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An Investigation on In-Process SPC Sampling Frequency for Shaft Bearing Turning Process

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AN INVESTIGATION ON IN-PROCESS SPC SAMPLING FREQUENCY FOR SHAFT BEARING TURNING PROCESS

An Abstract of a Thesis
Submitted
in Partial Fulfillment
of the Requirements for the Degree
Master of Science

Jiaxu Chu
University of Northern Iowa
July 2012

ABSTRACT

The machining processes have the uniqueness of discrete production and flexible scheduling as cellular manufacturing design. To ensure the quality of machining products, control charts for Statistical Process Control (SPC) has been implemented in the company to monitor the machining process. Sampling is an essential part of data collection for SPC control charts and sampling should be done for machining processes becomes an issue in the company.

This study focused on the investigation of SPC sampling frequencies for shaft bearing turning process using the technique of Average Production Length (APL). The purpose of the study was to identify the SPC sampling frequency for turning process. Study results indicated that the 100% inspection has significant shorter APL than 1/10 and 1/20 SPC sampling frequencies, and based on the study result, 1/10 is preferred over 1/20. The result provides insight on the SPC sampling frequencies in enabling the company to select an economical SPC sampling frequency for the machining manufacturing process.

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has been approved as meeting the thesis requirement for the

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CHAPTER I

INTRODUCTION

Quality is essential for organizations to succeed in today's competitive globalized market. Manufacturing firms are using different techniques and activities as tools to assure the quality of the products or services since the beginning, and the use of these tools are called Quality Control (QC). As a matter of fact, Quality Control is not a new invention, as many sources believed that it started thousands years ago when manufacturing was instituted by ancient civilizations (Besterfield, 2009; Duncan, 1986; NIST, 2010; Summers, 2010). However, comparing to the QC, the Statistical Quality Control (SQC) is a fairly new concept. SQC was first introduced in the 1920s by Walter A. Shewhart. Shewhart introduced control chart, kept updating on topic and published a book called "*Economic Control of Quality of Manufacturing Product*" on SQC in 1931 (NIST, 2010; Summers, 2010). After Shewhart, H.F. Dodge and H.G. Romig published articles about sampling inspection, and these three pioneers founded the modern SQC system (Dodge, 1943; Dodge & Romig, 1929; NIST, 2010). SQC uses statistical tools such as control charts, pareto charts, and cause-and-effect diagrams to ensure the quality of products (Besterfield, 2009; Summers, 2010).

Data collection is the essential procedure for gathering information in the SQC activities. After data has been collected, SQC proceeds to analyze and interpret the data by using statistical methods for quality support (Summers, 2010). In the industry, one hundred percent (100%) inspection in SQC is neither practical nor economical (Montgomery, 2005; Sarkadi & Vincze, 1974). 100% inspection requires more labor and production time during manufacturing process, especially in products with complex dimensions that require longer inspection time, or products that have to use destructive testing that cause more money to be wasted. Therefore, 100% inspection is not the preferred way of doing inspection in modern manufacturing.

Statistical Process Control (SPC) is part of SQC, but it focuses on using process control to ensure the quality of products. SPC uses control charts and process performance indexes as tools for quality control. Litsikas (1996) pointed out that correctly implemented Statistical Process Control (SPC) can dramatically reduced in-process inspection rate by 30% for manufacturers (Litsikas, 1996).

Background

A Mid-West based farming-equipment drivetrain components machining organization is experiencing incorrect SPC sampling frequency issue for gear and shaft machining division. There are more than one hundred machines in the firm's machining division. The machining division consists of various types of material removal processes. Including, but not limited to, turning, hobbing, shaving, shaping and grinding. The current SPC sampling plan has only two types of defined SPC frequencies: (1) 100% inspection, and (2) sampling inspection, in which inspection is performed on every three consecutive pieces of products for every four hours. First SPC sampling frequency of 100% inspection should be performed if the process capability index (C_{pk}) of the process is less than 1.33, and second SPC sampling frequency should implemented if C_{pk} of the process is equal to or larger than 1.33. Because the current SPC sampling plan is inflexible and rigid, it cannot reflect the complexity of the current manufacturing situation that involves different types of machining processes. For example, sometimes data that collected from performing 100% inspection was not necessary for analysis result. However, there is not a clear guideline on a proper SPC sampling method provided by theory or a practical rule of thumb. Therefore, there is a need to evaluate the current SPC sampling plan due to the incapability of the current SPC sampling plan. After consulting with the Quality Assurance Group in the company, the researcher selected two other sampling frequency options among machining processes. These selected frequencies are 1/10 and 1/20. The production departments are only willing to use fixed-time based SPC sampling frequencies. Upon completion and success, the result and findings of this study will be extended to the entire shaft bearing turning process for in entire company.

Statement of the Problem

The problem of this study is to determine the sampling frequency of Statistical Process Control for shaft bearing turning process.

Statement of Purpose

The purpose of this study is to evaluate the shaft turning process and provide an SPC sampling plan suggestion for the firm. The study will help the firm to establish knowledge for determining an economic SPC sampling frequency in the manufacturing processes that will meet the quality requirement. The result of this study will be documented as a reference for future related projects.

Need and Justification

The need/justification for the study was based on the following factors:

1. The current SPC sampling plan was developed when the company started the SPC implementation as daily production quality control tools a decade ago. The current SPC inspection plan does not have any statistical literatures to support it.
2. There is lack of empirical experiment studies on machining processes for determining SPC during process sampling frequency within the company.
3. In the firm, machine operators manually perform in-process inspections on manufacturing parts among all machining processes instead of automated inspection. Therefore, 100% SPC inspection is uneconomical. A practical and economical SPC inspection frequency needs to be implemented to reduce labor cost.

Statement of Hypothesis/Research Questions

It was hypothesized that, for a statistical process control process, 100% inspection of SPC is unnecessary and other sampling frequencies using SPC inspection frequency is preferable, such as 1/10 in subgroup size of one. The proposed alternate sampling frequency options including samples in subgroup size of 1 and at the sampling frequency (sampling interval) of 1/10 and 1/20.

The statement of hypothesis: there was no significant difference in the shaft bearing turning process in terms of Average Production Level (APL) when 1/10 sampling frequency and 1/20 sampling frequency was implemented. APL is the average production length between a mean shift is occurred and until a change in the process is detected

(Keats, Miskulin, & Runger, 1995). APL is directly related to SPC sampling frequencies and it is well explained in Chapter II. Review of Literature.

Assumptions

The following assumptions are made in pursuit of this study:

1. All the machines that are involved in this study are capable of performing jobs properly.
2. Raw forging materials are uniform and capable of producing parts.
3. Operators are capable of performing the turning operations with reproducibility.
4. Operators are capable of performing the inspection and data collection with reproducibility.
5. Inspection equipment is capable of performing the inspection with repeatability.
6. Working environment is stable and consistent.

Limitations

This study will be conducted in view of following limitations:

1. One machining cell in an agricultural equipment manufacturing company in the Mid-West of United States is under investigation.
2. The study will focus on one manufacturing cellular unit that has a shaft bearing turning process.

3. There are two identical computer numerical control (CNC) machines in the manufacturing cellular unit.
4. Data collection will be made from shaft bearing turning process with two operators, first shift and third shift for a period of time equaling two years.
5. In this study, measurements will only be performed by trained workers and workers will collect data manually.

Statement of Procedure

The procedure for this study was as follows:

1. Select machining process for the study
    a. Determine the research question
2. Review the past literatures that related to the SPC sampling frequency
3. Perform data collection on the selected process.
    a. Collect the history data of nonconforming products of selected process
    b. Filter out data from 100% inspection data population into 1/10 and 1/20. Researcher drew data points as 1/10 and 1/20 frequencies from 100% inspection data.
    c. Using APL as performance measurement to compare 100% inspection, 1/10 and 1/20 SPC sampling frequencies.
4. Perform data analysis and interpretation by using following tools:
    a. Analysis of variance
    b. Average Run Length Comparison
5. Provide study results and suggestion.

