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Analysis of a paleoglacier reconstruction model for valley glaciers of the Wind River Range, Wyoming

Taylor Rae Garton

University of Northern Iowa

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ANALYSIS OF A PALEOGLACIER RECONSTRUCTION MODEL FOR
VALLEY GLACIERS OF THE WIND RIVER RANGE, WYOMING.

An Abstract of a Thesis

Submitted

in Partial Fulfillment

of the Requirements for the Degree

Master of Arts

Taylor Rae Garton

University of Northern Iowa

May 2019
ABSTRACT

Various approaches, ranging from in-field morphological reconstructions to more technology-based modelling practices allow us to reconstruct and understand the long-term geomorphic evolution of landscapes. The study of paleo-environments by reconstruction can also give us profound insight into paleo-climate. A new model, GlaRe (Glacial Reconstruction), has been introduced into the alpine glacier modeling community to facilitate glacier reconstruction. In this research the mathematical equations representing the basal shear stress parameter are tested to determine the applicability of the GlaRe model to alpine glaciers in the American West. Known ages and extents of past glacier advances were used to test the reconstructions of two valley glaciers in the east flanks of the Wink River Range, Wyoming. GlaRe predicts the ice thickness of the glacier accurately when a variable basal shear stress is applied. If the default values programmed into the GlaRe toolbox are utilized the resulting outputs contain significant error. Models which have been vetted and validated by analytical solutions and field evidence can be used as a tool to facilitate insight into glacier observation. Validation of glacier models will enable accurate prediction of glacier dynamics in the present and for the future.
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Taylor Rae Garton
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May 2019
This Study by: Taylor Rae Garton

Entitled: Analysis of a paleoglacier reconstruction model for valley glaciers of the Wind River Range, Wyoming.

has been approved as meeting the thesis requirement for the

Degree of Master of Arts

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Date                                      Dr. Patrick Pease, Thesis Committee Member

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Date                                      Dr. Jennifer Waldron, Dean, Graduate College
DEDICATION

I would like to thank JMJ for always supporting me, whether I realized it or not. The constant support, consolation, and advice enabled me to complete this manuscript and to you I owe all my accomplishment. May you forever help me in my endeavors and teach me how to become better at whatever I choose to achieve.
ACKNOWLEDGEMENTS

This thesis was born out of stimulating conversations involving, and exploration of, the majestic landscape of the Wind River Range. I want to acknowledge and thank Professor Dr. Dennis Dahms for introducing me to this captivating landscape and allowing me to explore it. Secondly, I want to thank Professor Dr. James Dietrich for leading me through the methodology of my studies; the consolation, advice, and modelling puns will be warmly remembered.
TABLE OF CONTENTS

LIST OF TABLES ........................................................................................................................................ vii
LIST OF FIGURES ......................................................................................................................................... viii
CHAPTER 1 INTRODUCTION ...................................................................................................................... 1
CHAPTER 2 BACKGROUND .......................................................................................................................... 4
   Reconstruction Methodologies .................................................................................................................... 4
   Reconstruction Techniques ......................................................................................................................... 5
   Study Area ................................................................................................................................................ 6
CHAPTER 3 METHODS ................................................................................................................................ 8
   Shallow Ice Approximation ....................................................................................................................... 8
   Ice Extent Mapping ................................................................................................................................ 9
   Model Inputs .............................................................................................................................................. 10
   “Default” Shear Stress Modeling ............................................................................................................. 13
   Model Manipulation ................................................................................................................................. 19
   Model Comparisons ................................................................................................................................. 21
CHAPTER 4 RESULTS .................................................................................................................................. 23
   Comparison and Validation ....................................................................................................................... 23
   Ice Extent Mapping ................................................................................................................................ 23
   “Default” Shear Stress Modelling ........................................................................................................... 24
   Model Manipulation ................................................................................................................................. 26
   Model Comparison ................................................................................................................................. 31
CHAPTER 5 DISCUSSION ............................................................................................................................. 33
   Model Performance ................................................................................................................................. 33
LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1: Attribute output table for Sinks Canyon after the default run of 100kPa from the GlaRe tool: Ice Thickness Calculation.</td>
<td>12</td>
</tr>
<tr>
<td>Table 2: Table of the variables and equation components utilized within the GlaRe model.</td>
<td>15</td>
</tr>
<tr>
<td>Table 3: Attribute table for Sinks Canyon representing the initial 100kPa default model run and then the subsequent back-calculated model run with variable shear stresses. Slope and distance are included as these elements allowed for the calculation of shear stress.</td>
<td>21</td>
</tr>
<tr>
<td>Table 4: Volume calculations for Sinks and North Fork canyons. The 2-Dimensional measurement is included for scale when interpreting the total ice volume within the canyons.</td>
<td>31</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1: Location of the study sites, Sinks (Middle Popo Agie) and North Fork (North Fork Popo Agie) canyons, within the Wind River Range, Wy.</td>
<td>7</td>
</tr>
<tr>
<td>Figure 2: Generalized method flow chart leading from generation of GlaRe inputs to data and calculation manipulation. Outputs include all data generated from the methods including model derived and calculated shear stress and ice thicknesses.</td>
<td>10</td>
</tr>
<tr>
<td>Figure 3: Sinks and North Fork Canyon with all required inputs of the GlaRe toolbox: glacial extent, flowline, and moraine limit which represents the extent of glacial flow. Contour lines are also included to establish the dimensions of the canyons.</td>
<td>11</td>
</tr>
<tr>
<td>Figure 4: Top: Geomorphological mapping completed for both canyons to facilitate the GlaRe model. Bottom: The bed elevation and the moraine elevation from the Pinedale glaciation used as the ice surface.</td>
<td>24</td>
</tr>
<tr>
<td>Figure 5: Top: The default (100kPa) shear stress as viewed over the valley. The valley ice height and bed height are also included as references. Bottom: The difference in predicted ice height between valley ice extent and 100kPa default model run.</td>
<td>26</td>
</tr>
<tr>
<td>Figure 6: Valley calculated basal shear stress is shown in light gray with the ice height of the Pinedale glaciation and bed height overlain.</td>
<td>27</td>
</tr>
<tr>
<td>Figure 7: Top: Ice prediction comparison between the valley moraine derived ice extent, 100kPa GlaRe model run and the recalculated GlaRe variable shear stress model run. Bottom: The difference between the valley calculated ice height based off morphological evidence and the recalculated GlaRe model ice height.</td>
<td>28</td>
</tr>
<tr>
<td>Figure 8: Top: Comparison between the shear stresses of the canyon and back-calculated GlaRe stress values. Bottom: Difference in shear stress compared between the canyon ice moraine limits and the GlaRe back-calculated values.</td>
<td>29</td>
</tr>
<tr>
<td>Figure 9: Top: Comparison of ice thickness between valley ice heights, back-calculated GlaRe ice heights and the 100kPa heights. Bottom: The difference between the GlaRe model runs of back-calculated values and those from the 100kPa run.</td>
<td>30</td>
</tr>
<tr>
<td>Figure 10: 3-Dimensional perspective views showing ice thickness for the default (100kPa) run (first and third panel) and the back-calculated variable shear stress run (second and forth panel).</td>
<td>32</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

