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Abstract. This investigation showed that a true functional relationship, valid over a wide range of experimental conditions, does not exist between the results obtained by two methods of evaluating the strength of a cement-treated sandy soil. These two methods are the California Bearing Ratio test and the unconfined compressive strength test. However the equation, log CBR = 1.115 + 0.660 log UCS, can be used as a working relationship between the two tests under varying experimental conditions. It was concluded that a true functional relationship likely does exist for a given experimental condition, i.e., where only one experimental condition varied at a time.

Results obtained in this investigation suggest that a strength criterion of 250 psi for stabilized sand is unreasonable as it fails to take into account the strength gained due to the soil-cement being confined laterally.

One of the big difficulties in soil stabilization investigations is the general inadequacy of existing testing procedures for determining the exact performance rating of an improved soil for highway pavement design purposes. Probably the most commonly used criterion is the unconfined compressive strength (UCS) of the improved soil. The California Bearing Ratio (CBR) is also used as a design criterion for stabilized soils.

Although the UCS test is widely used in design, the exact minimum design criteria have yet to be established. The British Road Research Laboratory suggests a minimum 7-day strength value of 250 psi for soil-cement, in order to withstand the requirements of the American Society for Testing Materials (ASTM) durability test for wetting and drying or freezing and thawing (1). The Portland Cement Association states that soil-cement having an UCS of 300 psi after 7 days will usually pass the durability tests (2).

The CBR test is very much used in flexible pavement design. Some 14 of the 50 states now use the CBR value of a soil as their principal strength standard in highway design (3). The CBR test has been extensively correlated with the field performance of soils and it has been found that materials directly under the bituminous surface of a highway should have a CBR of at least 80%—this is equivalent to a laboratory CBR of about 120% (1).

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Lower CBR values are allowed at greater depths as the wheel-load stresses are more widely distributed.

Although the reliability of the CBR test for pavement design purposes is excellent, nevertheless the test has many disadvantages. Not only does the test require large quantities of soil and stabilization materials, but, in addition, it is relatively difficult and time-consuming to perform. The penetration test itself requires the services of two people for a minimum of ten minutes actual penetration time. On the other hand, the UCS test is simple to perform and requires small volumes of soil. To illustrate, in the work described here each CBR specimen took at least one hour to prepare and test, whereas each UCS sample required, on an average, a maximum of fifteen minutes to prepare and test. The amount of soil required for each CBR test was about ten pounds while only ½ pound was required for each UCS test.

The objective of this investigation was, therefore, to determine whether a tight functional relationship, valid over a wide range of experimental conditions, existed between these two methods of determining the strength of a cement stabilized soil. If such a relationship existed, then the CBR of the stabilized soil could be predicted from the more easily and quickly obtained UCS value.

Materials

The soil used in this investigation was a dune sand typical of those found in eastern Iowa. Sampling location and properties of the sample are given in Table 1.

The cement used was Type I normal Portland cement. Its properties are given in Table 2.

Table 1. Description and Properties of Sand

Location:	Benton County, Eastern Iowa Sect NW ¼, SE ¼, S-16 Twp. 86 N, Rn 10 W	Geological description	Wisconsin-age eolian sand, fine-grained, oxidized, leached
Soil series:	Carrington	Horizon:	C
Sampling depth, in feet:	6 – 11	IEES code number:	S–6–2
Textural composition, %:[a]		Mineral composition, %:[b]	
Gravel (2 mm.)	0.0	Total quartz	73.4
Sand (2 – 0.074 mm.)	94.4	Total feldspar	19.9
Silt (74 – 5μ)	1.6	Rock fragments	3.2
Clay (<5μ)	4.0	Calcite	0.2
Colloids (<1μ)	3.5	Mica	Trace
		Total heavy minerals	1.0
		Minus 0.044 mm. material	2.5
Predominant clay mineral:[c]	Montmorillonite and illite interlayer	Physical properties:	
		Liquid limit, %	19.0
		Plastic limit, %	. . .
Specific gravity 25C/4C:	2.64	Plasticity index	Non-plastic
Chemical properties:		Classification:	
Carbonates, %[d]	0.02	Textural	sand
pH	6.5	Engineering (A.A.S.H.O.)	A-3(0)
Organic matter %[d]	0.04		

[a] Dispersed by air-jet with sodium metaphospate dispersing agent. Coarse sand, 12.9%; fine sand, 81.5%.
[b] Material larger than 0.044 mm. (Per cent by volume of the whole sample)
[c] From X-ray analysis.
[d] Per cent by weight of oven-dry soil.