Definitions of Terms

The following terms are defined to clarify their use in the context of study:

1. Statistical Quality Control (SQC). SQC is the use of statistical tools and methods to produce quality products for customers (Derman & Ross, 1997).
2. Statistical Process Control (SPC). SPC is using a set of statistical tools to control processes (Griffith, 1996). SPC is “the application of statistical methods to the measurement and analysis of variation in any process” (Juran & Gryna, 1993, p. 377).
3. Under statistical process control. A process stays within statistical control limits after eliminate special causes (Griffith, 1996, p. 2).
4. Inspection. “A process of measuring, examining, testing, gaging or otherwise comparing the unit with the applicable requirements” (Griffith, 1996, p. 227). It “includes measurement of an output and comparison to specified requirements to determine conformity (Juran & Gryna, 1993).
5. Shaft bearing. A surface on the shaft that supports other parts (bearing, 2011).
6. Turning process. A form of machining removes material by rotating the part while feeding the part to the fixed cutting tool (Black & Kohser, 2008, p. 17).

CHAPTER II

REVIEW OF LITERATURE

Introduction

Previous literature on related issues has been reviewed. The review of literature has been categorized into several groups: (1) sampling inspection in Statistical Process Control (SPC), (2) continuous sampling inspection (3) average production length and (4) rectification for variables inspection in SPC.

Sampling Inspection in Statistical Quality Control

In SPC, sampling inspection is the method of collecting samples of data from a population, rather than 100 percent of population, and using it for data analysis, such as control charts (Guenther, 1977; Juran & Gryna, 1993). In 1920s, Dodge and Romig (1929) introduced the sampling inspection of quality control. They provided a theory that, based on statistics, performs inspections over samples that are selected from lots. Many sources indicated that the sampling inspection method from Dodge and Romig (1929) was the first to introduce statistical sampling inspection method in the quality history (Balamurali & Jun, 2006; Guenther, 1977; NIST, 2010).

Dodge and Romig (1929) noted that the purpose of the sampling inspection is to decide whether to accept accept the lot or reject it, not to measure the quality of a lot. There is an allowable percentage defective in the sampling inspection. Both Single Sampling inspection plan and the Double Sampling inspection plan, have a chance to accept bad lots and pass them to customers. This is normally called Type II error in statistic quality control. Type II error needs to be taking into consideration when organizations perform those sampling inspection methods.

Furthermore, Dodge and Romig's method has another limitation of lot based sampling, which does not correspond to all manufacturing processes, such as machining process that are discretely part-to-part based manufacturing process. This method considers only a batch of products after they are produced, but gear and shaft machining processes need in-process sampling inspection to ensure the process is under statistical control. Sampling inspection based on lots or batches does not provide statistical process control (SPC) on continuous production that consists of individual units. When this method was designed, it had no attention to provide a guideline on sampling frequency during a manufacturing process.

Continuous Sampling Inspection

Later on, Dodge (1943) published a sampling inspection method for continuous production on a Go-NoGo basis. Continuous production consists of individual units, such as parts in gear manufacturing. Dodge's inspection method also had been known as Continuous Sampling Plan (CSP-1).

The sampling inspection plan consists of two stages, (1) 100% inspection stage, after i consecutive units are free of nonconforming units, the next stage, (2) inspection will only be performed on a fraction f of units, as shown in Figure 1. When number of i consecutive non-defects parts have been reached, then the process can go to the next stage, which is sampling by fraction f, such as 1/10. In CSP-1, i and f can be determined by Average Outgoing Quality Level (AOQL).

Cellular manufacturing has been used in the company for years, and it involves production of parts families and limit quantity per production run from a single manufacturing cell unit. The limitation of CSP-1 is lack of flexibility, which is a very important criterion when a SPC study for cellular manufacturing of machining processes is planned and designed (Irani, 1999; Singh & Rajamani, 1996).

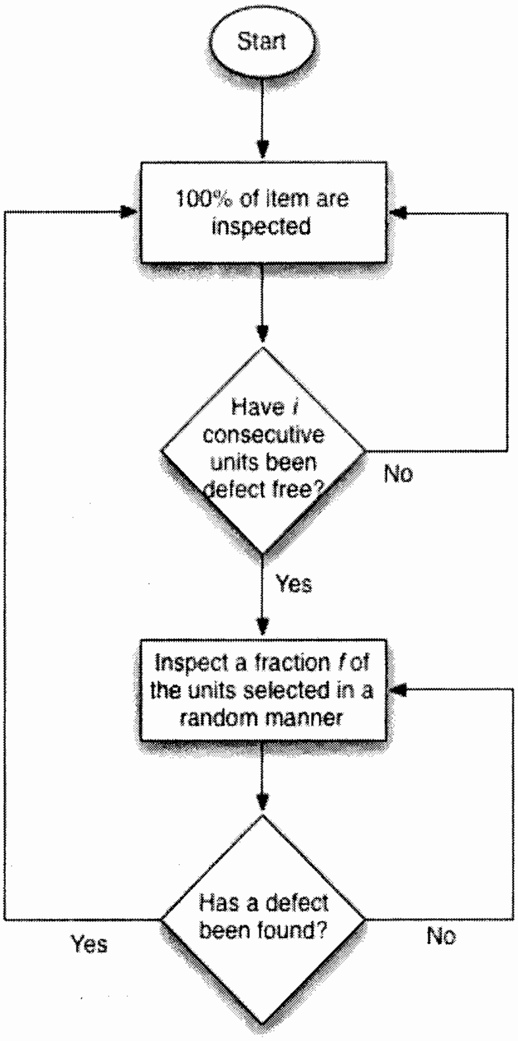

Figure 1. Procedure for CSP-1 Plan

Moreover, CSP-1 only considers nonconforming parts and modern SPC uses control limits in the control charts to perform process control. The difference between CSP-1 plan and SPC control charts is the same criteria for quality control.

Average Production Length (APL)

Traditionally, process control design methods suggest only a subgroup size of the sample and control chart limits width as the key elements of control chart design. Besides that, sampling size and sampling frequency are also mentioned in the traditional design, and Average Run Length (ARL) of the control chart is used to evaluate the sample size and sampling frequency. ARL provides the average number of points that must be plotted before a point indicates the process is out-of-control (Montgomery, 2005, p. 160).

$$\text{ARL} = \frac{1}{p}$$

In the formula of ARL, p is the probability that any point exceeds the control limits. ARL is also used as the evaluation of the performance of control charts (Keats et al., 1995). However, inspection frequency for control chart is not as important as subgroup size of samples and control chart limits (Dodge & Romig, 1929; Shewhart, 1931). ARL is not related to the sampling frequency in a SPC control charts design. Research that provides a statistical way of determine the inspection frequency as a quantified number other than percentage of the whole population (Dodge & Romig, 1929; Shewhart, 1931).

Keats et al. (1995) suggested three major things researchers should particularly take into consideration when the topic involves control chart design of SPC. They include SPC sampling frequency, subgroup size of samples and Average Production Length

(APL). Different from other researchers, Keats et al. (1995) promoted the idea of APL rather than ARL. APL is the average production length between a mean shift occurs and until a shift of the process is detected. Unlike APL, ARL is related with SPC sampling frequency. APL uses sampling frequency as one important variable for calculating the result and evaluating the process performance on targeting. Therefore, APL is a better choice determining the sampling frequency and sample subgroup size in SPC.

Whereas,

1. n = number of units in a sample subgroup.
2. S = the number of subgroup sampled between occurrence of a shift and a signal of the shift.
3. $E(S)$ = the expected number made before a signal occurs. It also equals to Average Run Length (ARL).
4. Z = the number of items produced between a shift and the first subgroup sampled after the shift.
5. $E(Z)$ = the expected number of units produced between a process shift and the next sample.
6. L = the production run length or total number of items produced between a shift and a signal.
7. h = the number of items produced between subgroups
8. r = sample rate, a ratio of sampled units to total units that were produced during that period as shown in Figure 3.
9. p = the probability that a change in the process mean is signaled.

10. Φ = the standard normal cumulative distribution function.

11. k = the control limit width parameter (upper control limit (UCL) = $\mu_0 + k\frac{\sigma}{\sqrt{n}}$ and lower control limit (LCL) = $\mu_0 - k\frac{\sigma}{\sqrt{n}}$).

