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The study of computational fluency

Cassandra Hart
University of Northern Iowa

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Abstract
Teachers, educators and researchers are still attempting to reach consensus on the kinds and amounts of computational fluency that are relevant for today's classroom. But one thing is certain, computational fluency is one of the most vital components of developing mathematical understanding. It is absolutely necessary that students have knowledge of basic number relationships and the ability to choose from a number of appropriate strategies. The question to be answered in this study is how can Computational Fluency strategies enhance student achievement in math and improve the current climate for learning? This study evaluated the implementation of the Computational Fluency Curriculum in the Waterloo Community School District.

The Study of Computational Fluency

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Cassandra Hart

University of Northern Iowa

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John E. Henning

Director of Paper

Victoria L. Robinson

Reader of Paper

John E. Henning

Advisor

Michael D. Waggoner

Department Head

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Introduction

Teachers, educators and researchers are still attempting to reach consensus on the kinds and amounts of computational fluency that are relevant for today's classroom. But one thing is certain, computational fluency is one of the most vital components of developing mathematical understanding. It is absolutely necessary that students have knowledge of basic number relationships and the ability to choose from a number of appropriate strategies. The question to be answered in this study is how can Computational Fluency strategies enhance student achievement in math and improve the current climate for learning? This study evaluated the implementation of the Computational Fluency Curriculum in the Waterloo Community School District

Mathematics learning is about making sense of mathematical ideas and acquiring skills and insights to solve problems (NCTM, 2000). The development of computational fluency is essential to the notion of mathematics learning mentioned above. Computational fluency involves accuracy, efficiency, generalization, and application of knowledge when solving problems (Wickett, 2003). What does it mean to be computationally fluent? Examining the implementation of instructional strategies in everyday math and learning more about the related elements of computational fluency will help us determine the best effective instructional strategies that can be used. "Mathematical learning is both about making ideas and about acquiring skills and insights to solve problems" (NCTM, 2000).

Significance

The Computational Fluency curriculum has been an important factor in the growth of instructional math strategies. This curriculum has increased student achievement and provided a better climate for learning. Computational fluency is instructionally efficient and focuses on helping students develop computational strategies. Computational fluency cognitively engages students and provides structured opportunities for problem solving. It is a competent way to learn basic number combinations.

The success and the importance of this curriculum, is determined by developing thinking strategies and opportunities for the students to try. The mini-lessons make it difficult for students to successfully use their informal more familiar strategies, thus offering some natural incentive for students to use other strategies (CFP 2004). Computational fluency encourages students to employ their sense-making and embrace challenge. Most students over time find these opportunities enjoyable and motivating. It also communicates positive beliefs and attitudes towards mathematics. This approach communicates to students that mathematics is more about solving problems and modeling situations than about memorizing meaningless facts and algorithms. This will lead more students to like math and eventually pursue more advanced math topics. One of the goals is to help children develop fluency with computation. Russell states that "Developing fluency requires a balance and connection between conceptual understanding and computational proficiency" (Russell, 2000).

Teaching the daily mini lessons from the Computation Curriculum will increase computation skills through out the year. Some of the lessons from the regular math curriculum may not be required. Reducing the time spent on computation lessons in the regular math class period will allow more instructional time to devote to other challenging mathematical topics. Also, the development of number sense and computational fluency should be an integral part of the mathematics. Other subjects in math such as data and measurement depend on these skills. Students must have a firm foundation in number sense.

Limitations

The limitations in this study include three first grade classrooms and two second grade classrooms. Only the second grade students were examined on during this study. Two first grade and one second grade teacher were interviewed. Each class had a variety of students with different types of needs. Only selected schools whose students scored below the 40^{th} percentile were offered this opportunity in math computation. Not every school in the district is following this curriculum. When new students entered the classroom they were often behind because there was only one cohort group.

Each teacher had to attend training for a week and meet twice a month during the school year. The only people who could teach the lessons were the teachers who went to the training. If a teacher from the program was gone, then a substitute teacher could not teach the lesson. Another teacher from the program would have to teach the lesson. During this research, it was discovered that many of the strategies did not emerge from the students. As students move toward fluency with greater numbers we are expecting that many different strategies might emerge.