Table 2. Cement Properties[a]

Cement type: I		
Chemical composition, %:		
Silica	21.62	
Alumina	5.04	
Iron oxide	2.97	
Lime	64.05	
Magnesia	2.90	
Sulfur trioxide	2.26	
Ignition loss	0.58	
Insoluble residue	0.16	
Physical properties:		
Fineness, turbidometer (Wagner), sq.cm./gm.		1855
Fineness, air permeability (Blaine), sq.cm./gm.		3395
Compressive strength (1:2.75 mortar)		
1 day, psi		....
3 day, psi		2269
7 day, psi		3721
28 day, psi		5625

[a] Data supplied by Penn-Dixie Cement Corporation, Des Moines, Iowa

Experimental Procedure

The first step consisted in preparing a large homogeneous master batch of the sandy soil from which 156 sub-batches were randomly selected. These were partitioned into 39-quadruple sub-batches. Members of the same quadruple were handled in the same manner, the same specified amounts of cement and water being added to each. Each such quadruple member was then sub-divided into three specimens, one large specimen for CBR testing and the two smaller specimens for UCS testing. These three specimens were cured in the same manner for a specified period of time before being tested.

The experimental origins of the CBR values and the UCS values are perhaps best illustrated schematically as in Figure 1.

CBR Test. All CBR specimens were prepared and tested according to the ASTM "Tentative Method of Test for Determining the Bearing Ratio (CBR) of Soils, 1959," with some exceutions as described now.

Due to the large number of CBR specimens that had to be reported and tested, it was felt to be impractical to us the CBR mold as described by ASTM. Instead a special CBR mold was devised. The mold is shown in Figure 2. It consists, simply, of a standard CBR mold cut on one side using a 1/16 in. mill. A 1/16 in. wide piece of steel, of such size and contour as to replace exactly the milled material, was inserted into the gap and soldered onto one side of the mold. The gap was closed or opened as required by means of the bolt attachment shown in Figure 2.

CBR specimens were prepared in the following manner. Using the bolt attachment, the gap on the side of the mold was closed

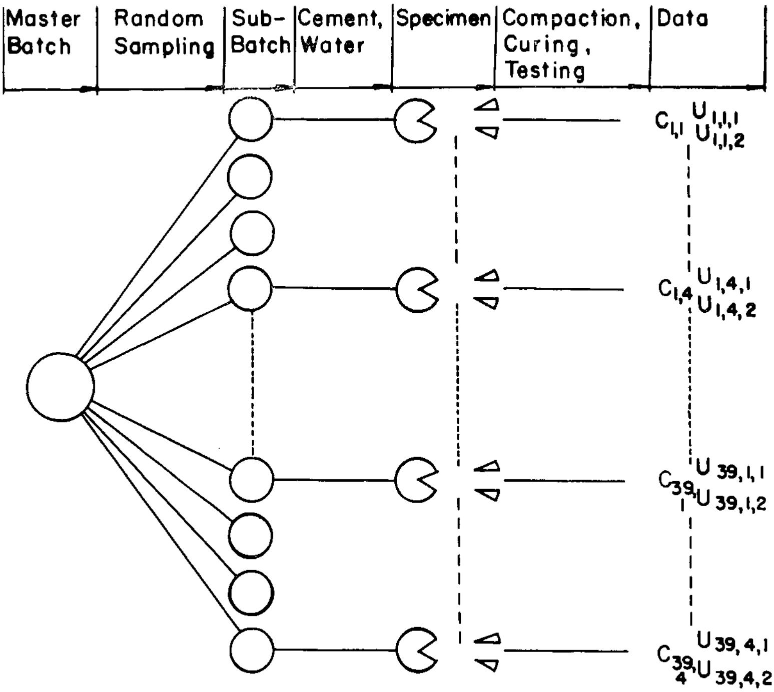

Figure 1. Structure of the experiment.

