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Abstract. This study was undertaken in an attempt to correct frequent student failure in understanding mathematical relationships describing physical phenomena.

The study involved two sections of University of Iowa Laboratory High School ninth graders enrolled in the course entitled Energy/Space. The proposed solution to the problem was to allow the students to experimentally develop their own mathematical expressions to fit specific physical situations. This procedure replaced the laboratory approach which merely called upon the students to verify relationships which had been previously introduced in classroom discussion.

The experimental approach, utilized by the students, involved a general discussion of the problem, materials needed, and an experimental design. From this point the students proceeded to conduct a series of trials, make observations, and draw conclusions based on these trials. Their experiments involved, for the most part, holding all of the variables constant save one in a series of steps, and finally experimentally deriving mathematical expressions to fit the physical situations.

The results of the study suggested that the students understood the mathematical relationships better under these conditions than they did when they merely verified the relationships. Problems of lack of self-direction on the part of students were soon overcome as the students performed a number of these activities.

In the eyes of the writer, one of the biggest stumbling blocks to the understanding of the physical world is the lack of understanding of mathematics. Galileo was quoted as saying that if he had his studies to begin over he would take the advice of Plato and start with mathematics (4). Nearly always in the physical sciences, and, to some extent in the life sciences, mathematics serves as the key for unlocking the secrets of the physical universe.

At the junior high school level, physical science is often little more than an unrewarding repetition of material that the students have most certainly encountered in their elementary science. If students are to be prepared to do the critical thinking required by the new national secondary curricula in science and mathematics, more than an unquantitative, descriptive course in physical science is necessary.

However, developing in students the capabilities necessary for encountering and successfully progressing in more advanced science is easier to talk about than it is to accomplish. Being able to define their problem, design their experiment, and then carry it through to some satisfactory end with recognition of the limits of application is not a simple task for any student.

This study involved two sections of ninth-grade students in science at the State University of Iowa Laboratory High School. The specific course is outlined in detail in the School's Curriculum Guide (8). It appeared to this writer, who served as instructor for these sections, that even at the expense of conducting a smaller number of laboratory exercises, it would be more profitable for the students to learn how to fit mathematical relationships to physical situations than merely to verify them.

Various Teaching Approaches

In teaching a true understanding of mathematical relationships describing physical phenomena, there are at least four approaches which may be used. They are (a) exposure to and blind acceptance of mathematical relationships, (b) teacher derivation of mathematical relationships, (c) student derivation of mathematical relationships, and (d) student development of mathematical relationships by experimentation. Points (c) and (d) are considered as separate approaches: experimentally developing mathematical expressions to fit physical situations is certainly different from merely deriving relationships mathematically, even when these are accompanied by laboratory verification.

Method (a) is undesirable for the obvious reason that little understanding and little or no learning takes place. Method (b) is somewhat better but still does not involve student participation. Method (c) is ideal fo rthe mathematically-inclined student, but may still fail to impart an understanding of the physical situation. Method (d) is better suited to the objectives of teaching an understanding of physical phenomena through mathematics. It was this latter method with which the writer concerned himself in this study.

The Traditional Teaching Approach

Traditionally, laboratory in the ninth-grade science has either been non-existent or of questionable value. Often if laboratory exercises are part of the ninth-grade course, they merely serve to verify, to some degree, physical relationships which have previously been stated and sometimes demonstrated by the instructor. For example, the relationship of force, radius, mass, and velocity in the centripetal force expression is often first discussed and then stated mathematically in the following manner:

$$F = \frac{m V^2}{r}$$

The students, then, given the proper equipment and directions (which quite often are not only too structured but also too specific), follow a step-by-step outline and then to some extent verify the physical relationship. It is not uncommon even after completing such a laboratory exercise for the students to reveal through class discussion or on written examinations that very little was understood.

The Teaching Approach Used in This Study

The approach employed in this study was to introduce the problem only in general terms and then allow the students to proceed to develop experimentally in mathematical expression describing the physical situation.

Among the activities conducted by the students in establishing mathematical relationships were the following: (a) centripetal force, (b) force of friction in a simple machine, (c) force of friction on a sliding block, (d) temperature conversion relationship between the standard Celsius scale and the scale of a home-made thermometer, (e) thermal expansion relationship, (f) Boyle's law relationship, and (g) Ohm's law relationship. Only the activity concerning the centripetal force relationship will be described here in detail.[1]

With specific reference to the centripetal force relationship discussed above, understanding of Newton's second law in reference to linear motion was assumed on the part of the students since this topic had been previously studied. It was then suggested that circular motion be described by some mathematical relationship and that Newton's law with some variation might still apply. Practical applications of the problem were discussed in terms of artificial satellites orbiting the earth and the orbital motions of planets and their moons.

