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In rcent years there has been a marked increase in the application of the methods of factor analysis to data associated with studies of learning phenomena or of practice effects. The problems are usually posed as investigations of changes in factorial composition as a function of practice with the variables of the correlation matrix including two or more trials on one or more tasks, or two or more parallel forms of some test. The first purpose of this paper is to present an application of a recently developed procedure for analyzing such correlational data.(2) In contrast to the analyses previously offered, the present procedure is logically correct and computationally feasible.

The recommended procedure not only reduces the influence of correlated error and specific factors, but also provides for a separation of the variables into two classes in terms of the hypotheses under consideration. This differentiation involves the specification of certain variables as the reference variables with the remaining variables classified and treated as experimental variables. The experimental variables are the ones about which statements are made in advance with reference to the hypotheses under consideration. This two-class, differentiation, while implicit in some factor analysis studies, has been explicitly stated in only a few cases. The method of factor analysis has been used most commonly as a method of defining, by response consistencies, variables which would then be used in more conventional experimental applications. It is the second purpose of this paper to indicate how available factor analysis methods may be used directly to investigate specific hypotheses related to the factor analysis formulations.

The data to which these procedures will be applied were obtained by Owens in his investigation of age and mental abilities (3). The initial observations were obtained from Iowa State College men to whom the Army Alpha, Form 6, was administered in January, 1919. During the period 1949-1950, the test was readministered to 127 of these cases. A principal axes analysis by McHugh and Owens of seven of the eight subtests of the first administration has been reported by Thompson (4). The present study analyzes the eight subtests of the first administration together with the eight subtests of the second administration by two procedures based on the centroid method of analyis and rotation to simple structure (6).

Two conventional analyses of the data for the 1919 and 1950 test administrations are presented in Table 1. The analysis was carried to two cycles to stabilize the diagonal values of less than unity

Table 1

Separate Factor Analyses of Army Alpha, Form 6, 1919 Data and 1950 Data (Centroid Method and Rotation to Simple Structure)

a: projections of the eight variables on the rotated normals

	1919 Data				1950 Data			
Code	A_1	B_1	h^2	Code	A_2	B_2	C_2	h^2
1	.06	.40	.25	1′	.00	—.08	.62	.55
2	—.01	.60	.49	2′	.07	.49	—.02	.54
3	.48	.19	.48	3′	.49	—.07	.09	.43
4	.58	.09	.54	4′	.62	.02	—.04	.69
5	.49	.15	.46	5′	.41	.08	.19	.63
6	—.07	.67	.56	6′	—.07	.64	.01	.73
7	.05	.54	.45	7′	.32	.12	.27	.67
8	.67	—.10	.53	8′	.60	—.02	—.08	.54

b: cosines of the angles between the normals after rotation

	1919 Data			1950 Data		
	A_1	B_1		A_2	B_2	C_2
A_1	1.00	—.52	A_2	1.00	—.46	—.25
B_1	—.52	1.00	B_2	—.46	1.00	—.42
.....	.....	.....	C_2	—.25	—.42	1.00

(communalities). Three of the approximate tests of significance presented by Thomson were used to determine when to stop factoring (5). Only two significant factors were found in the 1919 data, while three significant factors were indicated in the 1950 data. It should also be noted that the results of the 1919 analysis herein presented differ from those of the McHugh and Owens unpublished analysis only to the extent consistent with the differences in procedures. These differences are the inclusion of variable 1, the use of diagonal values of less than unity, and the use of an oblique simple structure.

The differences between the projections in Column A_1 and A_2 of Table 1 are comparable to those reported as sampling fluctuations. Even the somewhat greater discrepancies between Columns B_1 and B_2 might still be the basis for a judgment of "the same factor" in some factor studies. The reliance on judgment rather than on objective or analytical operations for the identification of factors represents the basic weakness of factor analysis today. Tucker's procedure for identifying factors in separate studies on the basis of an index of similarity of factor loadings reduces the area of judgment somewhat (7). However, in studies of the kind under consideration a completely objective procedure can be used. This procedure utilizes a single set of defining operations to specify a factor for both sets of data and thereby eliminates the judgmental processes of attempting to relate A_2 to A_1 and so forth.

The first step in an appropriate solution to this problem is to specify the reference variables and the experimental variables. Since the prior administration of the 1919 data provides a logical starting point, we may indicate the desired separation of the data by the following question—"Can the factors of the 1919 data account for the intercorrelations of the 1950 data and for the intercorrelations between the 1919 data and the 1950 data?" This formulation specifies three steps: first, the definition of the factors by the 1919 data; second, the determination of the factor loadings of the 1950 data in terms of the 1919 data; and third, the testing of the entire set of residuals for any additional significant factors. The computational procedures of the multiple group method provide the direct solution for these three steps. This projection aspect of the multiple group method has been used previously, but usually without the clear separation of one set of variables as the reference variables and another set as the experimental variables (1). Our procedure differs from that recently recommended by Tucker on this basic point (8). At issue are both the rank of the matrix of inter-correlations of the reference and experimental variables and the prior definition of the reference variables.

