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Astronomical Limitations on Detecting Habitable Planets Beyond the Solar System

SATOSHI MATSUSHIMA¹

Abstract. As a possible method of detecting an extrasolar planet, astrometric spectroscopic, and photometric methods are considered. Interest has been focused on the search for a planet whose physical conditions appear to favor biological evolution that could lead to the development of life as advanced as that on the Earth. In order to avoid speculative arguments, the set of assumptions introduced consists of only those that are acceptable at the present stage of our scientific knowledge. These basic assumptions are: (1) The speed of biological evolution is about the same as on the Earth; (2) a planet of our interest has to be within a certain distance from the parent star, so that the planetary temperature can be kept on a range where life can exist; and (3) atmospheric conditions, and the chemical composition of the air, are similar to those on the Earth and such conditions depend only on the ratio of mean thermal to escape velocities. Theoretically computed planetary models are used in determining mass-radius relations.

It has been found that the photometric method is far superior to the other two. In fact, it is the only means of detecting a planet of the proposed kind, and as a consequence, the conclusions do not strongly depend on the basic assumptions. Since the probability of success in the search appears to be quite high and various by-products are anticipated, the author proposes that an extensive photometric survey of late type stars be planned before much effort is expended on any radio astronomical search.

In recent years, considerable interest among radio astronomers has been focused on the possibility of interstellar communication with intelligent beings elsewhere in the universe. F. Drake (1960) has organized the project "OZMA" for the purpose of searching for artificial signals from stars outside the Solar System by means of an 85-foot radio telescope at the National Radio Astronomy Observatory. Such a telescope could detect artificially produced radio signals of the order of 10^{10} watts radiating power transmitted from distances within 10 light years. According to Su-Shu Huang (1959), however, within such a short distance there exist only two stars which appear to be able to support the development of an advanced form of life on planets accompanying them. Huang estimates that less than 5 per cent of the stars near the Sun would be able to support inhabitable planets. A more thorough discussion of this problem has been published

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by S. von Hoerner (1961). Since most of the factors determining the existence of intelligent life on other planets are unknown, von Hoerner assumed average values for those properties which are now unobservable. He has calculated the average distance to the nearest 10 civilizations to be at least the order of 10^4 light years. Following Drake's method, we find it requires a radio antenna of the order of 30 km in diameter to detect a signal from such an enormous distance. Therefore, it would be an extraordinary coincidence if there exist intelligent beings within the distances reachable by our radio telescopes.

In the light of the rather unpromising possibility of detecting interstellar radio signals, Otto Struve has pointed out that there should be a systematic search for planetary bodies accompanying solar-type stars or stars having other physical properties that may yet be amenable to life. He proposes three methods by which such a survey can be carried out with optical telescopes. The first two are based on detecting the periodic motion of a star due to perturbations produced by the revolution of an unseen planet accompanying it. This motion can be detected through small deviations from the rectilinear motion of the star across the line-of-sight (hereafter we call this the "Astrometric Method"). Or, the periodic change of the star's radial velocity might be observable through the Doppler shifts of spectral lines as in the case of a spectroscopic binary star (Radial Velocity Method). The last method is to find a small decrease in brightness of a star at a regular interval of time, due to the occultation of a portion of the star's surface by a planet in transit (Photometric Method).

The purpose of this paper is to examine quantitatively the possibilities of the above methods as astronomical means of detecting a possibly inhabited planet outside the Solar System. A comparison of these methods with radio astronomical search may be made by calculating the maximum distance at which such a planet can be detected within the accuracy of our observational techniques.