From this point on, it was necessary for the students to analyze the problem, plan a method of procedure, design an experiment, and proceed to establish a mathematical relationship to fit the motion of an object moving in a circular path about some point.

Materials that could be used were also suggested, but in as general terms as possible. It appeared that it was most profitable for the students to solve as many of their own problems and do as much as possible in designing their own experiment.

A Sample Experimental Procedure

The following paragraphs roughly parallel the procedure followed ,the observations made, and the conclusions drawn by

[1] The writer has on file dittoed copies of all of the above mentioned activities.

some of the students performing this activity. Due to the nature of the activity, variation in procedure results: some of these variations will be mentioned.

After the students discuss with the instructor the general problem of relating the variable quantities to the force involved in the circular revolution of a body about a point, the students set out to plan a procedure for solving their problem. With a minimum of assistance from the instructor they design an experiment and decide upon the materials that are needed. The students also initially attempt to determine the variables involved before they proceed, but often these variables are not all self-evident and are recognized only after the activity has been initiated.

A typical approach is to first determine the relationship of the mass to the force. One way this may be accomplished is by successively attaching a strong rubber band, about one foot in length, to three rubber stoppers whose masses are in a ratio of 1:2:3. Based upon their observations from rotating these stoppers at a constant velocity, the conclusion drawn is that the stretch of the rubber band is directly proportional to the mass of the revolving stopper. Stated mathematically this is:

$$\text{Stretch}\ \alpha\ \text{m}$$

Since an increased stretch of the rubber band is directly related to an increased balancing force, the students may wish to state this relationship in terms of the balancing force or:

$$\text{F}\ \alpha\ \text{m}$$

Problems often arise here due to the fact that the radius of revolution is not being kept constant when using the rubber band. However, the students initially attempt only to determine roughly a relationship and it is often at this point that they become aware of the other variable quantities involved.

The next step could be to vary the velocity of revolution while holding the mass constant and thus to relate this quantity to the force. The students will need to know or discover how to measure the velocity of a revolving body. This may be accomplished quite easily by the students, by timing twenty to thirty revolutions of the stopper with a stop watch. Since the stopper revolves a distance of $2\pi r$ units per revolution and the average velocity is equal to the distance traveled per unit time the relationship is:

$$v = \frac{d}{t} = \frac{(2\pi r)\ (\#\ \text{of rev})}{t}$$

This relationship may itself be experimentally derived by the students at this time or may have been derived by them at a previous time.

If at least one pair of velocities which are in a ratio of 1:2 are used, the students can conclude that the velocity is not directly proportional to the force, since doubling the velocity results in a force which increases more than two times. The students will not know whether the force is directly related to some multiple or to some power of the velocity. Thus they will probably need to state that:

$$F \alpha xV^y$$

Here x and y are, hopefully, integers; at least one of them may be equal to unity.

At this stage alert students will have noticed that the radius of the circle described by the revolving stopper varies with both the mass and the velocity. Also it will become apparent to them that to hold the radius constant necessitates something other than a stretchable rubber band for equipment. A length of string appears as if it would serve the purpose of holding the radius constant and some students choose to tie a spring balance to one end of the string and the rubber stopper to the other. Other students choose to use the spring balance and the string throughout the experiment rather than the rubber band. A more elaborate method involves the use of a rubber stopper, a length of string, a short piece of fire-polished glass tubing, and a small sand pail. The stopper and sand pail are tied to opposite ends of the string after it has been passed through the short piece of glass tubing. The glass tubing is then held vertically by the hand and serves as the center of radius of horizontal revolution. The force may be varied by adding sand to or taking it from a small pail which hangs directly below the student's hand.