Reconstructions of alpine valley glaciers have long been used to examine the implications of rapid climate shifts (Carr, Lukas, and Mills 2010). Reconstructions of past glaciated alpine environments allow us to establish paleo-climatic histories which can help us better understand past climatic variations. Once paleo-environments are reconstructed paleoglacier Equilibrium Line Altitudes (ELAs) can be determined which allows us to construct paleo-climatic interpretations. ELA is described as the position where mass balance equals approximately zero and is considered critical in examining glacier dynamics because of its link with the local climate (Carr and Coleman 2007). Evidence of ELA change is thus considered an important indicator of glacial response to climate change and helps to establish the dynamics associated with glacial extents. Paleoglacier reconstructions are most valuable and robust when they are based on morphological features because this allows for accurate reconstructions of former glacier geometry.

Paleoglacier surface and volume reconstructions, in particular, rely heavily on landscape morphology such as the terminal and lateral moraines, trimlines, and other ice contact features (Pellitero et al. 2016). Paterson (1994a) noted that climate conditions, along with the physical characteristics of ice, determines the breadth and behavior of a glacier (Carr and Coleman 2007; Paterson 1994a). These relationships provide the rationale for examining paleoglacier dynamics, for, in understanding the behavior of a glacier we gain additional insight into the glacier/climate response. By understanding the
connectivity of glaciers and climate we develop further understanding of future climate change and global cryosphere behavior (Rowan 2011).

Reconstructing ice masses generate boundary conditions which act as anchor points for glacier models to build up from. The determination of paleoglacier geometries is essential to constrain, test and refine numerical models that then can be used to predict and validate past climate change and, through extrapolation, future glacier and climate responses to changing climates (Lukas and Bradwell 2010). Modern paleoglaciological approaches establish three-dimensional glacier form from models (numerical, dynamic, etc.) which use evidence from glaciologists’ reconstruction of paleo-ice masses. As paleoglacier reconstructions become increasingly automated it becomes important to the validity of glacier models to test their ability to predict ice thickness and glacier extent. The increase in the quantity of geospatial data allows for improved glacier reconstructions through modelling approaches. Despite this progress, methods and sources of uncertainty are rarely stated and leave much to be discovered by the user (Pearce et al. 2017). Pearce et al. (2017) emphasizes the significance of modelling to paleoglacier reconstructions because many modern field sites may never be visited or revisited. There exists extraordinary potential for glacier models, but as (Benn and Evans 2010, 147) assert: “they are not widely appreciated beyond the confines of the modelling community”. This occurs, in part, because modelling glacier movement is difficult to perform consistently due to changing user interpretations of moraine limit and ice extent. However, this does not diminish a paleo-model’s potential effectiveness. Spatial models offer researchers the opportunity to not only reconstruct past landscapes and how they
have changed to the present, but also to extrapolate out into the future. This is what makes glacier models invaluable to the scientific community.

The aim of this research is to test the applicability of the glacier reconstruction model, GlaRe, to alpine valley glaciers in the American West. Specifically, testing the accuracy of ice extent and volume predictions through an examination of the mathematical equations that represent the basal shear stress parameter embedded in the GlaRe model. Shear stress was chosen as the focus because it exerts a first-order control on the modeled glacier 3D surface and thereby influences all aspects of the model’s performance (Pellitero et al. 2016). This study also aims to understand the implications of using a theoretical constant value for basal shear stress on a glacial reconstruction. The use of field and remotely sensed data will facilitate constraints on the model for valley glaciers in two glaciated canyons in the Wind River Range (WWR), Wyoming, USA (Figure 1). This study’s focus is on mid-latitude alpine valley glaciers because these glaciers directly reflect the regional environment and climate and allow researchers to create a direct relationship between glaciers and climate. The need to understand the effects on glaciated environments from future climate change is mounting as the advance and retreat of mid-latitude alpine glaciers is perhaps the most obvious proof of climate change during the Quaternary (~2.6 Ma to present; Carr, Lukas, and Mills 2010).
CHAPTER 2
BACKGROUND