Literature Review

Introduction

This section will present instructional practices that promote the development of computational fluency for teachers. There are three important increments of Computational Fluency that will provide the theoretical framework for this study. The first increment describes understanding and teaching computational fluency. The second component describes the characteristics of Computational Fluency and its impact on assessment. The third increment focuses on the influences of problem solving and number sense, which will impact computational fluency.

Teaching Practices

The common strategies create a framework for children to examine number relationships as they share and discuss strategies. This section will also discuss knowledge that will assist teachers understand the children's thinking. Teachers have discussed several different kinds of subtraction and addition problems and analyzed different approaches. One of the challenges is teaching these strategies to student who may not attain them on their own. Instead, teachers carefully nourish the knowledge that underlies the strategies by exposing children to a series of games and activities that naturally promote development.

When teachers begin to think about the mathematical understanding that is required in creating these approaches, they become more comfortable thinking about number relationships, and they are better able to question and guide students. Teachers begin to appreciate other benefits of helping children develop these approaches, including the following: (a) provide valuable diagnostic information to teachers, such as recognizing sophisticated methods; (b) improve communication, including speaking, listening, and symbolizing; and (c) develop mental computation ability simultaneously with greater facility with written computation.

Teachers must listen carefully so they can understand their students thinking and evaluate whether that thinking is logical. Then, on the basis of this analysis teachers must formulate new questions or tasks to dispel misconceptions, deepen understanding, and move children to more efficient or sophisticated methods (Flowers, 2003). Preparing teacher's directly in ways of thinking that build computational fluency and then analyze, generalize and extend children's strategies helps teachers achieve many goals. They gain flexibility, deepen their understanding of our number system, become stronger problem-solvers, and begin to appreciate the benefits of developing computational fluency in students (Flowers, 2003). By working toward our own fluency with each of these strategies, we have a better understanding of what to expect from our students, what significant advances in reasoning to look and listen for, and what to highlight as we analyze and compare strategies (Huinker, 2003). These are reasons why this research is important and how it can benefit everyone. This study has found that students have increased achievement

Teaching for both skill and understanding are crucial and learned together, not separately. Teaching in a way that helps students develop both mathematical understanding and efficient procedures is complex. It requires that teachers understand the basic mathematical ideas and recognize opportunities in students' work to focus on these ideas. Understanding algorithms is central to developing computational fluency.

Teachers must combine a renewed appreciation of the contributions algorithm make to mathematical proficiency with a design of approaches to teaching and learning that can develop both understanding and skill (Bass, 2003).

As educators and parents our job is to facilitate students' movement along the scale of computational fluency to in a depth mathematical understanding. When planning to teach arithmetic, it is important to keep in mind computation, problem solving, and number sense. These components are not separate but fundamentally intertwined.

What is Computational Fluency?

The NCTM Principle and Standards of School Mathematics (2000) define computational fluency as having efficient, flexible and accurate methods for computing. It is important to note, however, that it does not say computing is the same as using paper and pencil methods. Students need to be fluent in mental math, paper and pencil methods and using technology such as a calculator in computing answers to situations involving whole numbers as well as fractions and decimals. In fact, one often overlooked or underdeveloped aspect of computational fluency is not only being able to compute in all three ways but also knowing which method is best based on the given task. In addition, students must be able to determine if an exact answer or a close approximation (estimate) is sufficient.

Another key idea is that it is not enough to just compute approximate answers. Students must be able to compute accurately in all three methods, such as problem solving, computation and number sense. Each student will need to know when to use the appropriate operation. Students need time to develop facility with numbers in order to solve problems and think with numbers (Postlewait, 2000). In other words, they must be able to solve problems that involve numbers. By working towards our own fluency with each of these strategies, we have a better understanding of what to expect for our students (Huinker, 2000). Research tells us there is a scale of strategies through which students develop computational fluency with basic facts and multi-digit numbers in all four operations. If a student has memorized without the opportunity to develop through the process, and then forgets the fact, he or she will have no way to solve the problem. Further, there is growing evidence that once students have memorized and practiced procedures without understanding, they have difficulty learning to bring meaning to their work (Hiebert, 1999).