12. d = the deviation or shift of the process mean from the target value of mean in multiples of the process standard deviation, $d = \frac{|\mu-\mu_0|}{\sigma}$.

From the Figure 2, the relation between L, Z, S, n, h, and i are showed clearly for SPC sampling on a manufacturing process. Then, the equation can be described as this:

$$L = Z + hS - h + nS$$

The relation of n and h can be showed as following, n is sample size and h is sampling interval:

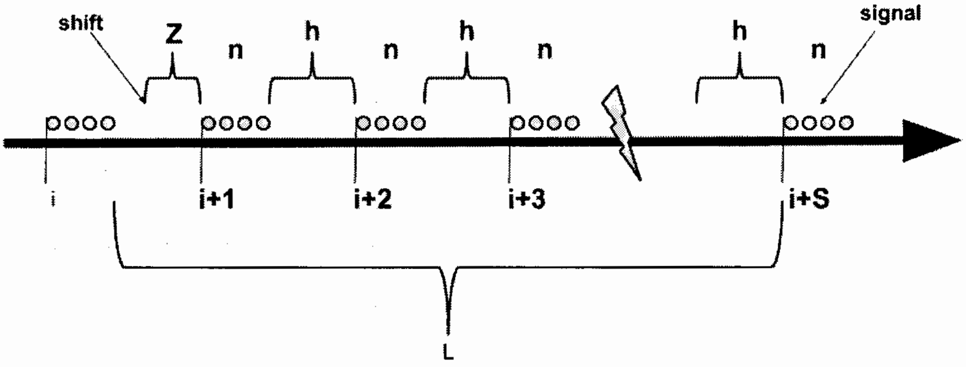

Figure 2. Process Shift, Signal, and Sampling Over Time

Sampling rate r can be calculated by the following equation:

$$r = \frac{n}{n+h}$$

Hence, APL can be described by the following equation:

$$\text{APL} = E(Z) + E(S)\frac{n}{r} - \frac{n}{r} + n$$

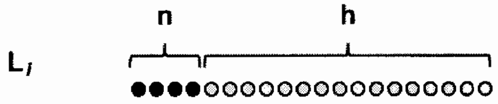

Figure 3. Sampling Rate with Sample Size

In the equation of APL, *E*(*Z*) represents the expected number of units produced between a process shift occurrence and the next sample unit. Rynolds, Amin, Arnold and Nachlas (1988) simulated *E*(*Z*) based on a model created by Duncan (1956). Their work showed a very robust way to calculate *E*(*Z*) as showed below.

$$E(Z) = \frac{h+n}{2} = \frac{\left[\left(\frac{n}{r}\right) - n + n\right]}{2} = \frac{n}{2r}$$

For an $\bar{X}$ control chart system, Keats et al. (1995) found out that *E*(*S*) is the same with ARL, and it can be transformed as the standard normal cumulative distribution function as showed below,

$$E(S) = ARL = \frac{1}{p} = \frac{1}{1 - \Phi(k - d\sqrt{n}) + \Phi(-k - d\sqrt{n})}$$

Ultimately, the notation of APL can be substituted by *E*(*Z*) and *E*(*S*) with $d \geq 0$, where APL can also can be introduced as APL$_d$.

$$APL_d = \frac{n}{2r} + \frac{n}{r}\left(\frac{1}{p}\right) - \frac{n}{r} + n = \frac{n}{r\left[1 - \Phi(k - d\sqrt{n}) + \Phi(-k - d\sqrt{n})\right]} - \frac{n}{2r} + n$$

Therefore, APL_d should be used as the measuring metrics for determining the effectiveness of selected sampling frequencies in this study. The smaller the APL_d is, the better sampling frequencies for the process (Keats et al., 1995).

Summary of Literatures Recommendations

This study focused on evaluating the effectiveness of an SPC sampling frequency. Unlike the ARL, which is not related to sampling frequency at all, APL considers sampling frequency as an important characteristic of SPC sampling plan. Therefore, using APL to evaluate the effectiveness of different SPC sampling frequencies would satisfy the needs of this study.

CHAPTER III

METHODOLOGY

In this chapter, the methodology of this study is explained. The chapter is structured under three categories: (1) development of statistical process control (SPC) sampling plans (2) data collection and sample selection, and (3) data analysis.

Development of SPC Sampling Plan

This study focuses on only one process, which is the shaft bearing turning process. The manufacturing cellular setup of the process has two identical lathes side by side. Figure 4 shows the physical setup of the working environment located at the organization's manufacturing plant.

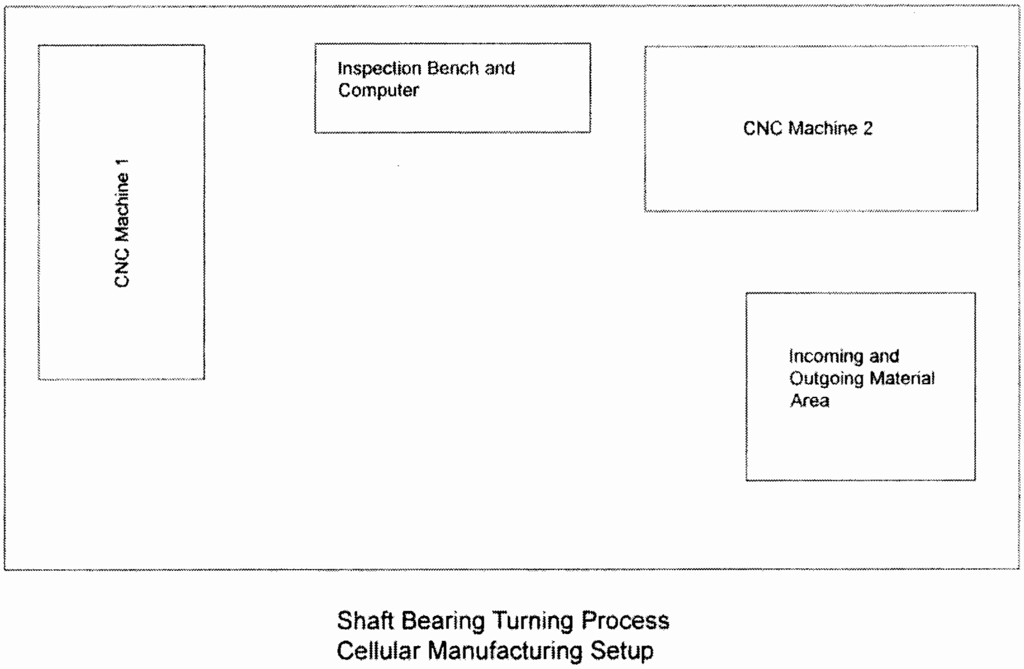

Figure 4. Cellular Manufacturing Setup of the Shaft Bearing Turning Process

Machines were operated by two operators, one operator was the 1st shift operator and the other operator was the 3rd shift operator. Table 1 shows the structure of the shaft bearing turning process. For this study, two parts have been selected for data collection, Part A and Part B as shown in Figure 5 and 6.

Table 1

Manufacturing Cell Structure of the Shaft Bearing Turning Process

		Shaft Bearing Turning Process			
		Machine 1		Machine 2	
		Part A	Part B	Part A	Part B
Operator	1st shift	Head Bearing	Head Bearing	Head Bearing	Head Bearing
		Tail Bearing	Tail Bearing	Tail Bearing	Tail Bearing
	3rd shift	Head Bearing	Head Bearing	Head Bearing	Head Bearing
		Tail Bearing	Tail Bearing	Tail Bearing	Tail Bearing

The criterion for parts selection was based on the yearly production of the parts, and the critical level. Production quantity for Part A and Part B are both over 1000 units per year per machine per shift. Both parts are differential drive shafts in farming equipment, therefore, they are critical components in the final product to customers. Each part has two similar features that turned in the same process by the same machine, (1) head bearing and (2) tailing bearing. Figure 5 and 6 show the three-dimensional (3D) draft models of part A and B. The diameter of the bearing is the studied feature of this study. Trained operators collected the bearing diameter data by using snap gages with wireless transmitters. The transmitters send inspection reading of outside diameter size to a computer with the specialized SPC data collection program software, and the computer receives and stores the information to the company's SPC database.