Reconstruction Methodologies

Glacier reconstruction is employed to understand and uncover the dynamic processes that drive a glacier by viewing its evolution through millennial time scales (Rowan 2011). Although, changes occurring in present day glaciers offer a clear guideline for the study of climate change, reconstructions of paleo-glaciations offer increased insight into paleo-climatic conditions. Reconstruction, therefore, allows for a more robust understanding of the interactions between ice and climate (Benn and Evans 2010). Several methods for reproducing the evolution of glaciers have been used that range from simplistic mathematical calculations based on geomorphological mapping and simple numerical modeling (Benn and Hulton 2010) to even more complex spatial models utilizing geographic information systems (GIS) (Pellitero et al. 2016). The best practice in reconstruction is to combine the available landform evidence with numerically derived reconstructions (Benn and Hulton 2010; S. Carr and Coleman 2007; Lukas and Bradwell 2010; Pellitero et al. 2016; Züst et al. 2014), thereby establishing the reconstruction in observed landforms and equations for glaciers that have been well vetted e.g. (Nye 1952). Examples of these types of models include: Benn and Hulton’s (2010) Profiler excel spreadsheet and Pellitero et al.’s (2016) paleo-reconstruction GIS toolbox named GlaRe. Both of these models are numerical approaches based on an iterative solution to the perfect plasticity assumption for ice rheology. Pellitero et al. (2016) mentions that the ideal situation is to run ice surface at 100,000 Pa (default value
for shear stress for paleoglaciers) and then to tune the shear stress parameter to fit the reconstructed 3D glacier surface. In order to tune the shear stress parameter, one must use geomorphological constraints or evidence of vertical ice thicknesses (e.g., valley depth or lateral moraines). Experts then tune the model to achieve better results through a manual input option to further influence and manipulate the toolbox, thus grounding the model in the natural environment through observation and expert input. Ground truthing allows users to help the computer model “build up the glacier”, with the assumption that it will help constrain the model to the natural limits that are observed on the ground. However, these ideas hold true only when the glacier dynamics/equations programmed within the model being implemented are thoroughly understood.

Reconstruction Techniques

The significance of using alpine paleo-glaciers stems from their records, being spans of time much longer than any observable record permits. Mountain glacier morphologies are heavily influenced by regional trends in climate and geography because by themselves alpine glaciers do not constitute a large enough portion of the earth’s water budget to alter global climate patterns. We observe modern glaciers melting and can measure the rate of change occurring on the glaciers by use of mass balance parameters. It is widely accepted by the scientific community that modern air temperatures are increasing, and climate is changing, so what does that mean for modern mountain glaciers? How much will they change in the future? Through model reconstructions we observe how mountain glaciers responded to past changes in hope to uncover details that
will help to understand how they will change under conditions predicted for the future (Davies and Holloway 2014).

**Study Area**

The Wind River Range (WWR) is located in the Middle Rocky Mountains of Wyoming, USA (Figure 1). The Pleistocene glacial succession is well-established for the WWR (Dahms et al. 2018; Fabel et al. 2004). Sinks and North Fork canyons are located on the southeastern flanks of the range, above Lander, Wyoming. For this study, the morphostratigraphic unit that represents the maximum advance of glaciers in these valleys during the LGM (~22ka) was chosen to facilitate the extent for glacier modeling. These glacial deposits have been mapped as belonging to the Pinedale glacial advance and are represented in Sinks and North Fork canyons by a sequence of terminal, recessional and lateral moraines (Dahms et al. 2018; Dahms 2004; Züst et al. 2014). Lateral moraines are characterized by sharp crests which mark the approximate ice elevation during the LGM and for each recessional stage (Dahms et al. 2018; Züst et al. 2014). Sinks Canyon contains the Middle Popo Agie River and occupies an ice shed area of 149km² (Dahms et al. 2018). Sinks Canyon is characterized by a typical U-shaped cross section (Fabel et al. 2004). The area of interest is the lower canyon behind the deposits of the Pinedale glaciation which encompasses an area of 23km². North Fork Canyon contains the North Popo Agie River and is located north of and adjacent to Sinks Canyon. North Fork Canyon also contains terminal and lateral moraines that have been radiometrically dated to the Pinedale glaciation (Dahms et al. 2018). The ice shed of the North Fork drains an area of 102km² (Dahms et al. 2018) and the area of interest is the
main canyon which covers an area of 23km$^2$. Sinks and North Fork canyons, being extensively studied with published numeric ages and mapped ice extents, are ideal localities on which to perform validation of the GlaRe model.

Figure 1: Location of the study sites, Sinks (Middle Popo Agie) and North Fork (North Fork Popo Agie) canyons, within the Wind River Range, Wy.
CHAPTER 3

METHODS

Shallow Ice Approximation

Establishment of morphological evidence for paleoglaciers generates paleo-glacier surface and volume reconstructions through varying methods. The most common approach is the Shallow Ice Approximation (SIA). SIA was first introduced by Fowler et al. (1978) and modified by Hutter, Legerer, and Spring (1981). SIA creates approximations based on the fact that the horizontal extent of an ice sheet is large compared with its thickness; this relationship allows for the determination of a glacier’s characteristics such as its movement (Hooke 2005). SIA represents a simplified version of the full Stokes Equation which incorporates a complete description of the stresses acting on an ice mass. Models of glacier flow attempt to duplicate this relationship through complex sets of equations, which can be solved to determine how an ice mass (the system) will respond to different sets of parameters. This implies that the best we can achieve is only an approximation of the glacier’s dynamics. The only stresses considered within SIA (not being the full Stokes) are shear stresses or the stress acting horizontal to the ground plane. Shear stress can be equated to tractions that are parallel and also acting in opposite directions (Benn and Evans 2010). Thus, although we must consider basal shear stress, there is no accepted way to determine the basal shear stress for paleoglaciers. One approach is to assume a single value or range of values, while another is to use dated and observed lateral moraines to back-calculate basal shear to match the geographical evidence in a valley.
Ice Extent Mapping

To produce a systematic assessment of the glacial geomorphology for Sinks and North Fork canyons, I follow a methodology similar to that which utilizes a combination of field data/observation and remotely sensed datasets (aerial photography and DEMs) (Boston 2012; Pearce et al. 2017) (Figure 2). For Sinks Canyon, the field data I used consists of GPS control points taken from four different transect lines collected by Züst et al. (2014), with the additional cross-sections of Dahms et al. (2018) and Fabel et al. (2004) to document the valley morphology. The field data was overlaid on 2017 NAIP aerial photography (Wyoming Geospatial Hub) and a USGS National Elevation Dataset 1-arcsecond DEM (~10 m cell size). Using this data, I digitized the Pinedale moraines and trimlines on the northern side of Sinks and North Fork canyons. The DEM elevations along the digitized line provide a longitudinal elevation profile of the maximum Pinedale ice extent. Less field data was available for North Fork Canyon, where the primary data source is radiometrically age-dated boulders from Dahms et al. (2018) and a secondary source from Dahms’ unpublished field maps (2019, personal communication). The controls for the Pinedale ice limit and extent in North Fork canyon ice relied heavily on the aerial photography and DEM interpretation to digitize the ice extent lines, again using the northern side of the valley. For both sites, the north side ice features were used as the primary source of the maximum ice extent with the assumption that the ice surface was approximately planar across the valley.
Model Inputs