as tightly as possible with a hand wrench. The inside of the mold was lightly coated with oil. The mold—with collar attached—was clamped to the base plate and the spacer disk was inserted into the mold. Two circular layers of wax paper, each just under 6 in. in diameter, were placed on top of the disk. The soil, cement and water mixture was compacted in the mold in accordance with the standard procedure (4). After compaction the extension collar was removed and the compacted material was carefully trimmed so as to be even with the top of the mold. The spacer disk and base plate were removed and the mold plus the compacted material weighed. A piece of wax paper, approximately 7½ in. square, was placed on each end of the mold and fastened in place by means of tight elastic bands. The mold was then placed in the curing room for 24 ± 3 hours. Care was taken that the mold rested on the end at which the soil cement was trimmed level with the lip of the mold. After this curing period, a mark was made on the mold lip and a similar adjacent mark made on the soil-cement specimen. The bolt attachment on the outside of the mold was loosened, allowing the mold to open about ¼ in. Usually this was sufficient to allow the mold to be withdrawn from about the specimen. The specimen was carefully wrapped in wax paper, sealed with adhesive tape, and replaced in its original position in the curing room.

In order to perform the penetration test, the CBR specimen was unwrapped and replaced in its original mold so as to fit its original contour. This was checked by having a mark on the specimen line up with a similar mark on the mold. The mold was closed about the specimen using a hand wrench to tighten the bolt attachment as tightly as possible. The penetration test was immediately carried out upon the specimen and the CBR value read at 0.10 inch penetration.

Some specimens were soaked before being tested. In such cases, the CBR specimens were taken from the curing room,

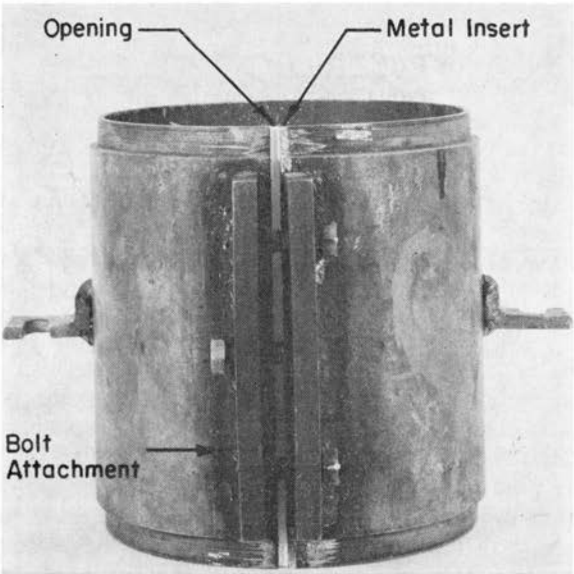

Figure 2. Modified CBR mold.

unwrapped and immersed in distilled water. Care was taken that the water surface remained a constant 1½ in. above the top of each specimen. The soaking period for such specimens was 24 ± 2 hours.

UCS Test. Specimens used in this test were 2 in. in diameter by 2 in. high. They were molded and compacted using a drop-hammer molding apparatus developed by Davidson and Chu (5). After compaction, specimens were ejected from the molds with a hydraulic jack. Each specimen was weighed to the nearest 0.1 g and its height measured to the nearest 0.001 in. A height tolerance of ±0.05 in. was maintained on all specimens.

Each specimen was wrapped in wax paper and sealed with adhesive tape before being placed in the curing room. After curing, the unconfined compressive strength of each specimen was obtained by means of a testing machine of the proving ring type. Load was applied to each specimen, the rate of deformation being 0.10 in. per minute, until complete failure was reached. The maximum load in pounds was divided by the cross-sectional area of the 2 in. diameter specimen and the result, in psi, reported as the unconfined compressive strength of the specimen.

Certain 2 in. diameter by 2 in. specimens required soaking prior to testing. Such specimens were unwrapped and immersed in distilled water for 24 ± 2 hours. Care was taken that, at all times, the surface of the water was ¼ in. above the top of each sample.