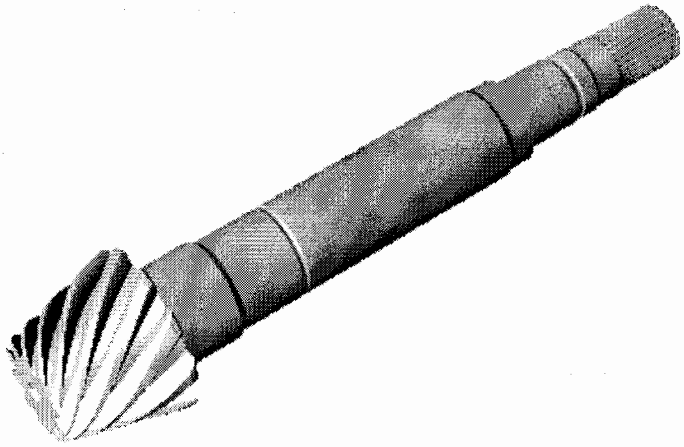

Figure 5. 3D Model of Part A

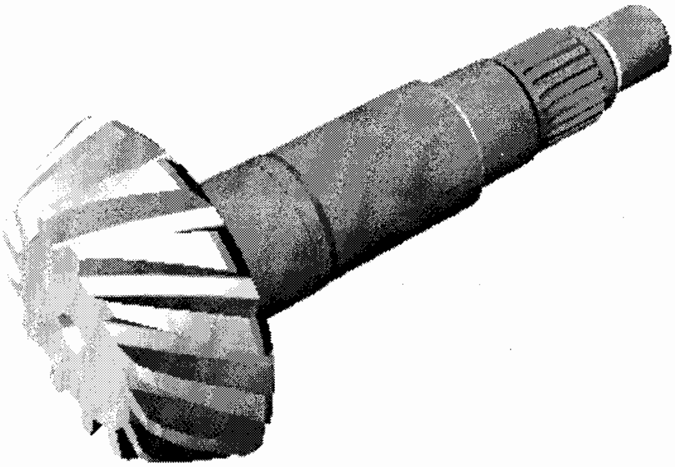

Figure 6. 3D Model of Part B

The experiment focused on the comparison between 1/10 sampling frequency and 1/20 sampling frequency for SPC inspection. When sampling was performed, only one subgroup size was used for this study, with the subgroup size of one. The reason for choosing the subgroup size as one is because current subgroup size is implemented for processes that have the process capability index (C_{pk}) of 1.33 or above. The current SPC scheme (sampling frequencies and subgroup size) was determined by company's manufacturing engineers and quality engineers based on the feasibility of implementation of daily manufacturing production.

Data Collection and Sample Selection

In the past two years, 100% inspection data of head bearing diameter and tail bearing diameter of the shaft bearing turning process was performed. This study considered the data collected over the past two years as population data. The process of shaft bearing turning involves 27 parts that are from one part family. All 27 parts require similar tooling, machining, operations and fixtures during the manufacturing process. Also, the shaft shares the same mechanical functionality for the final assembled products of farming equipment drive trains. These two parts contributed for approximately 35% of entire process production population over the two years period, as part A contributed for 20.70% and part B contributes for 13.67% of overall population. Once the parts were selected, the experiment data for the study were extracted from already-existing population database. For samples comparing 1/10 and 1/20 SPC frequencies with 100% inspection, researcher drew every tenth and every twentieth subgroup as samples to include in the data analysis.

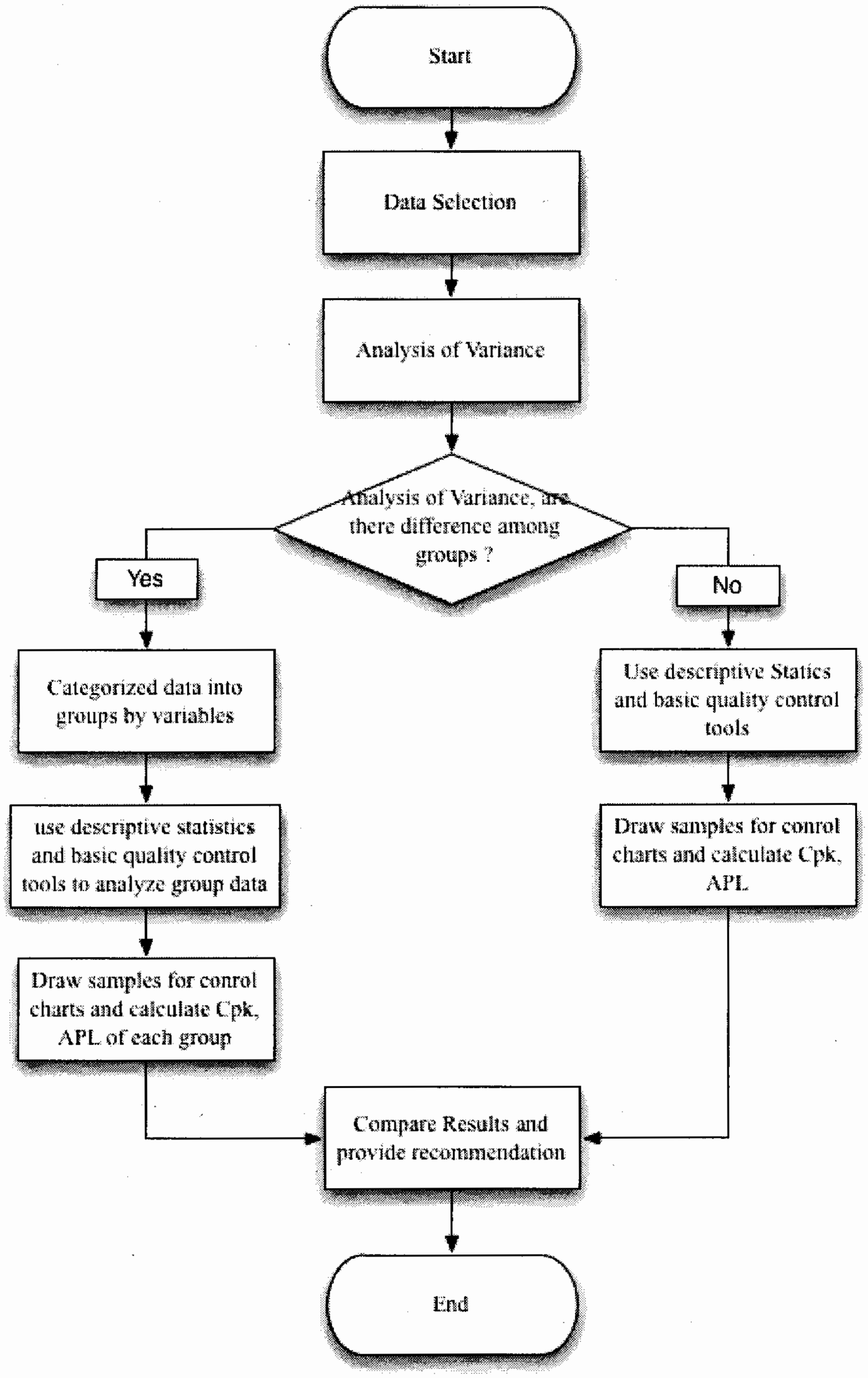

Figure 7. Flow Chart of Procedures for Analysis of Data

Data Analysis

Data analysis for the study followed the finish of the data collection phrase, where statistical tools applied to the collected data. There are four independent variables in this study. The independent variables have been used as factors for Analysis of Variance (ANOVA) to study the homogeneity of data. To answer the research question, a comparison of APL_d has been performed to discover which sampling frequency is better for the shaft bearing turning process. Figure 7 shows the procedures of data analysis in a flow chart. Results will be provided in Chapter IV of this study.