GlaRe, available as an ArgcGIS toolbox, is a series of Python scripts which provide a semi-automated method for generating glacier reconstructions (Pellitero et al. 2016). The model is globally accessible and available from the author. Such availability allows for input from a variety of backgrounds, thereby enabling a combination of model software development and expert insight. By utilizing a numerical approach, GlaRe works by using a minimal amount of morphological evidence, which for many glaciated valleys is advantageous. GlaRe requires only a DEM, minimal moraine evidence, a created flowline, and an indication of maximum ice extent (Figure 3). The GlaRe toolbox contains five sets of tools with the third being the Glacier Reconstruction toolset. This
toolset is used to calculate the ice thickness in each valley. The toolbox contains a sequence of tools to condition/setup the input data, run the ice thickness tool, and produce a GIS output based off the inputs into the model.

**Figure 3:** Sinks and North Fork Canyon with all required inputs of the GlaRe toolbox: glacial extent, flowline, and moraine limit which represents the extent of glacial flow. Contour lines are also included to establish the dimensions of the canyons.

The flowline for Sinks and North Fork canyons were constructed with the aid of the hydrology toolbox in ArcGIS. I edited the automatically generated flowline to smooth out sharp curves with the goal of keeping the flowline orthogonal to the valley. One of the preprocessing steps within the toolbox divided the centerline into 150-meter segments and created point features at each 150 m node (starting at the downstream end). The preprocessing tool also extracts the valley elevations at each point and adds the values as...
attributes. The ice extent was established from previous geomorphic mapping of moraine ridges and down valley ice limits (Dahms et al. 2018; Dahms 2004; Fabel et al. 2004; Züst et al. 2014). The valley glaciers in Sinks Canyon extended out to the Sinks and left a now-eroded terminal moraine at ~1990m (Dahms 2004; Züst et al. 2014). The Pinedale terminal moraine in North Fork Canyon is located near the canyon mouth at the Pine Bar Ranch locality (Dahms et al. 2018). The Ice Thickness tool within the GlaRe toolset was run on these parameters using the default setting for shear stress (100kPa). The output from the initial GlaRe model run was used as the default value run for shear stress and ice thickness analysis (Table 1). Each node, indicated by the FID number, has a calculated Ice Height and resulting Ice Thickness based on the shear stress input value and the Raster (DEM) value at each point. The distance represents the space between each node and is adjustable.

Table 1: Attribute output table for Sinks Canyon after the default run of 100kPa from the GlaRe tool: Ice Thickness Calculation.

<table>
<thead>
<tr>
<th>FID</th>
<th>Raster Value (m)</th>
<th>Distance (m)</th>
<th>Shear Stress (Pa)</th>
<th>Ice Height (m)</th>
<th>Ice Thickness (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>109</td>
<td>1941.75</td>
<td>150</td>
<td>100000</td>
<td>1941.75</td>
<td>0</td>
</tr>
<tr>
<td>110</td>
<td>1954.47</td>
<td>150</td>
<td>100000</td>
<td>2006.75</td>
<td>52</td>
</tr>
<tr>
<td>111</td>
<td>1966.03</td>
<td>150</td>
<td>100000</td>
<td>2034.82</td>
<td>69</td>
</tr>
</tbody>
</table>
“Default” Shear Stress Modeling

Basal shear stress is calculated from ice density, acceleration due to gravity, ice thickness and the surface slope of the ice. This stress plays a central role in the movement of a glacier. Basal shear stress is the result of the stresses accumulating at the base of the glacier mass which produces slippage of the ice and subsequent movement of the glacier (Benn and Evans 2010). Nye (1952) and Schilling and Hollin (1981) established a description for effective basal shear stress which is still widely accepted today. Effective basal stress is equal to strength at failure of 100 kPa as determined both theoretically and empirically (Locke 1995). This simplification of the natural environment is necessary for computer models to run effectively. Locke (1995), as with many other glaciologists, supports the development of a theoretical glacier profile model to increase our understanding of landform and glacier evolution through time.

The GlaRe model utilizes the principals of perfect plasticity rheology to establish itself in dynamic equations for glacier ice thickness prediction. Perfect plasticity assumes the glacier is in steady state, which implies that it is in equilibrium with such factors as temperature and accumulation. The theoretical assumptions of a steady state glacier result in major simplification of glacier dynamics as the basal shear stress is equated to a constant yield stress and the longitudinal stress gradients are ignored (Li et al. 2012). The theory works well for ice sheets and simple (little to no slope) glaciers. The glaciers model interpretation deteriorates when this theory is applied to valley glaciers. When bedrock slopes vary from $\alpha = 0.1$ to $\alpha = 0.5$ the SIA method is affected and validity of the method comes into question as the slope of the bedrock is seen to account for
discrepancies in the stress equations (Le Meur et al. 2004). By not accounting for the lateral drag (an influential factor in valley glaciers), the SIA flow is not held back which leads to a faster flow prediction and a smaller average cross-sectional area, when considering 3-dimensional models (Le Meur et al. 2004). Within 2D simulations the slope is the most stringent criterion for the applicability of the SIA method (Le Meur et al. 2004). The GlaRe model is particularly adapted to suite cirque and valley glaciers as it is based on perfect plasticity rheology for simplification of glacial dynamic equations. The GlaRe model (Pellitero et al. 2016) includes and extensively tests the f-factor because it is this parameter that accounts for the cross-sectional area and lateral drag of the glacier. Pellitero et al.’s (2016) validation of the f-factor parameter allows me to look at the shear stress parameter to further understand its influence on all resulting outputs generated by the model.