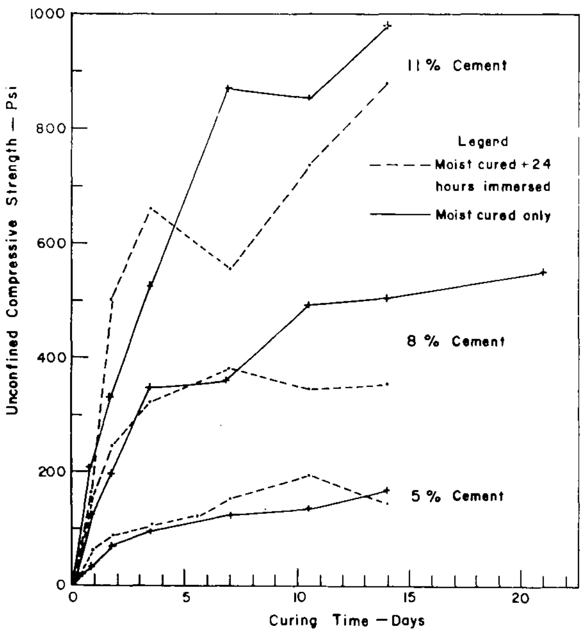

Figure 3. Unconfined compressive strengths versus curing times at varying cement contents and methods of curing.

Dry Densities. One CBR specimen and two UCS specimens were prepared from each sub-batch. A moisture sample was taken immediately prior to the preparation of the first specimen and immediately after the compaction of the last specimen. The average of these two moisture contents was then used to calculate the dry densities of the three specimens prepared from that particular sub-batch. The dry densities of the two types of specimens were within acceptable limits of variation. The average dry densities obtained at varying cement contents are shown in Table 3.

Table 3. Average Dry Densities of Soil-Cement Specimens in Pounds Per Cubic Foot

Type of specimen	Cement content, percent of total mix 5	8	11
UCS	107.2	111.0	112.1
CBR	105.0	108.7	110.8

Curing. Each CBR specimen and its corresponding pair of UCS specimens were placed side by side in the curing room. The temperature in the curing room was maintained at 70°F and relative humidity at 90%. Moist curing periods varied from 10 hours to 21 days. Approximately half of the specimens were cured for a further 24 ± 2 hours by immersing them in distilled water.

Cement Contents. Specimens were prepared using cement contents of 5%, 8% and 11% by weight of total mix.

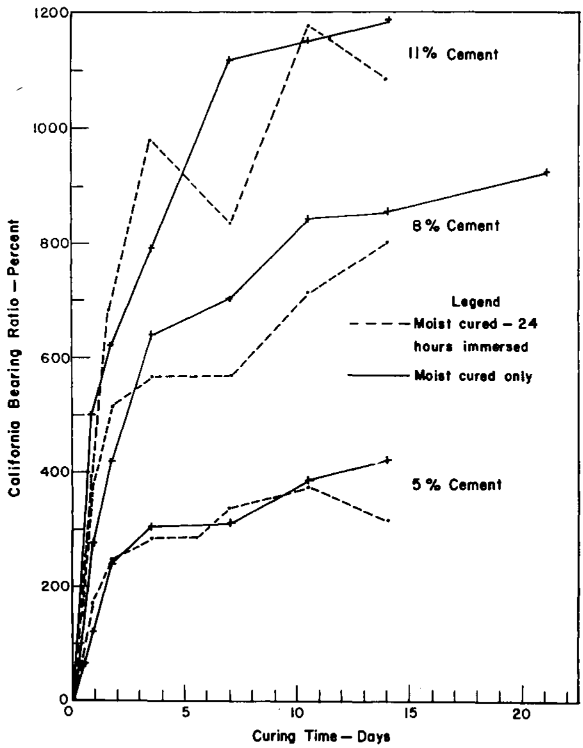

Figure 4. California Bearing Ratios versus curing times at varying cement contents and methods of curing.