Different Components of Computation

A student who is proficient with computation can calculate accurately and efficiently. Accuracy and efficiency doesn't mean that all students must compute in the same way or use the same procedure for all problems. Students reason in different ways, develop preferences for different approaches, and should learn that different methods are suitable for different numbers. Computing mentally is often hard to assess in school, perhaps because it doesn't produce written evidence, lead to homework assignments, or is measured on standardized tests. For students to become proficient with computation, they should be able to compute in their heads without resorting to paper and pencil. When problems are too complex or awkward to solve mentally, it's appropriate for students to figure answers using paper and pencils or manipulative.

This does not imply, however, that students must use the standard algorithm or any one specific procedure. Conforming to one particular method does not indicate proficiency. Rather, students should use paper and pencil to keep track of how they reason and also to provide a record of how they are thinking. "Being able to calculate in multiple ways means that one has transcended the formality of the algorithm and reached the essence of the numerical operations--the underlying mathematical ideas and principles (Russell, 1999). The reason that one problem can be solved in multiple ways is that mathematics does not consist of isolated rules, but connected ideas. Calculating mentally or with paper and pencil requires having basic number facts committed to memory. However, memorization should follow not lead, instruction that builds children's understanding. The emphasis of learning in mathematics must always be on thinking, reasoning, and making sense.

As the Standards for Grades 3-5 in *Principles and Standards* (NCTM, 2000) state: "Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently. The computational methods that a student uses should be based on mathematical ideas that the student understands well, including the structure of the base-ten number system, properties of multiplication and division [and, of course, of the other operations, as well], and number relationships" (p. 152).

Problem Solving

Being able to compute accurately and efficiently is certainly important. It's also essential that students learn to apply computation skills to problem-solving situations. When a problem calls for numerical calculations, students should be able to choose the correct operations needed, decide on the numbers to use, do the necessary calculations, and then appropriately interpret the results. While routine word problems, in which the situation is translated into an arithmetic operation or series of operations and then solved, have long been the standard in math instruction, students should also learn to solve non-routine problems. Non-routine problems require thinking and reasoning, not just translation, and may have more than one solution and more than one way to arrive at those solutions.

However, it is critical that students also learn to examine strategies--to analyze them, critique them, and find relationships among them. They need to develop the habit of explaining and justifying their solution strategies and the facility for analyzing incorrect procedures (Russell, 1999). The goal is not simply to come up with lots of strategies and compare them, but--over time--to identify and use general mathematical principles that lead to consistent, reliable solutions for a class of problems.

Number Sense

The development of number sense and computational fluency should be an integral part of the mathematics curriculum. Because other areas of the curriculum, such as data and measurement, are closely related to and sometimes dependent on these skills, students must have a firm foundation in number sense (Postlewait, 2003). Students with number sense have good numerical intuition. When applying computation skills to solve problems, they can judge if solutions are reasonable. Should they forget a procedure or part of a procedure, students with number sense can reason their way to a solution. They see numbers as tools, not barriers. They're curious about numbers and comfortable with them. Students with number sense can solve any routine math problem by calculating mentally.

Summary

Computation, problem solving, and number sense are not discrete and separate topics. Developing number sense depends on learning to compute. Solving problems also depends on computation. Deciding whether an answer makes sense, is an important aspect of problem solving that, draws on a student's number sense. Fluency means that students can flexibly choose computational strategies to solve problems, understand and explain their methods, and produce accurate answers efficiently. Thinking about probability gives students a chance to theorize about numbers. Analyzing statistical data calls for numerical reasoning and figuring out ways to make sense of unfamiliar computations calls for estimating. Students do need procedures to solve problems, but procedures alone are not enough. Both for their current work in the elementary grades and for future work in mathematics, students must learn about the structure of the base ten system and about the properties of operations as the foundation of computational fluency.