Before this simple computational procedure can be applied, one additional problem must be solved. This problem arises from the correlated specific and error factors associated with the administration of the variables under consideration. A solution to this problem has been developed (2). In effect, the test-retest correlations in the correlation matrix are considered as unknowns. The solution of these unknown values is comparable to the solution of the communality problem in a conventional factor analysis. The computational procedure, employing the multiple group method of factoring, leads to an iterative solution based on an assumed rank of the correlation matrix. The same procedure provides a stabilized set of values for the communalities, as well as a stable set of values to replace the observed test-retest correlations.

Table 2 presents the results of an application of the recommended procedures to the current data. The observed test-retest correlations are either equal to or greater than the coefficients estimated from the factors defined by the 1919 data. Had the observed values been used in the analyses, the likely result would be an improper increase in rank and changes in structure from doublet factors defined by test-retest pairs.

With the stabilized estimates of the test-retest values and of the communalities inserted in the correlation matrix, we may consider directly the prior question as to the relation between the 1919 data and the 1950 data. Two significant factors were defined by the centroids of the 1919 data. Since the residuals of the intercorrelations of the 1950 data were significant, two additional factors were

Table 2

Titles of Army Alpha, Form 6, Subtest Variables and Values of Observed Test-Retest Correlations and the Estimated Correlations Used in the Projection Procedure

Titles of Variables	Variables Correlated	Observed Values	Estimated Values
Following Directions	1, 1′	.30	.19
Arithmetic Problems	2, 2′	.69	.53
Practical Judgment	3, 3′	.56	.37
Synonym-Antonym	4, 4′	.64	.50
Disarranged Sentences	5, 5′	.48	.48
Number Series	6, 6′	.62	.51
Analogies	7, 7′	.56	.43
Information	8, 8′	.63	.43

determined from the 1950 data, and the communalities of all sixteen tests stabilized over these four factors. These four factors were then rotated in accordance with the principles of simple structure. Table 3 presents the results after rotation together with the stabilized communalities of the sixteen variables. Changes between the values for variables 1 to 8 in Table 3 and Table 1 can be accounted for largely by differences in the simple structure defined by sixteen rather than by eight variables and located in a four-dimensional space rather than a two-dimensional space. The values in each column of Table 3, a, represent the projections of the 16 variables on a single refer-

Table 3

Factor Analysis of Army Alpha 1919 and 1950 Data Using the Multiple Group or Projection Method and Estimated Test-Retest Correlations

a: projection of the two sets of eight subtests on the rotated normals

	Test	A_3	B_3	C_3	D_3	h^2
1919	1	.06	.25	.05	.01	.24
	2	—.03	.43	.01	.09	.51
	3	.45	.07	.03	.19	.49
	4	.54	.01	—.04	.28	.55
	5	.49	.05	.00	.13	.46
	6	—.09	.56	—.05	—.08	.58
	7	.05	.46	—.05	—.10	.46
	8	.70	—.08	—.07	.07	.59
1950	1′	—.09	—.08	.51	.31	.51
	2′	.10	.37	—.06	.28	.62
	3′	.37	—.10	.37	.04	.44
	4′	.41	.07	.32	—.03	.68
	5′	.36	—.03	.38	.26	.67
	6′	—.02	.34	.08	.20	.52
	7′	.16	.08	.47	.10	.66
	8′	.47	—.05	.33	—.09	.56

b: cosines of the angles between the normals after rotation

	A_3	B_3	C_3	D_3
A_3	1.00	—.49	.01	.23
B_3	—.49	1.00	—.53	—.42
C_3	.01	—.53	1.00	.03
D_3	.23	—.42	.03	1.00

ence normal. There is no logical problem or judgmental process involved as was the case with the data of Table 1.

The highest values for factor A_3 of Table 3 are for variables 3, 4, 5, and 8. The titles of these variables are given in Table 2. This factor, insofar as the identification with more adequate previous studies can be made, appears to correspond to the verbal skill factor.

The second column of Table 3, B_3, shows variables (2) and (6) with high loadings for both administrations. Variables (1) and (7) from the first administration also appeared on B_3. These results are consistent with a tentative identification of factor B_3 as a composite reasoning factor. The apparent tendency for the 1950 data to have somewhat lower loadings than the 1919 data on factors A_3 and B_3 may be considered intrinsic to the data and not characteristic of the method as such, since in other analyses using these methods such differences were not obtained. No adequate test of significance of such difference is available.

The fourth factor, denoted D_3, cannot be identified with any previously defined factor. However, with only 8 variables not specifically designed for a factor analysis investigation, no interpretation or further discussion of this result seems warranted at the moment.

The most exciting result of this analysis is represented by factor C_3. It will be noted that only the 1950 data have projections of any magnitude. These results clearly suggest that with the passage of time, not only will changes in the factor loadings on the reference factors such as A_3 and B_3 be found, but also new factors will appear. No attempt can be made from this limited study to identify the variables in the intervening time interval that could be considered as associated with these systematic changes in the factor structure. The investigation of changes in the factorial structure as a function of practice effects or of specific treatment conditions must be systematically carried out if the possible contributions of factor analysis to the accurate description and prediction of behavior is to be obtained. The procedures discussed in this paper will provide a logically correct and computationally feasible method of analysis of hypotheses regarding the effects of such treatments in a factor analysis.

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