Statistical software package STATISTICA 10 and IBM SPSS Statistics 20 have been used as the tools for statistical analysis, and also Microsoft Excel has been utilized for raw data filtering and formatting. STATISTICA was developed by the company StatSoft, Inc. It provides functions of data analysis, quality control packages, and data visualization. IBM SPSS Statistics 20 owned by IBM, a well-known statistical tool for researchers in different fields. Use of STATISTICA and IBM SPSS Statistics 20 helped statistical data interpretation and visualization of this study significantly.

CHAPTER IV

ANALYSIS OF THE DATA

This study was performed to determine the Statistical Process Control (SPC) sampling frequency of the hard turning process based on two years of population data. Population data represented the shaft bearings' outside diameter sizes (OD sizes) of two similar parts collected by two operators from two identical Computer Numerical Control (CNC) machines.

As the previous chapters mentioned, the collected data of shaft bearing out side diameter sizes was categorized into four independent variables: (1) operator, (2) machine, (3) part and (4) bearing locations.

Analysis of Variance (ANOVA)

A four-way ANOVA design was conducted to evaluate the interactions of four independent variables on the dependent variable. Before the ANOVA test, a normal probability plot was performed to examine the normality of collected data. Figure 8 shows the data is normally distributed. Table 2 showes the result of the four-way ANOVA with alpha level $\alpha = 0.05$. The result of p-value for effect of Operator*Machine*Part*Location is equal to 0.00, which is less than 0.05, showing the significant difference of variance with effect of all variables.

The collected data is based on offsets of target on the outside diameter specification value. Only the actual offset from the target value is recorded. In order to evaluate the group means without ignoring the data dispersion, an additional four-way

ANOVA test has been performed with all collected data transformed to their absolute values. Table 3 showed the four-way ANOVA result with absolute value of OD sizes.

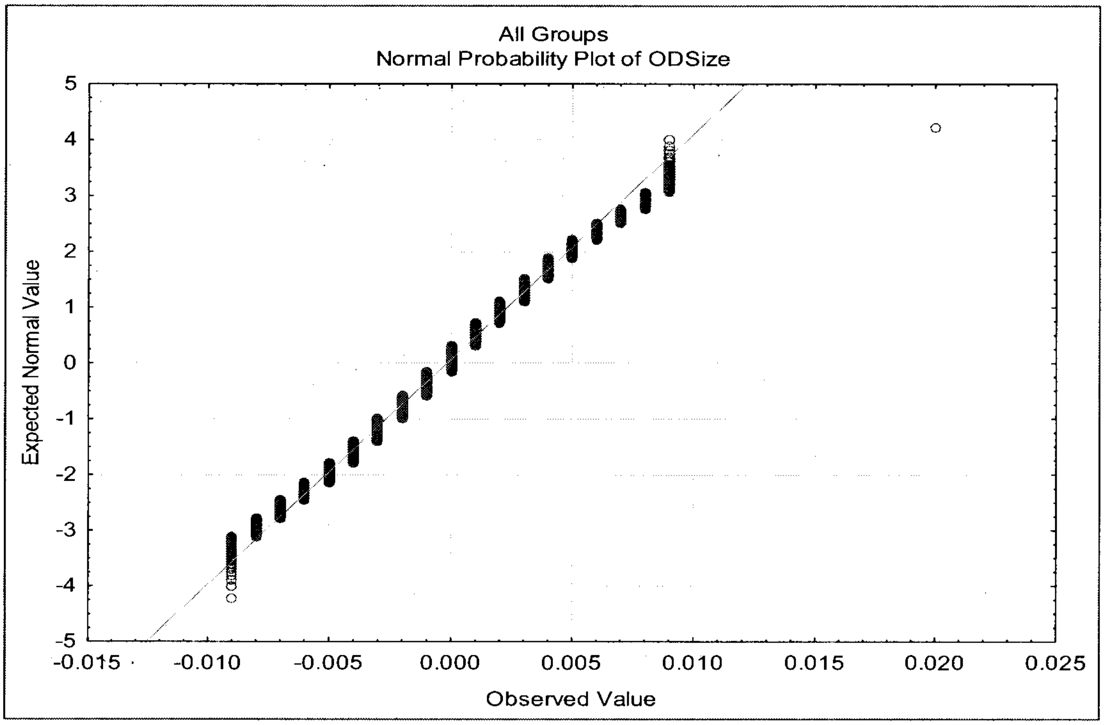

Figure 8. Normal Probability Plot of All Data

The interaction between all four variables: Operator*Machine*Part*Bearing Location had a p-value = 0.48, which indicate no significant difference of means. However, for all the three-interaction p-values, there was only one interaction that has a p-value greater than 0.05. This indicated that a one-way ANOVA should be performed with a group setting of all four variable interactions taken into consideration. Even this interaction resulted in a p-value greater than 0.05.

The four-way ANOVA with absolute value result confirmed there were differences among each variable and we need to study each variable individually. It also meant the data cannot be treated as one group for further tests in this paper. Therefore, the data was categorized into 16 groups and another one-way ANOVA was performed to test the variance among all groups. The categorization of these 16 groups showed in Table 4.

Table 2

4-Way ANOVA of Factorial design of Variables: Operator, Machine, Parts and Bearing Location

	SS	Degr. of	MS	F	p
Intercept	0.000146	1	0.000146	25.300	0.000000
(1)Operator	0.000232	1	0.000232	40.185	0.000000
(2)Machine	0.000213	1	0.000213	36.832	0.000000
(3)Part	0.001171	1	0.001171	202.937	0.000000
(4)Location	0.006772	1	0.006772	1173.491	0.000000
Operator*Machine	0.000082	1	0.000082	14.269	0.000159
Operator*Part	0.000012	1	0.000012	2.091	0.148146
Machine*Part	0.000574	1	0.000574	99.408	0.000000
Operator*Location	0.000730	1	0.000730	126.438	0.000000
Machine*Location	0.001388	1	0.001388	240.549	0.000000
Part*Location	0.000063	1	0.000063	10.839	0.000994
Operator*Machine*Part	0.000289	1	0.000289	50.113	0.000000
Operator*Machine*Location	0.000122	1	0.000122	21.124	0.000004
Operator*Part*Location	0.000076	1	0.000076	13.127	0.000291
Machine*Part*Location	0.000025	1	0.000025	4.289	0.038365
1*2*3*4	0.000389	1	0.000389	67.324	0.000000
Error	0.308632	53478	0.000006		

Table 3

4-Way ANOVA of Factorial Design of Variables with Absolute Values

	SS	Degr. of	MS	F	p
Intercept	0.124333	1	0.124333	49922.49	0.000000
(1)Operator	0.000030	1	0.000030	11.86	0.000575
(2)Machine	0.000280	1	0.000280	112.57	0.000000
(3)Part	0.000089	1	0.000089	35.64	0.000000
(4)Location	0.000125	1	0.000125	50.08	0.000000
Operator*Machine	0.000054	1	0.000054	21.71	0.000003
Operator*Part	0.000000	1	0.000000	0.04	0.833876
Machine*Part	0.000009	1	0.000009	3.78	0.051957
Operator*Location	0.000010	1	0.000010	4.06	0.044021
Machine*Location	0.000032	1	0.000032	12.99	0.000313
Part*Location	0.000006	1	0.000006	2.30	0.129515
Operator*Machine*Part	0.000016	1	0.000016	6.28	0.012244
Operator*Machine*Location	0.000051	1	0.000051	20.54	0.000006
Operator*Part*Location	0.000000	1	0.000000	0.00	1.000000
Machine*Part*Location	0.000070	1	0.000070	28.04	0.000000
1*2*3*4	0.000001	1	0.000001	0.48	0.489092
Error	0.133188	53478	0.000002		

Table 4

Data Groups by Combination of Variables: Operator, Machine, parts and Bearing Location

Group	Operator	Machine	Part	Bearing Location
1	1st shift	Machine 1	A	Head Bearing
2	1st shift	Machine 1	A	Tail Bearing
3	1st shift	Machine 1	B	Head Bearing
4	1st shift	Machine 1	B	Tail Bearing
5	1st shift	Machine 2	A	Head Bearing
6	1st shift	Machine 2	A	Tail Bearing
7	1st shift	Machine 2	B	Head Bearing
8	1st shift	Machine 2	B	Tail Bearing
9	3rd shift	Machine 1	A	Head Bearing
10	3rd shift	Machine 1	A	Tail Bearing
11	3rd shift	Machine 1	B	Head Bearing
12	3rd shift	Machine 1	B	Tail Bearing
13	3rd shift	Machine 2	A	Head Bearing
14	3rd shift	Machine 2	A	Tail Bearing
15	3rd shift	Machine 2	B	Head Bearing
16	3rd shift	Machine 2	B	Tail Bearing

One-Way ANOVA was performed on all groups first and also on each part groups separately. One-way ANOVA result in Tables 5, and 6 clearly indicate that there was evidence of significant difference among the means of all 16 groups. In other words, these 16 groups needed to be studied individually.