To get at ice thickness, GlaRe uses basal shear stress as one of its primary inputs the other being the shape factor (f-factor). No adjustment for the shape factor was introduced because the aim of this study is to isolate the shear stress parameter and to observe its overall effects on the validity of the GlaRe model. Pellitero et al. (2016) explicitly investigated the effects of f-factor but did not isolate the influence of shear stress on all the resulting tools. Pellitero et al. (2016, 78) notes that “The model requires the glacier basal shear stress as a primary input because this parameter exerts a first-order control on the output of glacier 3D surfaces”. The shear stress used in the model is a critical factor because the dynamics of the model are entirely controlled by the introduced stress. The default shear stress value suggested by a number of studies is 100kPa
(Pellittero et al. 2016; Rea and Evans 2007). This value is applied when there is a lack of evidence or no expert knowledge because glaciers typically are of the order of 100kPa (Nye 1952; Paterson 1994b; Schilling and Hollin 1981). GlaRe was run using the 100kPa default value to assess its effectiveness as a predictive reconstructive model.

*Table 2: Table of the variables and equation components utilized within the GlaRe model.*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equation Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_i$</td>
<td>Ice Thickness at some Initial Step (m)</td>
</tr>
<tr>
<td>$H$</td>
<td>Glacier Thickness (m)</td>
</tr>
<tr>
<td>$h_i$</td>
<td>Ice Surface Elevation (m)</td>
</tr>
<tr>
<td>$h_{i+1}$</td>
<td>Ice Surface Elevation at next upstream step (m)</td>
</tr>
<tr>
<td>$B_i$</td>
<td>Elevation of the Bed (m)</td>
</tr>
<tr>
<td>$B_{i+1}$</td>
<td>Bed Surface Elevation at next upstream step (m)</td>
</tr>
</tbody>
</table>

While there is lack of reliable published data on analyzing the nature of glacier flow, mainly because the data on the flow conditions within and beneath glaciers are exceptionally difficult to obtain (Carr, Lukas, and Mills 2010), equations work to facilitate models in reconstructing ice masses. To calculate ice thickness GlaRe utilizes a series of equations based on those developed by Benn and Hulton (2010). The Excel-based ‘Profilier’ created by Benn and Hulton (2010) was built on the assumption that ice deforms in response to the driving stress ($\tau_D$), which is a result of weight and the surface gradient of ice when a specified yield stress ($\tau_Y$) is reached (Table 2). The full relationship of driving to yield stress as well as development of basal shear stress and f-
factor equations are found in Benn and Hulton (2010). Since mountainous regions have varying bed slopes, Benn and Hulton (2010) found it necessary to calculate glacier surface in a succession of discrete steps. The equation being rewritten so that the ice surface elevation is calculated in steps moving upstream, where \( i \) is current step and \( i + 1 \) is the next upstream (up glacier) step:

\[
\text{Equation 1}
\]

\[
h_{i+1} = h_i + \left(\frac{\tau_y}{H}\right)_i \frac{\Delta x}{\rho g}
\]

where \( H \) equals the ice thickness (ice elevation \( h_i \) minus the elevation of the bed \( B_i \)), \( \Delta x \) is the Euclidean distance between steps, \( \rho \) is the density of ice (900 kg m\(^3\)), and \( g \) is the acceleration due to gravity (9.81 m s\(^2\)). This calculation is done iteratively starting at the terminus of the glacier and working up glacier to the user-defined end. The above equation was first introduced by Shilling and Hollin (1981) and Van der Veen (1999) later manipulated the equation to represent the interval between step \( i \) and \( i + 1 \) \((i + \frac{1}{2})\).

The adjustment of the equation was necessary to curb risk that errors would propagate upstream, meaning the reconstructed ice surface could represent an artefact of the numerical method. Thus, the values are calculated for the mid-point of the interval \( i \) to \( i + 1 \), rather than one end of the interval. This yields the equation:

\[
\text{Equation 2}
\]

\[
h_{i+1}^2 - h_{i+1}(B_i + B_{i+1}) + h_i(B_{i+1} - H_i) - \frac{2\Delta x \tau_y}{\rho g} = 0
\]
where \( \overline{\tau_Y} \) indicates that the yield stress represents the average for each interval. These equations are utilized within GlaRe. The model implemented within GlaRe then produces an equilibrium profile of a valley glacier in 2-dimensions, along the valley centerline (Pellitero et al. 2016). This is derived mathematically where \( h \) is ice surface elevation, \( \tau_{av} \) is basal shear stress, \( F_i \) is a shape factor, \( \rho \) is ice density, \( g \) is the acceleration due to gravity, \( \Delta x \) is step length, \( H_i \) is ice thickness and \( i \) refers to the iteration (step) number.

\[
\text{Equation 3} \\
h_{i+1} = h_i + \frac{\tau_{av} \Delta x}{F_i \rho g H_i}
\]

This numerical approach was written particularly for cirque and valley glaciers. The user is given the ability to adjust the input parameters: the basal shear stress (\( \tau \)), the shape factor (f-factor) (\( F \)), and the interpolation procedure (Pellitero et al. 2016). Looking more in depth at the equation; it is generated from the well-known Nye (1952) formula for the calculation of shear stress at the base of a glacier:

\[
\text{Equation 4} \\
\tau = \rho g H s
\]

where \( \tau \) is the basal shear stress and \( s \) is the ice surface slope. This standard equation was first used to establish valley shear stress for the Pinedale glacier in both Sinks and North Fork canyons. The values were calculated at the nodes created from the ice thickness tool as described above. The calculated basic shear stress values were then used to compare
between the default (100kPa) model run and the adjusted (user interpreted) run of the GlaRe model. Equation (2) is a standard quadratic equation (Benn and Hulton 2010) and as such can be solved by:

\[
\text{Equation 5} \\
y^2 + by + c = 0
\]