THE UNSEEN COMPANION OF BARNARD'S STAR

Among the three techniques mentioned above, the Astrometric Method is essentially the same as that which has actually been carried on by astronomers with a 24-inch refractor at the Sproul Observatory for many years. So far they have detected three unseen companion bodies whose masses are too small to be considered as stars. Especially the one discovered by Peter van de Kamp (1963) in 1963 is believed to have a mass as small as that of Jupiter. After careful analyses of data accumulated for more than twenty years, van de Kamp has concluded that the

unseen companion of Barnard's Star, the second nearest to the Sun, must be a planet of Jupiter's size revolving in a highly eccentric orbit with a period of 24 years at a mean distance of 4.4 Astronomical Units from the parent star. Unfortunately, however, there is little possibility of finding life on this planet, as its surface temperature seems to be much too low to permit any known type of biological evolution. The highly eccentric orbit would also make it unlikely that the planet could retain a physically stable atmosphere.

PHYSICAL CONDITIONS ON HABITABLE PLANETS

Ages of Parent Stars

Since at the present stage of our knowledge it is not possible to establish the time-scale of biological evolution, we follow Su-Shu-Huang's assumption that the life on other planets evolves at about the same speed as on the Earth and that only the stars which remain in the state of thermal equilibrium at least 3×10^9 years could be accompanied by planets supporting life. From the present knowledge of stellar evolution, only the stars on the main sequence—an evolutionary stage in which the source of energy supply is mainly due to thermonuclear reactions converting hydrogen into helium—in F or later spectral classes are believed to maintain the equilibrium for such a long period of time. The ages on the main sequence stage for stars of various spectral classes are compared in the second column in Table 1. In the following three columns, we note that the large luminous

Table 1. Physical characteristics of the Main Sequence Stars.

Spectral Type	Main Seq. Stage (x 10^9 Years)	Luminosity L(Sun=1)	Mass M(Sun=1)	Radius R(Sun=1)	Rotational Velocity (km/sec)
B0	0.008	29,000	17.0	7.6	200
B5	0.08	980	7.1	4.0	210
A0	0.4	100	3.55	2.6	190
A5	2	12.0	2.20	1.78	160
F0	4	7.1	1.78	1.35	95
F5	6	3.3	1.41	1.20	25
G0	11	1.3	1.07	1.05	<12
G2(Sun)	13	1.0	1.00	1.00	2
G5	17	0.69	0.93	0.93	<10
K0	28	0.24	0.81	0.85	<10
K5	70	0.105	0.69	0.74	<10
M0	>100	0.055	0.48	0.63	<10
M5	>100	0.009	0.22	0.32	<10

stars have to be excluded from our consideration and the upper limits of the luminosity, mass, and radius must be set at F0 stars. The values listed in Table 1 are in units of the solar values (Harris 1963, Allen 1963).

An interesting point has been noted in the mean rotational velocities of the stars listed in the last column, where a sharp decrease is observed between A and F stars. Struve (1960) pointed out that the drop in rotational velocity may be ex-

plained if we assume that at the time of stellar formation a large portion of the angular momentum was transferred to unobservable planets surrounding the late type stars. Hence, it may be considered as a supporting evidence for our fundamental assumption.

Temperature of Planets

It is unlikely that an advanced form of life could evolve under a temperature range much different from that on the Earth. Since the temperature of a planetary surface varies from region to region, depending on the inclination of the rotational axis with respect to the orbit, we shall consider mean values. Other than possible differences in the blanketing effect due to an atmosphere, the mean temperature, T , will depend mostly on distance, a , between a star of luminosity L and a planet. If we assume the planetary radiation follows the law of black body, we have the following relation:

$$T^4 = L \cdot a^{-2} \quad (1)$$

where T (measured in the absolute temperature scale) is expressed in units of the Earth's mean temperature, and L and a are given in units of the luminosity of the Sun and Astronomical Units (mean distance between the Sun and the Earth), respectively. If we take 290° K as the mean temperature of the Earth, the above equation gives 240° K for Mars and 340° K for Venus, in good agreement with recent observations.