By using a fixed mass and holding the velocity constant, the students discover that the balancing force decreases when the radius of revolution increases. Being careful to maintain a constant velocity the students generally measure the balancing forces necessary when two radii in a ratio of 1:2 are used. A few students have difficulty in representing it in a mathematical expression. Nevertheless, they generally conclude that:

$$F \alpha \frac{1}{r}$$

In the process of experimentally determining the relationships among quantities, the students must determine the number of variables involved. Often the students attempt to prematurely establish their final relationship before they have considered all of the variables. For instance, some students, after establishing the relationship between the force and the velocity, will state their relationship as follows:

$$F \alpha m (xV^y)$$

However, when they discover that the radius of revolution is also a variable and they establish its relationship to the force, the expression is stated as follows:

$$F \alpha\, m\ (1/r)\ (xV^y)$$

When the students decide that all of the variable have been considered, they attempt to verify the mathematical relationship and establish it in equation form. First, however, they need to establish the values of x and y which is generally accomplished by guessing simple values for them. Rather careful measurement will lead the students to best fit the experimental results. The students thus will usually establish the following mathematical expression which best fits the physical situation, that is:

$$F = \frac{m\ V^2}{r}$$

Interpretation and Discussion

Verifying mathematical relationships is one thing, experimentally developing the relationships is quite another. The inherent difficulty in such a procedure soon becomes quite obvious. It calls for a certain degree of mathematical sophistication on the part of the student. It also demands that the students be able to recognize the variables and then be able to set up an experimental design in which they are able to hold all of these variables constant save one.

Until the students have developed some skill in experimental design and procedure, they will need a considerable amount of aid from the instructor. However, after a few exercises of this sort, it was found that most of the students became more independent in their approach and procedure, and they needed fewer "hints" to make adequate progress and complete their exercises.

This writer also found that what he thought would be a problem, *i.e.*, that the students would find the mathematical expressions stated in their texts or another reference and then merely attempt to verify them, was no problem at all. Most of the students seemed to accept the activities as a challenge, and if it did become apparent to them that it was possible to look up the relationships, they did not seem to do so. Possibly they did not recognize the relationships when they saw them or maybe they felt that this wold be a form of "cheating" and realized that most of the joy of succeeding in the activities would be lost.

For nearly all of the activities used, this writer found it administratively efficient to allow the students to work in pairs. This saved time in the sense that two minds in operation on one problem seemed to contribute more than one. The students were also able to teach each other while working together. Many of the activities required more than one person to perform, especially ones in which time was a measured quantity. Others could be conducted by one student alone, and if this was possible it was encouraged but not demanded of the students.

Those students that conducted laboratory activities in which they experimentally derived mathematical relationships displayed without exception a more thorough understanding of the relationships than did those students who did not carry out the activities. All of the students did not have the opportunity to conduct all of the activities due to a lack of time.

Slower students did not seem to benefit as greatly from the activities as did the better students. However, these same slower students, who conducted specific activities of this type, possessed a better understanding of the mathematical relationships than did the better students who had not conducted the same exercises in the same manner.

The implication of this study is that this method of laboratory approach could be successfully expanded to encompass a major portion of the laboratory work conducted by ninth-grade students in physical science. The experimental derivations of mathematical expressions to fit physical situations are of varying difficulty. Some relationships, such as the temperature conversion relationship, are quite simple and most students have little difficulty experimentally deriving them. Individual differences among students may be allowed for in this manner.

Summary

This study was undertaken in an attempt to correct frequent student failure in understanding mathematical relationships describing physical phenomena.

The study involved two sections of University of Iowa Laboratory High School ninth-graders enrolled in a course emphasizing physical science. The proposed solution to the problem was to allow the students to experimentally develop their own mathematical expressions to fit specific physical situations. This procedure replaced the laboratory approach which merely called upon the students to verify relationships which had been previously introduced in classroom discussion.

The experimental approach, utilized by the students, involved a general discussion of the problem, materials needed, and an experimental design. From this point the students proceeded to conduct a series of trials, make observations, and draw conclusions based on these trials. Their experiment involved, for the most part, holding all of the variables constant save one in a series of steps and in this way experimentally deriving a mathematical expression to fit the physical situation.

The results of the study suggested that the students understood the mathematical relationships and the physical situations which were represented better than was the case when they merely verified the relationships. Problems of lack of self-direction on the part of the students were soon overcome as the students performed a number of these activities.

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Demonstration of the Transition from the Particle Approach to the Wave Approach[1]

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Abstract. This article describes a demonstration, suitable for elementary physics, to show the transition from the particle to the wave approach. Standing waves in a continuous string are approximated by particles fastened to a much lower density or "massless" string. The agreement between the continuous and discrete systems is good, and the wave approach is shown to be only another way of describing systems.

The purpose of this paper is to present a demonstration to be used at the elementary physics level to illustrate the transition from the particle approach to the wave approach as the number of particles in a system increases. In most elementary texts, the student first uses the particle approach to study such problems as blocks on inclined planes and the simple pendulum. Then in a separate section of the text, he is introduced to the wave approach and the continuous string. It is the author's feeling that this causes the student to consider the two approaches

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