All the values required for the variables within the quadratic equation can be found in equations 8-10 of Benn and Hulton (2010). When solving the quadratic equation with variables it yields:

\[
\text{Equation 6} \\
y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

where the ice surface elevation at each step is found by inserting the variable a, b and c into the solved quadratic and then iterating up-glacier along the ice flowline. Since the equation is quadratic and two solutions will be found, it is directed that the solution where \( h > B \) should be used as it shows the physical reality of the reconstructed landscape. The other solution, therefore, can be ignored. Building off of Equation (5) new variables were created for the quadratic solution derived by Benn and Hulton (2010). These new variables allowed further manipulation of the glacier surface equation found within the GlaRe model.
Equation 7
\[ b = -(B_i + B_{i+1}) \]

Equation 8
\[ c = h_i (B_{i+1} - (h_i - B_i)) - \left( \frac{2d\tau}{\rho g} \right) \]

Equation 9
\[ \frac{(-b \pm \sqrt{b^2 - 4c})}{2} = h_{i+1} \]

Model Manipulation

I hypothesized that the default shear stress (100kpa) was not going to match the mapped ice extent, because some of the ice surface slopes and valley slopes suggested that basal shear stress would likely be higher and lower than 100kPa. For reference, I calculated the approximate shear stress of the ice, at maximum ice extent, using the basic shear stress equation (Equation 4 as \( \tau = \rho g h s \)). The elevations of the digitized ice extent line are the basis for the calculation of \( h \) and the change in ice extent elevation between points over the sample distance (150 m) produced the ice surface slope \( (s) \). The density of ice and acceleration of gravity are the same as above. To connect the centerline points with the appropriate known ice extent elevations, I created valley spanning cross-section lines at each sample node that were orthogonal to the valley flowline and where the cross-section intersected the ice extent the DEM elevation was added to the sample point as an additional attribute.
The equations included in the GlaRe model are complex when compared with the basic shear stress equation. I decided to test what shear stresses would need to be input into GlaRe to achieve ice thicknesses that would match the mapped ice extents. To do this, I inverted Eqs. 7-9 to solve for basal shear stress ($\tau$):

**Equation 10**

$$\tau = \frac{g\rho h_i^2 - g\rho h_{i+1}^2 - g\rho h_{i+1}B_{i+1} + g\rho h_iB_{i+1} - g\rho h_{i+1}B_i + g\rho h_iB_i}{2d}$$

The result of the equation is a spatial variable shear stress along the flowline. In effect, the new values of shear stress should match the moraine evidence in the valley since they were calculated from the valleys morphostratigraphic features. Utilizing the GlaRe preprocessing tools, the newly derived model shear stress was incorporated, and the *Ice Thickness* tool was re-run to calculate ice thicknesses that are closer to the mapped elevations (Table 3).
Table 3: Attribute table for Sinks Canyon representing the initial 100kPa default model run and then the subsequent back-calculated model run with variable shear stresses. Slope and distance are included as these elements allowed for the calculation of shear stress.

<table>
<thead>
<tr>
<th>Raster Value (m)</th>
<th>Dis. (m)</th>
<th>Shear Stress (Pa)</th>
<th>Ice Height (m)</th>
<th>Ice Thick (m)</th>
<th>DEM (m)</th>
<th>Ice change over Distance (150 m)</th>
<th>Slope of Ice (°)</th>
<th>DEM Shear (Pa)</th>
<th>DEM Ice Thick (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1941.75</td>
<td>150</td>
<td>100000</td>
<td>1941.75</td>
<td>0</td>
<td>1972.1</td>
<td>8.41</td>
<td>0.06</td>
<td>15023</td>
<td>0</td>
</tr>
<tr>
<td>1954.47</td>
<td>150</td>
<td>100000</td>
<td>2006.75</td>
<td>52</td>
<td>1980.51</td>
<td>19.56</td>
<td>0.13</td>
<td>29975</td>
<td>30.3</td>
</tr>
<tr>
<td>1966.03</td>
<td>150</td>
<td>100000</td>
<td>2034.82</td>
<td>69</td>
<td>2000.07</td>
<td>19.37</td>
<td>0.13</td>
<td>38812</td>
<td>26.0</td>
</tr>
</tbody>
</table>

Model Comparisons

One way to validate is by comparing model outputs to observations (Hutton 2012). The testing of models uncovers errors made in the calibration of the model or else help to validate further the accuracy of the model results. Another common way to begin the validation process is by setting up parameters in a model in such a way that the model is constrained to duplicate a situation for which there is an analytical solution. This form of validation has been used in understanding other ice sheet numerical models (Hooke 2005). However, this has not been seriously attempted for valley glaciers which require their own set of specific parameterizations and evolved mathematical interpretations of shear stress and bed slope. This is especially true when applied to basal shear stress where little is known of its spatial and temporal distribution (Brædstrup et al. 2016). Comparisons of ice thickness and shear stress were primarily done in one-dimension.
along the valley centerline. The goal of these comparisons is to evaluate the correctness of the GlaRe outputs (modelled ice thickness versus mapped ice extent) and to explore the effects of a spatially variable shear stress. Because glaciers are more than one-dimensional features on the landscape, I also calculated the surface areas and 3D volumes of the different ice surfaces to compare how the full extents of the different calculations compare to each other.
CHAPTER 4

RESULTS

Comparison and Validation

Ice Extent Mapping

Using the geomorphological mapping approach outlined above allowed me to reconstruct the valley ice heights derived from the modern digital elevation model (DEM) in Sinks and North Fork canyons (Figure 4). The features used to infer ice height were the moraine limits for the Pinedale glaciation marked in Figure 4. The height of the valley ice derived from morphological features, in this case lateral moraines, operated as the baseline for all other outputs generated from the GlaRe model.
“Default” Shear Stress Modelling