Discussion of Data

Figures 3 and 4 indicate the strength results obtained under varying conditions. As expected, strength values increased with increasing cement contents and increasing lengths of curing. In addition, as the length of curing increased, the rate of strength gain decreased. It is interesting to compare the immersed specimens with the unimmersed specimens. The immersed specimens each had an extra day of curing and as a result gained extra strength. On the other hand, the immersed specimens lost a certain amount of strength due to being immersed. For the 5% cement specimen the strength values are close to each other, indicating that the strength gained due to the extra day's curing is essentially nullified by the strength loss due to being immersed. With the higher cement contents however, the immersion effect appears to be much more severe. It would seem as if this effect is mainly a function of length of curing. At low curing periods, immersion has little or no effect on strengths. At such times, the rate of strength gain is so fast that the extra day's curing tends to outweigh or balance the loss in

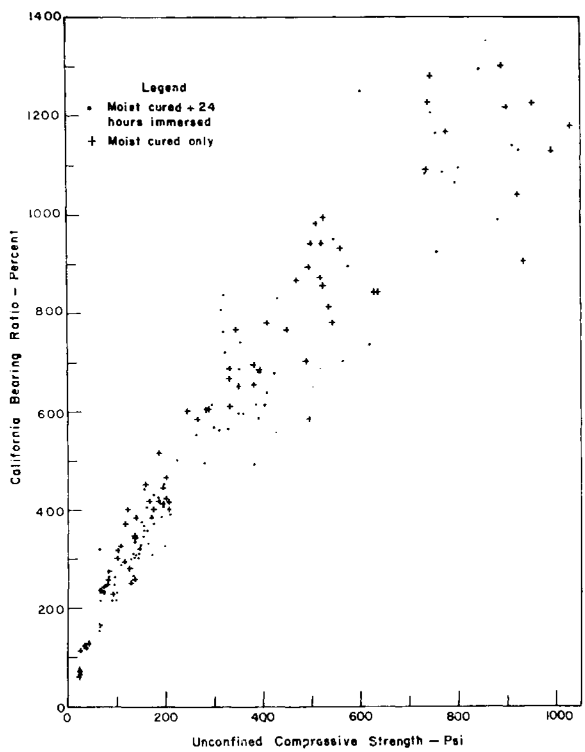

Figure 5. California Bearing Ratio versus unconfined compressive strengths.

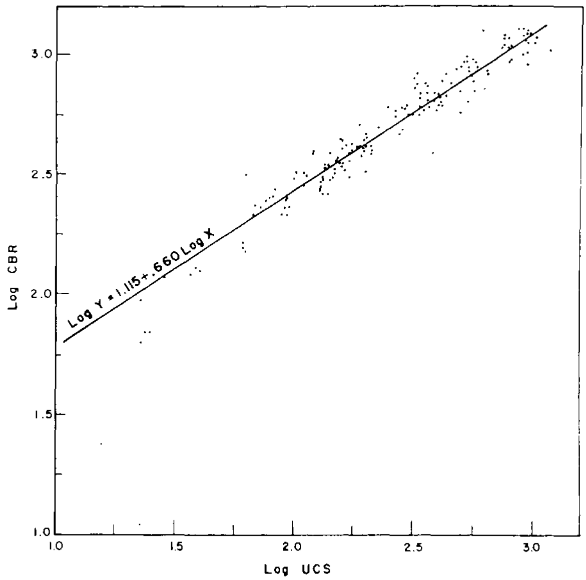

Figure 6. Log California Bearing Ratios versus log unconfined compressive strengths.

strength due to being immersed. However as the curing time increases, the rate of strength increase decreases and hence the strength loss due to being immersed is much more apparent.

Figure 5 shows all of the CBR values plotted against their corresponding UCS values. The grouping of the data suggested that a relationship seemed to exist between the two tests. When the data were transformed by means of logarithims, as shown in Figure 6, there were indications that possibly there might be a straight line relationship between log CBR and log UCS. The data were then analyzed in a statistical manner.

Summary of Statistical Analysis of Data

The data were first examined for inhomogeneity, in three different respects:

1. deviations of quadruple averages from trends exhibited in Figures 3 and 4
2. deviations of quadruple variances from trends exhibited in Figures 5 and 6, and
3. unusual consistency in quadruple configurations.

No quadruple was suspect in more than one respect, so that no quadruple was eliminated on the basis of these considerations.