The question is to determine the two extreme values of the temperature range in which evolution of life is possible. Although it is difficult to set sharp limits at the present stage of our biological knowledge, we may reasonably assume that an advanced form of life cannot evolve under a temperature much lower than, say, -40° C and much higher than 40° C . If we further assume that the regional deviations from the mean temperature and the atmospheric effect on a planet can be as much as $\pm 70^\circ$, we may introduce the following restrictions:

$$0.55 < T < 1.30 \quad (2)$$

after choosing 290° K for the Earth's mean temperature. It should be noted that the percentage error involved in an estimate in the centigrade scale is not as large as it appears in the Kelvin scale.

Masses of Planets

The most difficult question is to find an appropriate mass to be taken for the assumed planet. As we shall see later, the probability of success in our search depends on the mass and radius of a planet. In a single sample of the planetary system in the Uni-

verse, we observe that stars of solar type may be accompanied by planets whose masses range from 0.05 to 318 times the mass of the Earth. However, the progress of biological evolution must also depend on the atmospheric conditions of a planet. For example, Su-Shu Huang states that a planet of too large a mass would be likely to possess a reducing atmosphere which does not favor the development of life. Hence, it seems that there exists an upper limit on the mass of a planet in order that it may be inhabitable.

Without having much detailed knowledge on the formation and time-variation of planetary atmospheres, the only means to estimate the chemical composition of an atmosphere is to calculate the velocity of escape of the assumed planet. Let us introduce the assumption, in order to keep the atmospheric constituents similar to those on the Earth, that the ratio of the thermal velocity of an element to its escape velocity has to be the same order of magnitude as that on the Earth. The velocity of escape, V_e , in units of that of the Earth is given by

$$V_e^2 = m \cdot r^{-1} \quad (3)$$

where m and r are the mass and radius of a planet, expressed in units of those of the Earth. The mean thermal velocity of a certain molecule, V_t , expressed in units of that of the same molecule on the Earth is

$$V_t^2 = T. \quad (4)$$

Hence, it follows that for the permitted range of T given by the equation (2), $m \cdot r^{-1}$ can take a value in the following range to yield the same $V_e - V_t$ relation as on the Earth:

$$0.55 < m \cdot r^{-1} < 1.30 \quad (5)$$

As we shall see later, the probability of success in our search is greater for a planet of larger mass. From the equation (5), we see that to have m large, r has to be large, or a planet of the least density will give the largest mass for a given V_e . Hence, an "ideal planet" for our search would be the one that is composed of pure hydrogen. In a review article on Planetary Interiors, Wildt (1961) discusses various detailed computations for planetary models of pure hydrogen. On the basis of theoretically derived equations of state in low temperature, De Marcus (1951, 1958) and Abrikosov (1954) have given mass-radius relations of equilibrium configurations of a hydrogen sphere. Their mass-radius diagrams with the units being converted to the Earth's values, are reproduced in Figure 1. Due to the differences in assumed equations of state, Abrikosov's curve (solid line) differs markedly from that of De Marcus (dotted line). A point