Table 5

One-Way ANOVA for OD Size Data in Groups

	SS	Degree of	MS	F	p
Intercept	0.000146	1	0.000146	25.3002	0.000000
Group	0.013922	15	0.000928	160.8218	0.000000
Error	0.308632	53478	0.000006		

Table 6

One-Way ANOVA for Absolute Value of OD Size Data in Groups

	SS	Degree of	MS	F	p
Intercept	0.124333	1	0.124333	49922.49	0.000000
Group	0.001289	15	0.000086	34.50	0.000000
Error	0.133188	53478	0.000002		

Multiple Comparison

One-way ANOVA only showed means are different between groups, but it did not show the exact comparison results between each pair of group means. Therefore, multiple comparison tests were used to identify the group mean differences in detail. Table 7 showed the Bonferroni test results from original data values of OD sizes data and Table 8 showed the Bonferroni test results from absolute value of OD sizes data. Results in Table 8 were based on absolute value of the original data to show the true variation of observed data. The actual results of the Bonferroni test could be found at Table A1 and A2 from Appendix A.

The concluded results showed in Table 9 and 10 correspond to Table 7 and 8. Table 9 showed Group 16 is different from the rest of groups, and Table 11 showed Group 16 has the largest mean among all groups with original data values. In Table 10, it showed Group 7, 5 and 3 are statistically different from most of the groups. Table 11 indicated that Group 7 and 5 had the smallest mean among all groups with calculation from absolute value of original date and Group 3 had the largest mean of all.

Table 7

Bonferroni Test Results for Original OD Sizes

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1																
2	X															
3	X	X														
4																
5																
6				X												
7																
8																
9	X	X	X				X									
10				X		X		X								
11	X	X	X				X		X							
12				X		X				X						
13																
14				X		X		X		X		X				
15	X	X	X		X				X		X					
16	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	

Note: X *represents the value that is less than 0.05*

Table 8

Bonferrroni Test Results for Absolute Values of OD Sizes

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1																
2																
3	X	X														
4			X													
5	X	X	X	X												
6	X	X	X	X												
7	X	X	X	X		X										
8			X		X	X	X									
9			X		X		X									
10					X	X	X									
11			X		X	X	X									
12					X	X	X									
13	X	X	X	X			X	X		X		X				
14		X	X	X	X		X	X		X		X				
15			X		X		X			X						
16					X	X	X		X				X	X	X	

Note: X *represents the value that is less than 0.05*

Table 9

Summary of Boneferroni Tests Results for Original Data

Group	Statistically Difference from Groups	Number of Groups
1	2,3,9,11,15,16	6
2	1,3,9,11,15,16	6
3	1,2,9,11,15,16	5
4	6,10,12,14,16	5
5	15,16	2
6	4,10,12,14,16	6
7	9.11.16	3
8	10,14,16	3
9	1,2,3,7,11,15,16	7
10	4,6,8,12,14,16	6
11	1,2,3,7,9,15,16	7
12	4,6,10,14,16	5
13	16	1
14	4,6,8,10,12,16	6
15	1,2,3,5,9,11,16	7
16	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15	15

Table 10

Summary of Bonferroni Test Results for Absolute Values of Original Data

Group	Statistically Difference from Groups	Number of Groups
1	3,5,6,7,13	5
2	3,5,6,7,13,14	6
3	1,2,4,5,6,7,8,9,11,13,14,15	12
4	3,5,6,7,13,14	6
5	1,2,3,4,8,9,10,11,12,14,15,16	12
6	1,2,3,4,7,8,10,11,12,16	10
7	1,2,3,4,6,8,9,10,11,12,13,14,15,16	14
8	3,4,6,7,13,14	6
9	3,5,7,16	4
10	5,6,7,13,14,15	6
11	3,5,6,7	4
12	5,6,7,13,14	5
13	1,2,3,4,7,8,10,12,16	9
14	2,3,4,5,7,8,10,12,16	9
15	3,5,7,10,16	5
16	5,6,7,9,13,14,15	7

Table 11

Group Means Table

Group	Mean	Mean of Absolute
1	-0.000375	0.001938
2	-0.000507	0.001962
3	-0.000489	0.002122
4	0.000271	0.001993
5	-0.000744	0.001650
6	0.000223	0.001748
7	-0.000175	0.001612
8	0.000617	0.001964
9	-0.000371	0.001857
10	0.000455	0.002079
11	-0.000313	0.001938
12	0.000178	0.002057
13	-0.001116	0.001774
14	0.000435	0.001808
15	-0.000492	0.001850
16	0.001359	0.002094

Average Production Length Calculation

The next step of data analysis was to calculate Average Production Length (APL_d) for each sampling frequency: 1/10, 1/20 and 100% inspection. As mentioned in previous chapters, 1/10 and 1/20 are two sampling frequencies that engineers are testing to replace the 100% inspection in SPC activities. The detailed description of APL_d equation could be found in Chapter II. APL_d was calculated for all 16 groups. In order to clarify the manufacturing process related study, parts are usually considered differently for analysis, therefore the researcher split up the APL_d results into two parts: Part A and Part B. All 16 groups were separated into two sets of 8 groups. Part A contains Group 1, 2, 5, 6, 9, 10, 13 and 14; Part B contains Group 3, 4, 7, 8, 11, 12, 15 and 16.

The sample mean (μ_0) of each group has been calculated from collected data, and the population mean (μ) is calculated from 100% inspection data of each group in a two-year period. From Table 6 and 7, APL_d results for SPC sampling frequency of 1/10 and 1/20 are all smaller than APL_d result for 100% inspection. It's clearly identified that 100% inspection for SPC activities could identify mean shift with a few parts production. However, the practical design of SPC sampling is aiming to eliminate 100% inspection for daily production (Guenther, 1977; Litsikas, 1996). And also, the APL_d is designed for mean shift, not for out of statistical control detection, therefore it supposed to be used as a reference to select SPC sampling frequencies to meet the production need (Keats et al., 1995). If one only looks at SPC sampling frequencies of 1/10 and 1/20, 1/10 can identify mean shift in a shorter production length than 1/20. From Table 12, 13, 14 and 15, a 1/10 SPC frequency need to perform inspection three times after the mean shift occurs and it will have a production run of about 30 units; a 1/20 SPC frequency needs to have a production run of about 60 units. Despite the accuracy of 100% inspection, SPC frequency of 1/10 is the better one to choose for the shaft bearing turning process, because it reduced 90% of the inspection time. However, Table 16 showed that 1/10 sampling frequency only can detect 10% of all non-conforming parts.