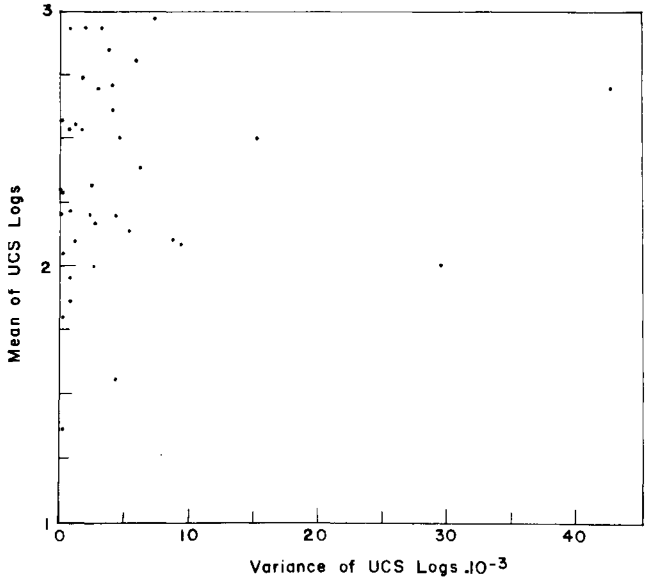

Figure 7. Variance versus mean for quadruples of unconfined compressive strength logs.

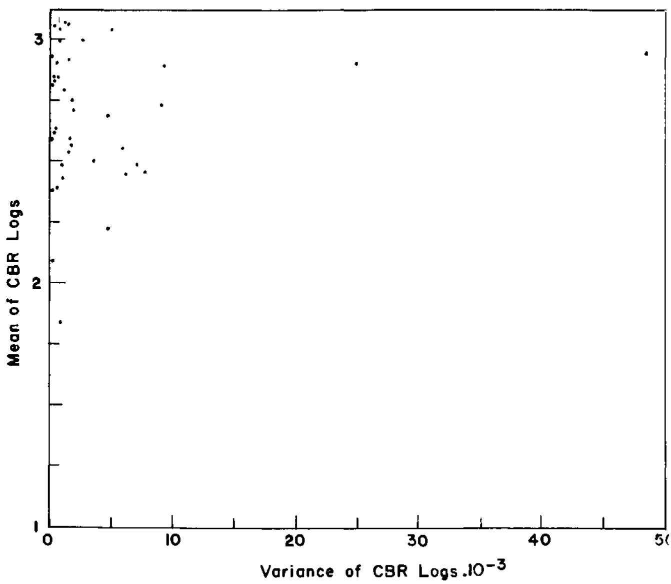

Figure 8. Variance versus mean for quadruples of California Bearing Ratio logs.

Figures 6, 7 and 8 now suggest that the following statistical model (*) will afford a reasonable description of the data if it is hypothesized that a simple functional relationship relates UCS logs to CBR in the absence of test errors.

$$X_{ij} = \log (UCS_{ij}) = \xi_i + \varepsilon_{ij}$$
(*)
$$Y_{ij} = \log (CBR_{ij}) = \alpha + \beta\xi_i + \eta_{ij}, \text{ i: } 1,2 \ldots\ldots.39, \text{ j:}1,2,3,4$$

where

ξ_i : errorless log UCS for the i th test condition,

$\alpha+\beta\xi_i$: errorless log CBR for the i th test condition, a linear function of ξ_i,

ε_{ij} : a normal error variable with mean zero and standard deviation $\sigma\varepsilon$

η_{ij} : a normal error variable with mean zero and standard deviation$\sigma\eta$

In addiion, it is assumed in model (*) that the 312 errors variables ε_{ij}

andη_{ij} are uncorrelated except for a constant correlation between ε_{ij}

and $\eta_{i'j'}$ when i = i' and j = j'.

Figures 6, 7, 8, 9 and 10 suggest that the chosen model (*) is valid for the following reasons.