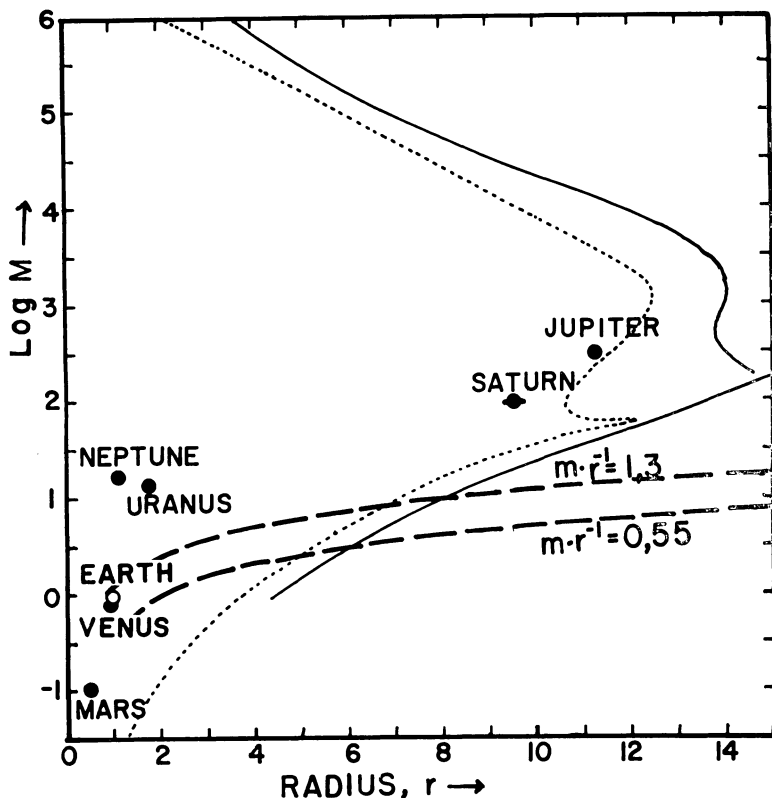


Figure 1. *Mass-Radius Relations Obtained from Theoretical Models of Pure Hydrogen Spheres.* Dotted line is by De Marcus (1951; 1958), and solid line by Abrikosov (1954). Two heavy dashed lines indicate the region in which (m-r) relation gives the same V_e-V_t ratio as on the Earth (see equations 3-5).

on the lower portion of the diagram represents an equilibrium configuration of planetary size for a set of mass and radius. In the upper portion of the diagram, the curves merge to the white dwarf configurations given by Chandrasekhar's theory.

Wildt states that since hydrogen is the least dense and most compressible solid material known, it is physically impossible to realize more extended configurations of solid matter and that the entire domain to the right of a diagram is a "forbidden region." The positions of the known planets are also indicated in the same figure. The horizontal distances of the planetary points from the m-r diagrams are some measure of the accumulation of elements heavier than hydrogen in the planets.

The two heavy dashed curves in Figure 1 define the relations, $m \cdot r^{-1} = 0.55$ and $m \cdot r^{-1} = 1.30$, which indicate the permissible range of m and r for a planet of our interest obtained in the

equation (5). Hence, from the points where the curves cross Abrikosov's diagram, we find

$$m = 10.5 \quad \text{and} \quad r = 8.1 \quad \text{for} \quad T = 1.3 \quad (6)$$

and

$$m = 3.3 \quad \text{and} \quad r = 6.0 \quad \text{for} \quad T = 0.55 \quad (7)$$

We note that De Marcus' diagram gives about 15 per cent smaller values for both m and r .

PLANETARY MOTION

The formulation of fundamental equations of a planetary motion may be simplified by expressing the physical quantities of a star-planet system in units of the corresponding quantities for the Sun-Earth system. Throughout the following discussion, therefore, we express the mass, M , and radius, R , of a star in units of the mass and radius of the Sun, respectively, and the mass, m , and radius, r , of a planet in terms of those of the Earth. We then note that

$$(\text{Unit of } m) = 3 \times 10^{-6} (\text{Unit of } M) \quad (8)$$

and

$$(\text{Unit of } r) = 0.92 \times 10^{-2} (\text{Unit of } R). \quad (9)$$

As we have noted before, a highly eccentric orbit will produce a rather damaging effect on a planet in supporting biological development. Hence, in the following formulation we consider a planet revolving in an orbit of small eccentricity as in the case of all the planets in the Solar System except Mercury and Pluto. In this case, the mean values of various quantities such as velocities and the distances from the central star will closely represent the values at any point in its orbit.

