15)$$

Similarly, to satisfy the second of boundary conditions (8) gives:

$$D_m = \frac{\frac{2}{m^5\pi^5}\left[\tanh\alpha_m - \frac{\alpha_m}{\cosh^2\alpha_m}\right] + 4\left(\frac{b}{a}\right)^5\sum_{n=1,3,5}^{\infty}\frac{D_n \tanh 2\alpha_n}{n\pi\left[1+\left(\frac{mb}{na}\right)^2\right]^2}}{\left(\frac{4\alpha_m}{\sinh 4\alpha_m}-1\right)}. \qquad (16)$$

Equations (15) and (16) give two sets of infinite number of equations to determine the coefficients D_m and D_n. After these coefficients are determined, the resultant deflection of the plate can be obtained by superimposing w_1 from equation (4) and w_2 from equation (5).

If the plate is subjected to any other type of loadings, solutions can be derived by following similar procedures. Of course, w_1 has to be changed to proper functions, but w_2 remains practically the same as equation (5). The equations to determine the coefficients of w_2 will be the same as equations (15) and (16) except the first terms in the numerators.

Example 2. Rectangular Plate with Two Adjacent Edges Clamped and the Others Free.

For this problem, a slight adjustment for both w_1 and w_2 is necessary in order to obtain the correct solution. If we follow the same procedures used for the first example, the function w_2 for the correction state is unable to eliminate both the distributed reactions along the free edges and the concentrated reaction at the free corner introduced by w_1. To avoid this difficulty, we consider w_1 as the deflection of a simply supported plate of double the dimensions of the original plate as shown in dotted lines on Fig. 2. Then the function for the correction state can be selected in similar form as before, and the coefficients of which can be adjusted so as to satisfy all the boundary conditions without difficulty.

Let us consider again the case of uniformly distributed load. If the coordinate axes are chosen as shown in Fig. 2, the deflection

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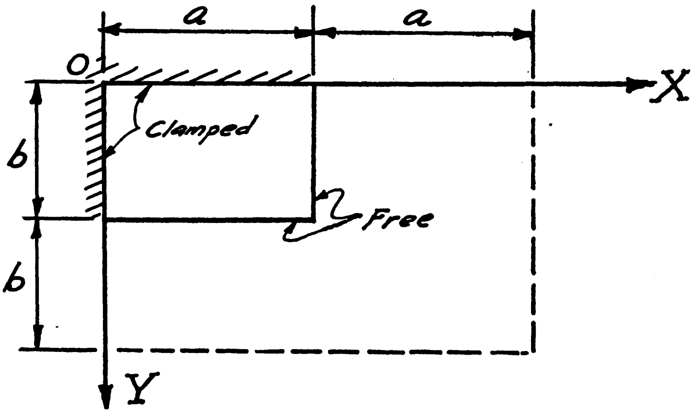

Fig. 2. Rectangular Plate with Two Adjacent Edges Clamped and the Others Free

of a simply supported plate of the size 2a x 2b subjected to a uniformly distributed load of intensity q_o is given by:

$$w_1 = \frac{64 q_o a^4}{D} \sum_{i=1,3,5}^{\infty} \frac{\sin \frac{i\pi x}{2a}}{i^5 \pi^5} \left[1 - \left(\frac{\alpha_i \tanh \alpha_i + 2}{2 \cosh \alpha_i} \right) \cosh \frac{i\pi(y-b)}{a} + \frac{i\pi(y-b)}{4a \cosh \alpha_i} \sinh \frac{i\pi(y-b)}{2} \right] \quad \text{.......... (17)}$$

w_2 is taken in the following form:

$$w_2 = \frac{16 q_o a^4}{D} \sum_{m=1,3,5}^{\infty} \sin \frac{m\pi x}{2a} \left(B_m \frac{m\pi y}{2a} \sinh \frac{m\pi y}{2a} + C_m \sinh \frac{m\pi y}{2a} + D_m \frac{m\pi y}{2a} \cosh \frac{m\pi y}{2a} \right)$$
$$+ \frac{16 q_o b^4}{D} \sum_{n=1,3,5}^{\infty} \sin \frac{n\pi y}{2b} \left(B_n \frac{n\pi x}{2b} \sinh \frac{n\pi x}{2b} + C_n \sinh \frac{n\pi x}{2b} + D_n \frac{n\pi x}{2b} \cosh \frac{n\pi x}{2b} \right) \quad \text{.......... (18)}$$

The boundary conditions are:

For clamped edges:

$$\left. \begin{aligned} w = O, \text{ when } x &= O \\ \text{and } y &= O \end{aligned} \right\} \quad \text{.......... (19)}$$

$$\left. \begin{aligned} \frac{\partial w}{\partial x} &= O, \text{ when } x = O \\ \frac{\partial w}{\partial y} &= O, \text{ when } y = O \end{aligned} \right\} \quad \text{.......... (20)}$$