After the establishment of valley ice heights, the GlaRe model ran using the default basal shear stress of 100kPa. The inputs are shown in the top panel of Figure 4 being the flowline, DEM surface and glaciation extent. The 100kPa glacier model run showed some striking differences in predicted ice thicknesses and mapped ice extent.
(Figure 5). The default shear stress model both over and under-predicted the ice thickness in both canyons. The default run in Sinks canyon began by overpredicting the ice thickness from 0 (the terminus) to 4500 m into the canyon. The error then switches to under-prediction from 4650 m to 14400 m into the canyon and the remainder of the canyon, back to 16200 m, is over-predicted. In North Fork canyon the default also begins by overpredicting the ice thickness from the terminus up to 2250 m. Beginning at 2400 m and continuing back into the canyon to 16950 m the default then under-predicts the ice thickness in the canyon. Some error is to be expected, since a model does not perfectly adhere to all the environmental constraints found in the natural world; however, the extent of mis-interpretation resulting from the 100kPa model run begs the question of a default values predictive ability and how this quantity used for shear stress affects the overall effectiveness of the GlaRe reconstruction. Subtracting the default model run from the mapped ice heights permits observation of the difference in ice thicknesses (bottom Figure 5).
Model Manipulation

To further test the model Equation 4 was used to calculate the approximate shear stresses based on the mapped ice extent (Figure 6). This confirmed my hypothesis that the 100kPa shear values were insufficient to describe the variability in shear stresses. As the ice surface changes rapidly, there are abrupt changes in the shear stress values. The first example of this in Sinks canyon is seen 1500 m along the valley when the ice height goes from 2095 to 2129 m which results from an equation-based shear stress of 183592 Pa. This drastic change is hard for the default to follow as it occurs rapidly with a

Figure 5: Top: The default (100kPa) shear stress as viewed over the valley. The valley ice height and bed height are also included as references. Bottom: The difference in predicted ice height between valley ice extent and 100kPa default model run.
significant rise in ice height. The same can be said of North Fork canyon whose first rapid shift occurs at 2400 m along the valley. The ice thickness changes from 2086 m to 2147 m with an exceptional large mathematical derived shear stress.

![Valley Shear Stress and Ice Prediction](image)

**Figure 6: Valley calculated basal shear stress is shown in light gray with the ice height of the Pinedale glaciation and bed height overlain.**

As we would expect by using the back-calculated shear stress values from Equation 10, the output values from GlaRe’s *Ice Thickness* tool are in better agreement with the mapped ice extent (Figure 7). The differences in predicted ice height are minor; the back-calculated ice thicknesses are within 5 meters of the mapped ice extent 44% of the area in Sinks canyon and 48% in North Fork canyon. The ice thickness difference between calculated and mapped ice height rose above 15 meters 11% of the total predicted heights in Sinks canyon and 13% in North Fork canyon.
The back-calculated shear stress values also vary from the basic shear stress values (Equation 4) (Figure 8). Shear stress in model and valley follow closely together but seem to deviate when there are steep slopes. Sinks Canyon exhibits a combination of negative and positive differences but more commonly has higher values around 40,000 Pa showing a tendency towards underestimation. North Fork has almost entirely negative values averaging at 20,000 Pa of difference which demonstrates overestimation of the valley shear stress.

Figure 7: Top: Ice prediction comparison between the valley moraine derived ice extent, 100kPa GlaRe model run and the recalculated GlaRe variable shear stress model run. Bottom: The difference between the valley calculated ice height based off morphological evidence and the recalculated GlaRe model ice height.

The back-calculated shear stress values also vary from the basic shear stress values (Equation 4) (Figure 8). Shear stress in model and valley follow closely together but seem to deviate when there are steep slopes. Sinks Canyon exhibits a combination of negative and positive differences but more commonly has higher values around 40,000 Pa showing a tendency towards underestimation. North Fork has almost entirely negative values averaging at 20,000 Pa of difference which demonstrates overestimation of the valley shear stress.
Comparing the ice thickness calculations for the default and back-calculated runs (Figure 9) demonstrates the deviation from realistic ice surface values in the 100kPa default outputs (Figure 9). The default run for Sinks canyon exhibits a maximum difference of -98.56 meters and has an average overestimation of ice of 44.6 m and underestimation of 34.1 m. In Sinks canyon the deviation from over to under prediction of ice thickness occurs at 4500 m (4.5 km) distance along the valley. The default run shifts to underpredicting in the middle and upper reaches for both of the canyons. In North Fork canyon the maximum difference in ice thickness between the back-calculated
values and the default run is 191.68 m. The middle and upper portion of the canyon have the most severe error in ice prediction with the overestimation averaging at 34.6m and the underestimation at 111.9 m. In North Fork canyon the deviation from over-to-under prediction of ice height occurs at 2250 m (2.25 km) distance along the valley.

Figure 9: Top: Comparison of ice thickness between valley ice heights, back-calculated GlaRe ice heights and the 100kPa heights. Bottom: The difference between the GlaRe model runs of back-calculated values and those from the 100kPa run.
Model Comparison

The longitudinal variations between the model runs illustrate that there is a significant difference between the different shear stress methods. Extending the modelled ice surfaces to the valley walls allows us to see how these differences would be expressed in 3-dimensional terms of surface area and ice volume (Figure 10 and Table 4). Sinks and North Fork canyons, when using the default (100kPa) model run, both show underestimation of the total surface area of ice extent and total ice volume. North Fork Canyon has more significant under-prediction of ice extent and volume as seen in the area difference between the default and back-calculated values. The change in area between the different model runs produced a 10.5 km² surface area difference and a 2.8 km³ volume difference for North Fork. The difference in surface area for Sinks Canyon is calculated as 4.1 km² with a volume difference of 0.98 km³.

Table 4: Volume calculations for Sinks and North Fork canyons. The 2-Dimensional measurement is included for scale when interpreting the total ice volume within the canyons.