Table 12

APL_d Values of 100% Inspection and Sampling Frequency 1/10 and 1/20 for Part A Groups

Sampling Freq	Group	N	μ_0	σ	n	r	k	d	APL_d
100%	1	6739	-0.000375	0.002521	1	1.0	1	0	3.65
100%	2	6739	-0.000507	0.002512	1	1.0	1	0	3.65
100%	5	5998	-0.000744	0.002003	1	1.0	1	0	3.65
100%	6	5998	0.000223	0.002279	1	1.0	1	0	3.65
100%	9	1133	-0.000371	0.002396	1	1.0	1	0	3.65
100%	10	1133	0.000455	0.002682	1	1.0	1	0	3.65
100%	13	2032	-0.001116	0.002011	1	1.0	1	0	3.65
100%	14	2032	0.000435	0.002302	1	1.0	1	0	3.65
1/10	1	674	-0.000312	0.002579	1	0.1	1	0.024585784	27.50
1/10	2	674	-0.000545	0.002366	1	0.1	1	0.015833962	27.51
1/10	5	600	-0.000730	0.002060	1	0.1	1	0.00683515	27.51
1/10	6	600	0.000207	0.002317	1	0.1	1	0.006938984	27.51
1/10	9	114	-0.000640	0.002410	1	0.1	1	0.111894659	27.22
1/10	10	114	0.000421	0.002899	1	0.1	1	0.011856971	27.51
1/10	13	204	-0.001132	0.002091	1	0.1	1	0.007988864	27.51
1/10	14	204	0.000515	0.002346	1	0.1	1	0.033962982	27.49
1/20	1	337	-0.000365	0.002483	1	0.05	1	0.004025338	54.03
1/20	2	337	-0.000469	0.002459	1	0.05	1	0.015536216	54.02
1/20	5	300	-0.000623	0.002109	1	0.05	1	0.057242492	53.87
1/20	6	300	0.000173	0.002210	1	0.05	1	0.022351468	54.01
1/20	9	57	-0.000930	0.002103	1	0.05	1	0.26584273	50.84
1/20	10	57	0.001158	0.002651	1	0.05	1	0.264974332	50.86
1/20	13	102	-0.001098	0.002160	1	0.05	1	0.008154741	54.03
1/20	14	102	0.000559	0.002464	1	0.05	1	0.050239283	53.91

Table 13

APL_d of 100% Inspection and Sampling Frequency 1/10 and 1.20 for Part A Groups

Sampling Freq	Group	N	μ_0	σ	n	r	k	d	APL_d
100%	3	4615	-0.000489	0.002727	1	1.0	1	0	3.65
100%	4	4615	0.000271	0.002597	1	1.0	1	0	3.65
100%	7	3644	-0.000175	0.002094	1	1.0	1	0	3.65
100%	8	3644	0.000617	0.002525	1	1.0	1	0	3.65
100%	11	1312	-0.000313	0.002498	1	1.0	1	0	3.65
100%	12	1312	0.000178	0.002619	1	1.0	1	0	3.65
100%	15	1274	-0.000492	0.002270	1	1.0	1	0	3.65
100%	16	1274	0.001359	0.002200	1	1.0	1	0	3.65
1/10	3	462	-0.000578	0.002674	1	0.1	1	0.03331178	27.49
1/10	4	462	0.000297	0.002579	1	0.1	1	0.010041463	27.51
1/10	7	365	-0.000145	0.002097	1	0.1	1	0.014244544	27.51
1/10	8	365	0.000600	0.002877	1	0.1	1	0.00606578	27.51
1/10	11	132	-0.000045	0.002710	1	0.1	1	0.098839983	27.28
1/10	12	132	0.000280	0.002645	1	0.1	1	0.038829183	27.48
1/10	15	128	-0.000484	0.002368	1	0.1	1	0.003284061	27.51
1/10	16	128	0.001328	0.002314	1	0.1	1	0.013555911	27.51
1/20	3	231	-0.000502	0.002747	1	0.05	1	0.004850064	54.03
1/20	4	231	0.000238	0.002579	1	0.05	1	0.012618345	54.02
1/20	7	183	-0.000180	0.002200	1	0.05	1	0.002384108	54.03
1/20	8	183	0.000459	0.002649	1	0.05	1	0.059802795	53.86
1/20	11	66	-0.000152	0.002702	1	0.05	1	0.059866222	53.86
1/20	12	66	0.000470	0.002413	1	0.05	1	0.121059326	53.33
1/20	15	64	-0.000375	0.002360	1	0.05	1	0.049631983	53.91
1/20	16	64	0.000969	0.002377	1	0.05	1	0.164391656	52.76

Table 14

APL_d of 100% Inspection and Sampling Frequency 1/10 and 1/20 for Part A Groups Absolute Value

Sampling Freq	Group	N	μ0	σ	n	r	k	d	APL_d
100%	1	6739	0.001938	0.001655	1	1.0	1	0	3.65
100%	2	6739	0.001962	0.001648	1	1.0	1	0	3.65
100%	5	5998	0.001650	0.001356	1	1.0	1	0	3.65
100%	6	5998	0.001748	0.001479	1	1.0	1	0	3.65
100%	9	1133	0.001857	0.001557	1	1.0	1	0	3.65
100%	10	1133	0.002079	0.001753	1	1.0	1	0	3.65
100%	13	2032	0.001774	0.001462	1	1.0	1	0	3.65
100%	14	2032	0.001808	0.001489	1	1.0	1	0	3.65
1/10	1	674	0.001947	0.001719	1	0.1	1	0.005271	27.51
1/10	2	674	0.001880	0.001535	1	0.1	1	0.053649	27.45
1/10	5	600	0.001653	0.001428	1	0.1	1	0.002065	27.51
1/10	6	600	0.001757	0.001522	1	0.1	1	0.005529	27.51
1/10	9	114	0.001921	0.001581	1	0.1	1	0.040515	27.48
1/10	10	114	0.002298	0.001804	1	0.1	1	0.121274	27.17
1/10	13	204	0.001819	0.001529	1	0.1	1	0.029122	27.49
1/10	14	204	0.001838	0.001540	1	0.1	1	0.019583	27.51
1/20	1	337	0.001861	0.001682	1	0.05	1	0.045778	53.93
1/20	2	337	0.001953	0.001563	1	0.05	1	0.006165	54.03
1/20	5	300	0.001637	0.001467	1	0.05	1	0.009351	54.03
1/20	6	300	0.001693	0.001428	1	0.05	1	0.038454	53.96
1/20	9	57	0.001772	0.001452	1	0.05	1	0.058607	53.87
1/20	10	57	0.002175	0.001891	1	0.05	1	0.050769	53.91
1/20	13	102	0.001882	0.001518	1	0.05	1	0.071326	53.79
1/20	14	102	0.001931	0.001618	1	0.05	1	0.07619	53.75

Table 15

APL_d of 100% Inspection and Sampling Frequency 1/10 and 1/20 for Part B Groups Absolute Value

Sampling Freq	Group	N	μ0	σ	n	r	k	d	APL_d
100%	3	4615	0.002122	0.001780	1	1.0	1	0	3.65
100%	4	4615	0.001993	0.001686	1	1.0	1	0	3.65
100%	7	3644	0.001612	0.001347	1	1.0	1	0	3.65
100%	8	3644	0.001964	0.001703	1	1.0	1	0	3.65
100%	11	1312	0.001938	0.001606	1	1.0	1	0	3.65
100%	12	1312	0.002057	0.001630	1	1.0	1	0	3.65
100%	15	1274	0.001850	0.001404	1	1.0	1	0	3.65
100%	16	1274	0.002094	0.001517	1	1.0	1	0	3.65
1/10	3	462	0.002093	0.001759	1	0.1	1	0.016561	27.51
1/10	4	462	0.001985	0.001671	1	0.1	1	0.004789	27.51
1/10	7	365	0.001647	0.001304	1	0.1	1	0.026532	27.50
1/10	8	365	0.002184	0.001964	1	0.1	1	0.111881	27.22
1/10	11	132	0.002121	0.001676	1	0.1	1	0.109146	27.23
1/10	12	132	0.002144	0.001564	1	0.1	1	0.055498	27.44
1/10	15	128	0.001906	0.001477	1	0.1	1	0.038043	27.48
1/10	16	128	0.002172	0.001543	1	0.1	1	0.050358	27.45
1/20	3	231	0.002130	0.001801	1	0.05	1	0.004253	54.03
1/20	4	231	0.001952	0.001697	1	0.05	1	0.023845	54.00
1/20	7	183	0.001765	0.001320	1	0.05	1	0.115997	53.39
1/20	8	183	0.002087	0.001688	1	0.05	1	0.073254	53.77
1/20	11	66	0.002152	0.001620	1	0.05	1	0.131677	53.21
1/20	12	66	0.001955	0.001472	1	0.05	1	0.069709	53.80
1/20	15	64	0.001906	0.001422	1	0.05	1	0.039495	53.95
1/20	16	64	0.001969	0.001633	1	0.05	1	0.076831	53.75