For free edges:

$$\left.\begin{aligned} \left.\frac{\partial^3 w}{\partial x^3} + (2-\nu)\frac{\partial^3 w}{\partial x \partial y^2}\right| &= O, \text{ when } x = a \\ \left.\frac{\partial^3 w}{\partial y^3} + (2-\nu)\frac{\partial^3 w}{\partial y \partial x^2}\right| &= O, \text{ when } y = b \end{aligned}\right\} \cdots\cdots\cdots (21)$$

$$\left.\begin{aligned} \frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2} &= O, \text{ when } x = a \\ \frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2} &= O, \text{ when } y = b \end{aligned}\right\} \cdots\cdots\cdots (22)$$

It is seen that boundary conditions (19) are already satisfied. Boundary conditions (21) will be satisfied, if

$$B_m = \frac{(1-\nu)(\cosh\alpha_m)\, C_m + \left[(1-\nu)\alpha_m \sinh\alpha_m - (1+\nu)\cosh\alpha_m\right] D_m}{(1+\nu)\sinh\alpha_m - (1-\nu)\alpha_m\cosh\alpha_m} \cdots\cdots\cdots (23)$$

and

$$B_n = \frac{(1-\nu)(\cosh\alpha_n)\, C_n + \left[(1-\nu)\alpha_n \sinh\alpha_n - (1+\nu)\cosh\alpha_n\right] D_n}{(1+\nu)\sinh\alpha_n - (1-\nu)\alpha_n\cosh\alpha_n} \cdots\cdots\cdots (24)$$

By similar procedures used in the first example, it is easy to verify that to satisfy the boundary conditions (20) and (22), we obtain the following equations:

$$64(\alpha_n)^4 \sum_{m=1,3,5}^{\infty} \frac{\left\{C_m\left[(-1)^{\frac{n-1}{2}}\left(\nu+\left(\frac{na}{mb}\right)^2\right)\sinh\alpha_m - \left(\frac{na}{mb}\right)(1-\nu)\right]\cosh\alpha_m + D_m\left[\left(\frac{na}{mb}\right)\left((1+\nu)\cosh\alpha_m - (1-\nu)\alpha_m\sinh\alpha_m\right) - \right.\right.}{\left[(1+\nu)\sinh\alpha_m - (1-\nu)\alpha_m\cosh\alpha_m\right]}$$

$$\frac{\left.\left.-(-1)^{\frac{n-1}{2}}\left(\nu+\left(\frac{na}{mb}\right)^2\right)\alpha_m\right]\right\}}{\left[1+\left(\frac{na}{mb}\right)^2\right]^2} + \frac{n^5\pi^5}{2}(C_n+D_n) + \frac{4}{\alpha_n}\sum_{i=1,3,5}^{\infty}\frac{1}{\left[1+\left(\frac{ib}{na}\right)^2\right]^2} = 0$$

$$64(\alpha_m)^4 \sum_{n=1,3,5}^{\infty} \frac{\left\{C_n\left[(-1)^{\frac{m-1}{2}}\left(\nu+\left(\frac{mb}{na}\right)^2\right)\sinh\alpha_n - \left(\frac{mb}{na}\right)(1-\nu)\right]\cosh\alpha_n + D_n\left[\left(\frac{mb}{na}\right)\left((1+\nu)\cosh\alpha_n - (1-\nu)\alpha_n\sinh\alpha_n\right)\right.\right.}{\left[(1+\nu)\sinh\alpha_n - (1-\nu)\alpha_n\cosh\alpha_n\right]}$$

$$\frac{\left.\left.-(-1)^{\frac{m-1}{2}}\left(\nu+\left(\frac{mb}{na}\right)^2\right)\alpha_n\right]\right\}}{\left[1+\left(\frac{mb}{na}\right)^2\right]^2} + \frac{m^5\pi^5}{2}(C_m+D_m) + \left[\tanh\alpha_m - \frac{1}{\cosh^2\alpha_m}\right] = 0$$

$$8n\pi\left(\frac{a}{b}\right)^2\sum_{m=1,3,5}^{\infty}\frac{(-1)^{\frac{m-1}{2}}(1-\nu)\left[\left\{(-1)^{\frac{n+1}{2}}\left(\frac{na}{mb}\right)\frac{\sinh 2\alpha_m}{2}+\cosh\alpha_m\right\}C_m+\left\{\alpha_m\sinh\alpha_m-\frac{(1+\nu)}{(1-\nu)}\cosh\alpha_m+(-1)^{\frac{n-1}{2}}\left(\frac{na}{mb}\right)\alpha_m\right\}D_m\right]}{\left[1+\left(\frac{na}{mb}\right)^2\right]^2\left[\sinh\alpha_m-\frac{(1-\nu)}{(1+\nu)}\alpha_m\cosh\alpha_m\right]}$$