<table>
<thead>
<tr>
<th>Volume Calculations</th>
<th>Surface Area (km²)</th>
<th>Volume (km³)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100kPa</td>
<td>Modelled</td>
</tr>
<tr>
<td>Sinks Canyon</td>
<td>40.609</td>
<td>44.680</td>
</tr>
<tr>
<td>North Fork Canyon</td>
<td>21.458</td>
<td>32.006</td>
</tr>
</tbody>
</table>
Figure 10: 3-Dimensional perspective views showing ice thickness for the default (100kPa) run (first and third panel) and the back-calculated variable shear stress run (second and forth panel).
CHAPTER 5
DISCUSSION

Model Performance

Glacier Parameters within the Model

With the use of recent and historic remote sensing techniques there is hope to establish accurate checks for the reconstructions of paleo-ice masses through the use of computer models. The tools available to compute past and future glacier change(s) range from the simple models, (Benn and Hulton 2010; Benn and Gemmell 1997), to more complex — geometry-aware — models, such as OGGM (Maussion et al. 2018) and GlaRe. Having been tested on two geomorphically dissimilar extant glaciers by Pellitero et al. (2016) and by this study, GlaRe is found to model accurately when constrained by the bed topography. These tests are extremely important to validate the model and ensure accurate representations of data when modelling glacial activity. These models can be used as a tool to facilitate insight into our observations when they are validated by analytical solutions and the field evidence.

Model Manipulation

The GlaRe model run which utilized spatial variability best predicted the natural field observations. A default value for basal shear stress produces a large error in the predicted ice thickness values. Obviously, the quality of the output is directly related to the quality of the input parameters when considering modelled reconstructions. The GlaRe model requires the basal shear stress to be the primary input since this parameter exerts a first-order control on the output 3-dimensional surface. The ideal situation, as
stated in Pellitero et al. (2016), is to initially reconstruct the ice surface using the standard value (100kPa) and then tune $\tau$ to fit the geomorphological constraints present in the valley. These natural constraints limit the ice height error by limiting the construction of the ice surface to the mapped moraines, crests, and trimlines. If the use of a default value cannot be overcome, then the tests will produce errors ranging from 10-to-100’s of meters (as shown in the results above), depending on the valley slope and shape. The model was found to have weaknesses when using only a default or singular value for the basal shear stress. Le Meur et al. (2004) predicted that the slope should be an influential factor when employing SIA methods. Benn and Hulton (2010) also noted that a steep backwall or an abrupt change in valley slope tends to create misrepresentation as the model does not intercept the backwall (as do real glaciers). The immensity of the underestimation may be due, in part, to the challenges of predicting stepped or variable slope changes in ice surfaces. The default value run, as shown in the 2-D results, is inhibited when it comes to predicting drastic slope changes. The volume further proves the default model runs limitations in regard to variable ice prediction and increases the need for ground evidence to establish the model in the glaciated landscape.

Models are assumptions and simplifications, so some distortion is allowable. The distortions and deviations from valley morphology in this study are most striking when running the default of 100 kPa. Using a different reference basal shear stress may reduce the error margin produced by the use of a constant basal shear stress value. Other studies (Züst et al. 2014), have used multiple constant basal shear stress values to reduce error through averaging. The default value of 100kPa propagates enough error which appears
to interfere with the reconstructions of glaciated landscapes. When comparing the valleys of Sinks and North Fork canyon to the model default estimation of ice surface we notice underestimation of ice surface beginning with steep slopes along the predicted ice profiles. As the model further predicts, each up-valley step or slope change appears to further propagate the under-estimation of ice surface. This pattern of error in ice-thickness variation is attributed to drastic bed surface slope changes. Thus, the default value for basal shear stress, when set to 100kPa, is not an accurate predictor of ice height in these valleys.

**Model Comparison**

When the ice thickness values are applied to the full valleys the volumes are also severely under-predicted, as shown in Table 4. These analyses show that expert knowledge is critical when defining the parameters of this model. The method of back-calculation to achieve ice thickness values mimicking those represented by the moraines in each valley shows that by tuning the basal shear stress the model can produces realistic results. However, without the constricting constraints of the user it is evident from the default value that the model would both over- or under-predict ice thickness and volume. Certain geomorphological features, such as a steep headwall or a rapid change in bed slope, seem to initiate error propagation throughout the rest of the interpretation. The GlaRe model, therefore, is most ideal in situations where there is observable evidence of ice extent (terminal and lateral moraines) which can be used to constrain the predictions of the Glare reconstruction. Otherwise a certain amount of error will be associated with the result since it was not rooted in field evidence or expert knowledge.
CHAPTER 6

CONCLUSION

The use of topographic control points for ice extent and ice height were used to establish a constraint upon the GlaRe model in order to reconstruct the volumes and extents of two LGM valley glaciers in the Wind River Range of Wyoming. The ability of the GlaRe model to recreate ice volumes to the moraine crests in each valley is relatively good. These results were established by the use of back-calculated model equations for shear stress derived from the previous models of Benn and Hulton (2010) and Pellitero et al. (2016). The accuracy of the GlaRe model diminishes when the value for shear stress is the default value of 100kPa. This study shows that a shear stress value of 100kPa does not allow the model to reconstruct a valley glacier accurately. Spatial variability of the shear stress is an essential parameter that is needed in order to accurately model the paleo-ice surfaces of these valleys. This demonstrates the need for field validation in order to avoid misinterpretation of model results which accumulate when reconstruction models are not tested. I suggest that future users of the GlaRe model are explicit in their methods and detailed in how they set up parameters within the model. Further investigation into the dynamics of shear stress are still needed to answer questions about how we should model ice. One possible avenue is to add variability to the default model runs based on the changes in slope of a valley floor. Similar to the problems encountered in calculations of river shear, the slope of the valley floor may act as a good predictor for the average shear stress, although this would pose a problem when considering multiple glaciations.
More and more, models are being used to predict, interpret and define past, present and future environments. The scientific community needs to guard against any misinterpretations which untested models may produce. Validation and testing of any particular model do not make it infallible and its usefulness should be seen in that light. To establish valid estimates of shear stress and other important features of ice dynamics we must continue to test and validate our models against field evidence, so that reliable predictions result from computer model simulations.
REFERENCES