1. Figure 6 indicates that if a functional relation exists be-

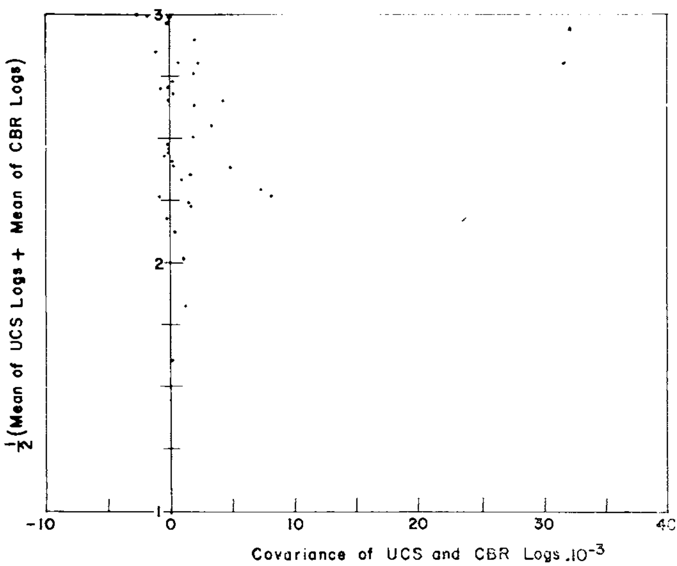

Figure 9. USC-CBR covariance versus UCS-CBR mean for quadruples of UCS-CBR pairs.

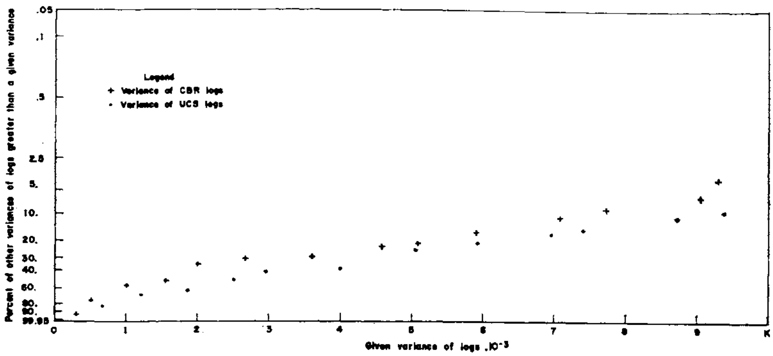

Figure 10. Variance of quadruples of unconfined compressive strength logs and variances of quadruples of California Bearing Ratio logs plotted on chi-square probability paper.

tween "true" UCS logs and "true" CBR logs, then it is very likely to be a linear relationship.

2. Figures 7 and 8 show that it is not unreasonable to assume a constant variance for ε_{ij} and η_{ij}.

3. Figure 9 indicates that a constant correlation exists between ε_{ij} and η_{ij}.

4. Figure 10 suggests that the normality assumption is not unwarranted. It indicates that, to within expected sampling variation, the CBR and UCS variances follow the chi-square distribution arising under normality.

Assuming the model (*) to be acceptable, it is now possible to proceed with the construction of a 5% test of the hypothesis:

$$H_0: "\alpha = \alpha_0, \beta = \beta_0"$$

Compute

1. $\sum_i \sum_j (Y_{ij} - \alpha_0 - \beta_0 X_{ij})^2$

2. $\sum_j (X_{ij} - \bar{X}_{i.})^2 / 3 \equiv S^2_{x,i}$,

$\sum_j (Y_{ij} - \bar{Y}_{i.})^2 / 3 \equiv S^2_{y,i}$, and

$\sum_j (X_{ij} - \bar{X}_{i.})(Y_{ij} - \bar{Y}_{i.}) / 3 \equiv S^2_{x,y,i}$

3 $S^2_x = \sum S^2_{x,i} / 39$,

$S^2_y = \sum S^2_{y,i} / 39$ and

$S^2_{xy} = \sum S^2_{x,y,i} / 39$

4. $\mathcal{S}_{\alpha_0, \beta_0} \equiv \sum_i \sum_j (Y_{ij} - \alpha_0 - \beta_0 X_{ij})^2 / S^2_y + \beta_0^2 S^2_x - 2\beta_0 S^2_{x,y}$

Compare

$$\mathcal{S}_{\alpha_0,\beta_0} \text{ with } 117 + \left[F_{39,117}(.05)\right]\left[39\right] \equiv K$$

$$\text{If } \mathcal{S}_{\alpha_0,\beta_0} < K, \text{ accept } H_0$$

$$\text{If } \mathcal{S}_{\alpha_0,\beta_0} \geqslant K, \text{ reject } H_0$$

The above is a 5% test for the following reason.