Considering a planetary system consisting of a star M and a planet m , let A and a be the distances (in Astronomical Units) from the center of mass to the star and to the planet, respectively. Since in general $a \gg A$, a can be approximated to be the distance between M and m . We then have a relation:

$$A = 3 \times 10^{-6} m \cdot M^{-1} \cdot a. \quad (10)$$

When such a system is at a distance of D parsecs from the Earth, A will subtend a small angle θ , where

$$\theta \cdot D = 3 \times 10^{-6} m \cdot M^{-1} \cdot a, \quad (11)$$

in which θ is given in units of seconds of arc; θ being the amplitude of the periodic deviation of the star from its proper motion as seen from the Earth, due to the perturbation caused by the revolution of the planet. The Astrometric Method is based on the detection of this small angle θ .

The period of the revolutionary motion, P, expressed in units of years will be given by

$$P^2 = a^3 \cdot M^{-1} . \tag{12}$$

From the equations (10) and (12), we have the velocity of the star's revolution:

$$V \text{ (in km/sec.)} = 9 \times 10^{-5} m \cdot M^{-\frac{1}{2}} \cdot a^{-\frac{1}{2}} . \tag{13}$$

The above equation gives the maximum radial velocity observable when the orbital plane is perpendicular to the line-of-sight. Hence, the Radial Velocity Method must detect a displacement, $\Delta\lambda$, of a spectral line, λ , the maximum amount of which is given by

$$\frac{\Delta\lambda}{\lambda} = 3.33 \times 10^{-4} V . \tag{14}$$

The basic relation governing the Photometric Method will be determined by the size of a planet relative to a star. The decrease in brightness of a star whose radius is R when it is occulted by a planet of radius r is

$$L = 0.85 \times 10^{-4} r^2 \cdot R^{-2} \cdot L , \tag{15}$$

or if we express the brightness-decrease on the stellar magnitude scale and denote it by $\Delta_{\text{mag}} L$, we have

$$\Delta_{\text{mag}} L = -2.5 \log (1 - 8.5 \times 10^{-5} r^2 \cdot R^{-2}) . \tag{16}$$

Thus, we see that $\Delta_{\text{mag}} L$ is independent of the brightness of the star itself.

RESULTS

Astrometric Method

From the equations (1) and (11), we write

$$\theta \cdot D = 3 \times 10^{-6} m \cdot M^{-1} \cdot L^{\frac{1}{2}} \cdot T^{-2} . \tag{17}$$

We recall that for the main sequence stars, L and M are related through the mass-luminosity law

$$L = M^n \tag{18}$$

where

$$n > 3.0 . \tag{19}$$

Hence, in the equation (17), we realize that

$$M^{-1} \cdot L^{\frac{1}{2}} = M^{(n-2)/2} > M^{\frac{1}{2}} , \tag{20}$$

so that $\theta \cdot D$ takes the maximum value for the largest M. It follows therefore that for a given θ , we can reach the furthest distance for FO stars through the astrometric method.

The value of θ is fixed by the accuracy of the present astrometric observations. According to van de Kamp (1962), the probable error in θ is

$$\Delta \theta = \pm 0.002 \text{ sec.} \tag{21}$$

From Table 1, we find the most favorable case for the astrometric method to be

$$L = 7.1 \quad \text{and} \quad M = 1.8 . \quad (22)$$

Substituting (6), (21), and (22) into (17), we have

$$D_{\max} = 1.4 \times 10^{-2} \quad \text{for} \quad T = 1.30, \quad (23)$$

and similarly from (7), (21), (22) and (17),

$$D_{\max} = 2.4 \times 10^{-2} \quad \text{for} \quad T = 0.55 . \quad (24)$$

It follows therefore that through the astrometric method, we can detect a probably inhabited planet only if it is within the distance of 0.01 parsec or one hundredth of the distance to the nearest star. Noting that D is proportional to m , therefore, we find it impossible to reach even a distance of the nearest star, unless the planet in question has a mass of the order of Jupiter's mass. We thus conclude that it is nearly impossible, by means of the astrometric method, to detect a planet that might bear life on it.

