Iowa Science Teachers Journal

Volume 13 | Number 1 Article 5

1976

Keeping up with the Sun

David R. Duncan
University of Northern Iowa

Bonnie H. Litwiller
University of Northern Iowa

Follow this and additional works at: https://scholarworks.uni.edu/istj

Part of the Science and Mathematics Education Commons

Recommended Citation

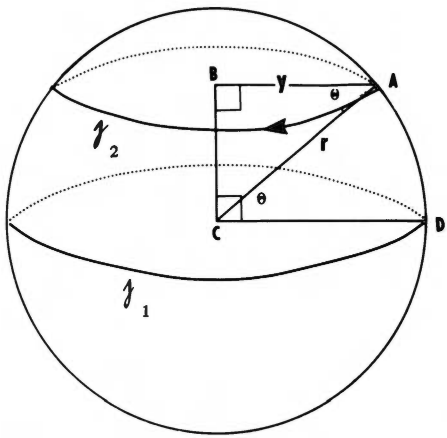
Duncan, David R. and Litwiller, Bonnie H. (1976) "Keeping up with the Sun," *Iowa Science Teachers Journal*: Vol. 13 : No. 1 , Article 5.
Available at: https://scholarworks.uni.edu/istj/vol13/iss1/5

This Article is brought to you for free and open access by UNI ScholarWorks. It has been accepted for inclusion in Iowa Science Teachers Journal by an authorized editor of UNI ScholarWorks. For more information, please contact scholarworks@uni.edu.

Keeping Up With the Sun

David R. Duncan and Bonnie H. Litwiller
Department of Mathematics
University of Northern Iowa
Cedar Falls, Iowa 50613

While flying from Minneapolis to a convention on the west coast we observed that it was becoming dark outside even though we took off in daylight and were flying west toward the sun. We asked ourselves the question, "How fast should we be flying to 'keep up with the sun'?" To examine this question, study Diagram 1 below.



Legend:

γ_1 is the equator.

C is the center of the earth.

A is the location of the airplane flying west.

γ_2 is the latitude circle being followed by the airplane.

B is the center of γ_2.

D is the intersection of the line through C parallel to $\overleftrightarrow{AB}$ so that $\angle$ ACD is acute (the measure of $\angle$ ACD is simply the latitude of point A and may be found in any geographic book of tables).

r (the length of $\overline{AC}$) is the radius of the earth (3960 miles).

Diagram 1 and legend.

We wish to calculate the *critical speed* at latitude θ — that is, the constant speed at which an observer traveling west along γ_2 from point A would return to point A in exactly 24 hours. Since γ_2 makes a complete rotation approximately every 24 hours, an observer traveling west along γ_2 at the *critical speed* would change time zones once every hour. He would return to A after one trip around γ_2 at the same clock time as when he left – but a day later. During the entire trip the sun (if visible) would appear to stay in approximately the same relative location in the sky the observer would seem to "keep up with the sun".

How can the *critical speed* at latitude θ be calculated? Since $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\angle BAC \cong \angle ACD$ (alternate-interior angles). Thus, $\cos\theta = \frac{y}{r}$ or $y = r\cos\theta$. Because AB is a radius of γ_2, the circumference of γ_2 is $2\pi y$ or $2\pi r\cos\theta$. γ_2 thus revolves at the speed of approximately $\frac{2\pi r\cos\theta}{24}$ mph (from west to east) or $\pi r\frac{\cos\theta}{12}$ mph. The critical speed at latitude θ is thus $\frac{\pi r\cos\theta}{12}$ mph (east to west). An observer traveling west from A along γ_2 at this speed would return to A in 24 hours; during the entire trip the sun would stay in the same relative location in the sky.

Let us calculate the critical speed for Minneapolis for which $\theta = 45.0°$

$$\text{C. S.} = \frac{\pi r\cos\theta}{12}$$

$$= \pi\frac{(3960)\cos 45.0°}{12}$$

$$= 733 \text{ mph}$$

Since the airplane (in our opening example) was traveling approximately 500 mph, it is evident why the sun set.

Find the critical speed for Quito, Equador which lies on the equator where $\theta = 0°$.

$$\text{C. S.} = \frac{\pi r\cos\theta}{12}$$

$$= \frac{\pi(3960)\cos 0°}{12}$$

$$= 1{,}037 \text{ mph}$$

In Table 1, we have calculated the *critical speed* for selected North American cities.

Table 1

City	θ (latitude)	Critical Speed
Mexico City, Mexico	19.4°	978 mph
Miami, Florida	25.8°	933 mph
Pheonix, Arizona	33.5°	865 mph
Peoria, Illinois	40.7°	786 mph
Waterloo, Iowa	42.5°	764 mph
Fargo, North Dakota	46.9°	708 mph
Winnepeg, Manitoba	49.9°	668 mph
Saskatoon, Saskatchewan (Canada)	52.1°	637 mph
Juneau, Alaska	58.3°	545 mph
Anchorage, Alaska	61.2°	499 mph
Fairbanks, Alaska	64.8°	441 mph
Pt. Barrow, Alaska (on North Slope)	71.3°	332 mph
Alert, Northwest Territories Canada (in Arctic Ocean)	82.5°	135 mph

Without calculating the critical speed, one might suspect that it would decrease by the same amount for each constant increase in latitude. But, this is not the case because cosine θ decreases more rapidly as θ approaches 90°; this is shown in Table 2.

Table 2

θ	Cos θ	C. S.
0°	1.000	1,037 mph
10°	.985	1,021 mph
20°	.940	974 mph
30°	.866	898 mph
40°	.766	794 mph
50°	.643	666 mph
60°	.500	518 mph
70°	.342	355 mph
80°	.174	180 mph
89°	.017	18 mph
89.5°	.009	9 mph

Teachers and students are encouraged to search for other applied situations which raise stimulating questions answerable only by using secondary school mathematics.