Define

$$z_{ij}(\alpha_0,\beta_0) \equiv (Y_{ij} - \alpha_0 - \beta_0 X_{ij})$$

Then

$$\mathcal{S}_{\alpha_0,\beta_0} = \sum_i\sum_j (Y_{ij} - \alpha_0 - \beta_0 X_{ij})^2 / s_Y^2 + \beta_0^2 s_X^2 - 2\beta_0 s_{X,Y}^2$$

$$= \frac{\sum_i\sum_j z_{ij}^2(\alpha_0,\beta_0)}{\dfrac{\sum_i\sum_j (Y_{ij}-\bar{Y}_{i.})^2 + \beta_0^2 \sum_i\sum_j (X_{ij}-\bar{X}_{i.})^2 - 2\beta_0 \sum_i\sum_j (X_{ij}-\bar{X}_{i.})(Y_{ij}-\bar{Y}_{i.})}{(3)(39)}}$$

$$= \frac{\sum_i\sum_j \left[z_{ij}(\alpha_0,\beta_0) - \bar{z}_{i.}(\alpha_0,\beta_0)\right]^2 + 4\sum_i \bar{z}_i^2(\alpha_0,\beta_0)}{\sum_i\sum_j \left[z_{ij}(\alpha_0,\beta_0) - \bar{z}_{i.}(\alpha_0,\beta_0)\right]^2 / (3)(39)}$$

$$= (3)(39) + (39)\left[\frac{4\sum_i \bar{z}_{i.}^2(\alpha_0,\beta_0)/(39)}{\sum_i\sum_j \left[z_{ij}(\alpha_0,\beta_0) - \bar{z}_{i.}(\alpha_0,\beta_0)\right]^2} \quad (3)(39)\right]$$

$= 117 + (39)$ (a statistic distributed as $F_{39,\,117}$ Under H_0), Q.E.D.

For the data at hand, it is found that $\min\limits_{\alpha,\beta} \mathcal{S}_{\alpha,\beta} > K$.

Hence, no hypothesis of form H_0 is acceptable at the 5% level. This then suggests that the model (*) is not realistic.

The most suspect feature of the model (*) would seem to be the hypothesizing of a functional relationship between the "true" USC logs and "true" CBR logs. This then is a feature calling for re-examination. Actually, it is somewhat more reasonable to think of a set of functional relationships, each corresponding to variation of but a single factor level, forming a two-dimensional configuration in the plane bounded by two envelopes. It follows that, unless one is willing to specify factor conditions rather exactly, it becomes rather difficult to bring rigorous statistics to bear on the problem of determining a confidence interval for CBR values corresponding to specified UCS values.

The above does not, of course, preclude the possibility of using to good advantage the strong correlation evidence in

Figure 6, at least until further investigation yields statistical recipies as functions of factor conditions. It is to this end that the following equation of fit is presented:

$$\log Y = 1.115 + .660 \log X,$$

where Y = CBR value and X = UCS value. The α and β values appearing in the equation, i.e. $\alpha = 1.115$ and $\beta = .660$, are in fact the α and β values that minimize $\mathcal{S}_{\alpha,\beta}$.

Conclusions

1. A true functional relationship, valid over a wide range of experimental conditions, does not exist between the unconfined compressive strength and the California Bearing Ratio.

2. The equation, log CBR = 1.115 + 0.660 log UCS, provides a working relationship between the California Bearing Ratio and the unconfined strength under varying experimental conditions.

3. A true functional relationship likely does not exist between the CBR and the UCS for a given experimental condition where only one factor is varied, e.g. if the soil, cement content and method of curing were kept constant and only the length of curing was varied. Thus the above equation, although it cannot be considered an estimate of a single true relationship, can be considered the "average" of many single factor relationships.

4. It may be that a true relationship exists between the CBR and the UCS of stabilized soils where the only variable is the soil type. It should be kept in mind however that soil type is not as well defined a factor as cement content, curing time or method of curing.

5. Based on this investigation, it appears that a sand-cement mixture with an unconfined compressive strength of 250 psi has a California Bearing Ratio of about 500%. Similarly, a sand-cement mixture with a CBR of 120% has a UCS of about 29 psi. These figures immediately suggest that a criterion of 250 psi for stabilized sand is unreasonable as it fails to take into account the strength gained due to being confined laterally.

Acknowledgements

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