Table 16

Number of Non-Conforming Parts List for 100% Inspection and Sampling Frequency 1/10 and 1/20

Group	Population	100%		1/10		1/20	
		NC	% of Population	NC	% of Population	NC	% of Population
1	6739	47	0.70%	5	0.07%	2	0.03%
2	6739	58	0.86%	3	0.04%	1	0.01%
3	4615	60	1.30%	6	0.13%	3	0.07%
4	4615	39	0.85%	2	0.04%	1	0.02%
5	5998	9	0.15%	2	0.03%	2	0.03%
6	5998	19	0.32%	1	0.02%	0	0.00%
7	3644	2	0.05%	0	0.00%	0	0.00%
8	3644	33	0.91%	6	0.16%	2	0.05%
9	1133	6	0.53%	1	0.09%	0	0.00%
10	1133	9	0.79%	0	0.00%	0	0.00%
11	1312	3	0.23%	0	0.00%	0	0.00%
12	1312	7	0.53%	0	0.00%	0	0.00%
13	2032	2	0.10%	1	0.05%	1	0.05%
14	2032	1	0.05%	0	0.00%	0	0.00%
15	1274	1	0.08%	0	0.00%	0	0.00%
16	1274	1	0.08%	0	0.00%	0	0.00%

CHAPTER V

CONCLUSION AND RECOMMENDATION

Conclusion

This thesis has investigated Statistical Process Control (SPC) sampling frequency for the selected shaft bearing turning process. 100% inspection has the smallest Average Production Length (APL_d), however 100% inspection has high labor costs than SPC sampling. Sampling frequency 1/10 can reduce 90% of the inspection time from 100% inspection, but it can only detect 10% of the non-conforming parts. A machining process is discrete manufacturing process which can only produce one part at one time from one machine. During SPC sampling frequency inspections, operator could miss non-conforming parts which produced within sampling intervals, but SPC was there to catch the process mean shift and also locate normal cause during process production. SPC can detect non-conforming parts from normal cause, and operators can check all the parts until the last sampled unit. Therefore, the result of APL_d for each sampling frequency evidently showed 1/10 is a better choice than 1/20 because the APL_d of 1/10 is smaller than APL_d of 1/20.

Recommendation

Further studies need to consider different sampling frequencies and different sampling size options such as $n = 3$ and 5 with different k value to calculate APL_d value for sampling frequency. Moreover, the researcher can employ the sampling frequency and collect SPC data of the process to test the performance of sampling frequency. A more flexible design of study can help evaluate impact of multiple criterions, such as sample size (n), control limits width parameter (k), and sampling rate (r).

For further study, rectification or removal of nonconforming parts in manufacturing processes can be taken into consideration. For gear machining continuous production, based on SPC inspection, either rectification or removal of nonconforming parts is necessary to insure the quality of work. In sampling stage of CSP-1 plan, Dodge recommended to use rectification and removal to ensure the quality of the product (Dodge, 1943). Furthermore, Vaughan (Vaughan, 2001) proposed a more detailed SPC-quarantined design. Both of researchers share the idea of rectification, which is doing rework on out of control parts for inspection, but Vaughan pushed it to the next level. SPC-quarantined in process control is when a part is found out of control, operator needs to checking the reverse order of the past production parts until the last in-control inspected part. In the focus of this study, SPC-quarantined design can have better overall outgoing quality than CSP-1 plan. Rectification can prevent Type II errors, which is to release nonconforming products to customers from a manufacturing point of view (Vaughan, 2001). Therefore, researchers need to include rectification or removal of nonconforming parts with sampling frequency to examine the process quality performance.

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APPENDIX A

TABLE OF MULTIPLE COMPARISON RESULTS

Table A1.

Benferroni Tests Results for Outside Diameter Data

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		0.1702	1.0000	0.0000	0.0000	0.0000	0.0062	0.0000	1.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000
2	0.1702		1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.9019	0.0000	0.0000	0.0000	1.0000	0.0000
3	1.0000	1.0000		0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000
4	0.0000	0.0000	0.0000		0.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0811	0.0000
6	0.0000	0.0000	0.0000	1.0000	0.0000		0.0000	0.0000	0.0000	0.3348	0.0000	1.0000	0.0000	0.0691	0.0000	0.0000
7	0.0062	0.0000	0.0000	0.0000	0.0000	0.0000		0.0000	1.0000	0.0000	1.0000	0.0006	0.0000	0.0000	0.0060	0.0000
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		0.0000	1.0000	0.0000	0.0000	0.0000	0.7318	0.0000	0.0000
9	1.0000	1.0000	1.0000	0.0000	0.0002	0.0000	1.0000	0.0000		0.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000
10	0.0000	0.0000	0.0000	1.0000	0.0000	0.3348	0.0000	1.0000	0.0000		0.0000	0.5221	0.0000	1.0000	0.0000	0.0000
11	1.0000	0.9019	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000	1.0000	0.0000		0.0000	0.0000	0.0000	1.0000	0.0000
12	0.0000	0.0000	0.0000	1.0000	0.0000	1.0000	0.0006	0.0000	0.0000	0.5221	0.0000		0.0000	0.2976	0.0000	0.0000
13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
14	0.0000	0.0000	0.0000	1.0000	0.0000	0.0691	0.0000	0.7318	0.0000	1.0000	0.0000	0.2976	0.0000		0.0000	0.0000
15	1.0000	1.0000	1.0000	0.0000	0.0811	0.0000	0.0060	0.0000	1.0000	0.0000	1.0000	0.0000	0.0000	0.0000		0.0000
16	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	

Table A2.

Benferroni Tests Results for Absolute Value of Outside Diameter Data

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		1.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000	0.6126	1.0000	1.0000	0.0052	0.1429	1.0000	0.1388
2	1.0000		0.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0003	0.0137	1.0000	0.7408
3	0.0000	0.0000		0.0099	0.0000	0.0000	0.0000	0.0007	0.0000	1.0000	0.0234	1.0000	0.0000	0.0000	0.0000	1.0000
4	1.0000	1.0000	0.0099		0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0013	0.5109	1.0000
5	0.0000	0.0000	0.0000	0.0000		0.0821	1.0000	0.0000	0.0064	0.0000	0.0000	0.0000	0.2707	0.0119	0.0049	0.0000
6	0.0000	0.0000	0.0000	0.0000	0.0821		0.0047	0.0000	1.0000	0.0000	0.0094	0.0000	1.0000	1.0000	1.0000	0.0000
7	0.0000	0.0000	0.0000	0.0000	1.0000	0.0047		0.0000	0.0006	0.0000	0.0000	0.0000	0.0248	0.0009	0.0004	0.0000
8	1.0000	1.0000	0.0007	1.0000	0.0000	0.0000	0.0000		1.0000	1.0000	1.0000	1.0000	0.0017	0.0439	1.0000	1.0000
9	1.0000	1.0000	0.0000	1.0000	0.0064	1.0000	0.0006	1.0000		0.0955	1.0000	0.2119	1.0000	1.0000	1.0000	0.0280
10	0.6126	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0955		1.0000	1.0000	0.0000	0.0004	0.0447	1.0000
11	1.0000	1.0000	0.0234	1.0000	0.0000	0.0094	0.0000	1.0000	1.0000	1.0000		1.0000	0.3980	1.0000	1.0000	1.0000
12	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.2119	1.0000	1.0000		0.0000	0.0010	0.1020	1.0000
13	0.0052	0.0003	0.0000	0.0000	0.2707	1.0000	0.0248	0.0017	1.0000	0.0000	0.3980	0.0000		1.0000	1.0000	0.0000
14	0.1429	0.0137	0.0000	0.0013	0.0119	1.0000	0.0009	0.0439	1.0000	0.0004	1.0000	0.0010	1.0000		1.0000	0.0000
15	1.0000	1.0000	0.0000	0.5109	0.0049	1.0000	0.0004	1.0000	1.0000	0.0447	1.0000	0.1020	1.0000	1.0000		0.0114
16	0.1388	0.7408	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0280	1.0000	1.0000	1.0000	0.0000	0.0000	0.0114	