Radial Velocity Method

From the equations (1) and (13), we obtain

$$V = 9 \times 10^{-5} m \cdot M^{-1/2} \cdot L^{-1/4} \cdot T , \quad (25)$$

in which we note that from (18) and (19)

$$M^{-1/2} \cdot L^{-1/4} = M^{-(2+n)/4} < M^{-5/4} \quad (26)$$

Hence, we see that a star of small mass should produce a large shift. Hence, taking the values, $L = 0.009$ and $M = 0.22$ for M5 star from Table 1 and $m = 10.5$ and $T = 1.3$ from (6), we find from equation (25) that the maximum velocity observable is

$$V = 0.014 \text{ km/sec.} \quad (27)$$

The probable error of spectroscopic measurements of radial velocity is (Petrie 1963),

$$\Delta V = \pm 0.12 \text{ km/sec.} \quad (28)$$

at the H_γ (4340 Å) line for a spectrum whose dispersion is 3.5 Å/mm. Hence, the largest radial velocity obtainable is still an order of magnitude smaller than the limit of the spectroscopic analysis. Since V is proportional to m , a planet of about 100 times the Earth's mass is required to be detectable.

Photometric Method

In this case, the accuracy of observations depends only on the size of a planet relative to its parent star. We notice in equation (16) that $\Delta_{\text{mag}} L$ is large when r is large and R is small. Hence, as the most favorable conditions for our purpose, we consider a system consisting of an M5 star ($R=0.32$) and a planet of mass 10.1 ($r=8.1$). Then, from equation (16), we

find the brightness-decrease expressed on the magnitude scale as

$$\Delta_{\text{mag}} L = 0.06 . \tag{29}$$

The accuracy of photoelectric photometry is around 0.01 magnitude under average sky conditions. Hence, the photometric method appears to be a very promising one, as the brightness-decrease predicted in (29) is well above the limiting accuracy. An important point to be noted here is the fact that the above result is obtained independently of T and m, and hence all of our basic assumptions, except for a chosen value of r, do not enter into the computation. The probability of success seems to be high even for small r. For example, if we solve the equation (16) for $\Delta_{\text{mag}} L = 0.01$ and $R = 0.32$, we find $r = 3.3$. Hence, a planet as small as three times the Earth in radius may well be detectable.

$\Delta_{\text{mag}} L$ computed for stars of various spectral classes being accompanied by a planet of $r = 8.1$ ($T = 1.3$) are listed in the first column in Table 2. The periods of the brightness-decrease computed from the equations (12) and (1) are given in the

Table 2. Stars Accompanied by a Planet of Radius = 8.1 (Earth's).

Spectral Type	Brightness-Decrease $\Delta_{\text{mag}} L$ (Magnitude)	Period P (Months)	Duration of Occultation t (Hours)	Vis. Absolute Magnitude M_v	Dist. Seen as 15th Mag. Star D (Parsecs)
F0	0.003	18	17	2.6	3020
F5	0.004	11	14	3.4	2100
G0	0.005	6.5	11	4.4	1320
G5	0.007	4.2	8.6	5.2	910
K0	0.008	2.4	7.7	5.9	660
K5	0.010	1.2	5.1	8.0	250
M0	0.015	0.8	4.2	9.2	145
M5	0.057	0.4	2.2	12.3	35

second column. We note that the periods of revolution of the planet thus computed are less than a year for most of the stars. The third column gives the time duration of the occultation period by a planet, which we denote by t. In units of hours, t may be computed from the following formula:

$$t \text{ (hours)} = 13 \cdot R \cdot M^{-\frac{1}{2}} \cdot L^{\frac{1}{4}} \cdot T^{-1} . \tag{30}$$

The numerical value of 13 corresponds to the time that the Earth spends in moving the distance equal to the diameter of the Sun. It should be noted that the fact that both P and t are small is a great advantage from the observational point of view.