$$+\frac{4\alpha_n^2(1-\nu)\left\{C_n\left[\frac{2\cosh^2\alpha_n}{(1+\nu)}+\sinh^2\alpha_n\right]-\left[\frac{2}{(1-\nu)}+\frac{(1-\nu)}{(1+\nu)}\alpha_n^2\right]D_n\right\}}{\sinh\alpha_n-\frac{(1-\nu)}{(1+\nu)}\alpha_n\cosh\alpha_n}$$

$$-\frac{16}{n\pi^4}\sum_{i=1,3,5}^{\infty}\frac{(-1)^{\frac{i-1}{2}}}{i^3}\left[\frac{1+\nu\left(\frac{na}{ib}\right)^2}{\left(1+\left(\frac{na}{ib}\right)^2\right)^2}\right]=0$$

$$8m\pi\left(\frac{b}{a}\right)^2\sum_{n=1,3,5}^{\infty}\frac{(-1)^{\frac{n-1}{2}}(1-\nu)\left[\left\{(-1)^{\frac{m+1}{2}}\left(\frac{mb}{na}\right)^2\frac{\sinh 2\alpha_n}{2}+\cosh\alpha_n\right\}C_n+\left\{\alpha_n\sinh\alpha_n-\frac{(1+\nu)}{(1-\nu)}\cosh\alpha_n+(-1)^{\frac{m-1}{2}}\left(\frac{ma}{nb}\right)\alpha_n\right\}D_n\right]}{\left[1+\left(\frac{mb}{na}\right)^2\right]^2\left[\sinh\alpha_n-\frac{(1-\nu)}{(1+\nu)}\alpha_n\cosh\alpha_n\right]}$$

$$+\frac{4\alpha_n^2(1-\nu)\left\{C_m\left[\frac{2\cosh^2\alpha_m}{(1+\nu)}+\sinh^2\alpha_m\right]-\left[\frac{2}{(1-\nu)}+\frac{(1-\nu)}{(1+\nu)}\alpha_m^2\right]D_m\right\}}{\sinh\alpha_m-\frac{(1-\nu)}{(1+\nu)}\alpha_m\cosh\alpha_m}$$

$$+\frac{2\left[\alpha_m\tanh\alpha_m(1-\nu)-2\nu(\cosh\alpha_m-1)\right]}{m^3\pi^3\cosh\alpha_m}=0 \qquad \text{......(28)}$$

Equations (25), (26), (27) and (28) give four sets of an infinite number of equations to determine the coefficients C_m, D_m, C_n, and D_n. B_m and B_n are determined from equations (23) and (24). After these coefficients are determined, the resultant deflection of the plate can be obtained by superimposing w_1 from equation (17) and w_2 from equation (18). If the plate is subjected to any other type of loadings, solutions can also be determined by following similar procedures.

Numerical Example:

Let us consider a square plate with two adjacent edges clamped and the others free subjected to a uniformly distributed load of intensity q_o. Equations (25), (26), (27) and (28) are reduced to two sets of an infinite number of equations for this case. In order to check the results by Huang and Conway (8), we consider the particular case with $\nu = 0$. If we use only one term of the series, we have the following simultaneous equations:

$$-0.26740633\ C_1 + 2.0388978\ D_1 = -0.00436351$$
$$-190.26898\ C_1 + 33.259486\ D_1 = 0.0740696$$

from which $C_1 = -0.0007813$ and $D_1 = -0.0022426$

Substituting these values in to equation (23), we have

$$B_1 = 0.0027072$$

The maximum deflection is at the free corner and is equal to:

$$(w)_{max} = +0.0376781\ \frac{q_o a^4}{D}$$

If two terms of the series are used, we obtain four simultaneous equations:

$$-0.26740643\ C_1 + 2.0388978\ D_1 - 3.2504609\ C_3 + 0.6926047\ D_3 = -0.00436351$$

$$37.412468\ C_1 - 6.8038228\ D_1 + 514.95619\ C_3 + 146.68875\ D_3 = -0.00326226$$

$$-190.26898\ C_1 + 33.259486\ D_1 - 192.48457\ C_3 + 41.014407\ D_3 = 0.0740696$$

$$-8.313881\ C_1 + 1.5119607\ D_1 - 3380.676\ C_3 + 26.280553\ D_3 = 0.0001844$$

Solving these equations, we have:

$C_1 = -0.00077191$, $D_1 = -0.00226133$

$C_3 = 0.00000134$, $D_3 = 0.00006505$

and $B_1 = 0.00270543$, $B_3 = -0.00006539$

Using these values, the maximum deflection is found to be

$$(w)_{max} = 0.036222\ \frac{q_0 a^4}{D}$$

which checks very closely with Huang and Conway's results.

If three terms of the series are used, the result will be affected only in the fourth decimal point.

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