As was noted earlier, the maximum distance to which the photometric method can reach with the same accuracy does not directly depend on the limiting accuracy of the method, but it depends on the intrinsic luminosity (or Absolute Magnitude) of a star itself. If we denote the absolute visual magnitude and the apparent visual magnitude of a star by M_v and m_v , respectively, the distance D in units of parsecs is given by

$$\log D = 1 + 1/5 (m_v - M_v) . \tag{31}$$

The last two columns in Table 2 give the absolute visual magnitude and the distance D , at which a star of each spectral class is seen with the apparent magnitude, $m_v = 15$. With a telescope of moderate size we can observe a star down to the 15th magnitudes with an accuracy of $\Delta_{\text{mag}} L = 0.01$. From equation (31), we see that if the limiting apparent brightness is increased by one magnitude, the corresponding increase in the maximum reachable distance is about 60 per cent and the number of stars to be included in observations would be about 4 times larger.

CONCLUSION

In the preceding discussion, we have compared the astrometric, spectroscopic, and photometric methods as a means to detect a possibly inhabitable planet. As a result, the photometric method has been found to be far superior to the other two. As far as we are interested only in searching for an inhabitable planet, the astrometric method is so inferior that it does not seem to be applicable under any circumstances. The spectroscopic method may become successful, if we do not have to place the assumed condition of the atmospheric compositions and if we can allow a planet of Jupiter's size. Such a case might arise if we accept the idea that an entirely different form of biological evolution might be possible on other planets. In this case, the argument becomes too speculative, and in fact, all the assumptions and limitations adopted in this paper may lose their fundamental reasons.

On the other hand, the photometric method appears to be a very promising one. In Table 2, we see that a habitable planet accompanying either K or M stars can well be detectable within the limit of presently available techniques of astronomical photometry. By repeating a statistical analysis of many observations, the accuracy may go up to cover all the solar type stars (G2). Furthermore, when a telescope on an orbiting observatory begins to function as on the ground, the photometric accuracy should reach 0.001 magnitude. Thus, covering the stars of all the spectral class in Table 2, we can reach a distance of the order of 10^4 parsecs. Even at the present limit ($D = 500$ parsecs), there will be as many as 10^4 stars within our reach. It should be emphasized that, as a consequence of the fact that the photometric method seems to be the only means of detecting a planet of the proposed kind, the conclusions do not directly depend on any of the assumptions adopted in the calculations. The only restriction imposed on the photometric search is that the assumed planet has to be larger than a few times the Earth in radius.

An important advantage in the photometric method in com-

parison with the others is that it is not based on a perturbing force of planetary bodies. The mass of a planet in which we are interested is less than 10 times the Earth's mass. Hence, if in the same system there exists another planet as large as Jupiter, the effect of smaller planets might be washed out by the larger one, thus making it impossible to detect this particular planet. The rather short periods of light variation listed in Table 2 are necessary consequences of our limitation on the temperature range, which appears to be another great advantage. The brightness-variations due to various activities on a star itself such as large sun spots and flares, etc. might amount to a comparable magnitude as that due to a planet. Such effects may be distinguished by continuing the observations for a period many times longer than the period of planetary revolution. It would therefore be extremely difficult if we were to look for a brightness change of the order of 1 per cent lasting only for a few hours and taking place every few years.

In conclusion, therefore, the author proposes that an extensive photometric survey of star brightness should be carried out first for M-type and later K-type stars before serious effort is expended on a radio astronomical search. Such a program can be carried out along with other photometric observations with a relatively small, say, 12 to 24 inch telescope. For example, a reflector of about 16 inches aperture may be sufficient to observe a star down to the 16th magnitude, thus reaching the distance shown in the last column of Table 2. Various important discoveries may be made as by-products of the project. For example, a number of flare stars will undoubtedly be discovered. The existence of sunspots or flare-type activities may be confirmed and their characteristics can be studied extensively.

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