A flower stands up tall because it's bound by the stem, and held up by roots which have a strong foundation in the soil. The flower must receive all the proper care and nutrients to survive. A plan for arithmetic instruction must incorporate attention to each flower or child so that students have a firm and well-balanced base on which to rest their arithmetic understanding. My goal is for students to demonstrate competence and confidence in everything they do especially in math.

Methods

Introduction

The question that needs to be answered is: How can computational fluency strategies improve student achievement in math, as well as the current climate for learning? To find out this research question three methods of studies were viewed. The first method of study was an interview in which teachers shared their thoughts and beliefs about the new curriculum. It described aspects of a curriculum that was designed to develop computational fluency. The second method was classroom analysis, which concentrated on differentiated instruction by developing computational fluency through using a game format. The third method was a Standardized Achievement Analysis based on the Iowa Test of Basic Skills (ITBS) math data.

Setting

Based on the ITBS scores, the second grade students through out the district scored below the 40th percentile in math. Five schools' were selected to participate in a research based project to increase student achievement through computational fluency. This curriculum is taught in addition to the new math curriculum that the district has adapted. This project focuses on developing new thinking strategies, creating more instructional time for teachers by reducing the number of mathematics lessons taught during the year that are devoted to computation and provoking transitions to new thinking strategies.

Participants

Interviews

There are three participants for this study. During the first study, three teachers were interviewed, two first grade teachers and one second grade teacher. Teacher one is a first year teacher who is teaching second grade. She graduated from the University of Northern Iowa with a B.A. in Education. Teacher two is a first grade teacher, who has been teaching for over thirty years. She is married with two sons. She has a B.A. and a Masters Degree Teacher three has been teaching 31 years in the Waterloo Community School District. She is now teaching first grade. She is married with two sons. She has a B.A. + 45 hours towards a Masters Degree.

Classroom Assessment

During the second study, there were two classes with 21 students. Each class had students with a wide range of scores. The first class had three students who were ELL, and one student who was blind. They were also 3 Hispanics, 9 African Americans, and 9 Caucasians students. The second class had 12 Caucasians, 9 African Americans, and 1 other nationally. Each class had diverse learners. There were a lot of Bosnian students who were identified as ELL students. This means that English is their second language. Those students were in the second class room. We found that to use differentiated instruction, would meet our learning goal and the students' needs.

Standardized Achievement Tests

The third source of data was a standardized achievement test. It focused on second grade students from Lowell Elementary. This study examined the math data form the Iowa Test of Basic Skills Test. Every student from second grade through 12th grade takes the test in the district. This does includes special needs students also. We just focused on the students at Lowell elementary. There were forty-eight second graders who took the ITBS Test. This included eight special needs students.

INSTRUMENTS

Interviews

The first study reviewed an additional math curriculum, entitled Computational Fluency. This new curriculum was incorporated to increase computational fluency, by developing thinking strategies through five-minute mini lessons. Teachers were required to attend a weeklong training and had to meet twice a month for two and half hours. The purpose of these interviews was to discover teachers' attitudes toward this curriculum. It is also important to find out what concerns, questions and opinions teachers' have about the computational fluency curriculum, how students were assessed, what skills should students obtain, and does this curriculum develop students into life long learners? During the interview a list of seven questions were asked. Some of them were structured and others unstructured. (To see a list of the question refer to Appendix A). The first interview took place in the classroom.

Classroom Assessment

The purpose of this classroom analysis is to determine if Computational Fluency can be increased through using a variety of hands on games instead of just using paper and pencil. This analysis will also determine if students were able to use the different strategies that they were introduced to. For this study, learning goals and assessments were created. We used the Chapter 2 pre and posttest from the math curriculum. It was a performance test with 13 problems. The test was worth 15 points, and if the students scored 12 or higher then they were proficient.

Standardized Achievement Tests

This part of the analysis is based on the Iowa Test of Basic Skills. The Iowa Test of Basic Skills is a standardized achievement assessment that is given to grades second through twelfth. This is mandated by the Waterloo Community School District to meet the No Child Left Behind guidelines. This test was taken in the fall of 2004.

This data will show what percentage of the second grade students scored in problem solving and computation. This analysis may answer several questions. Have student achievement increased, decreased or stayed the same? Does a correlation exist between computation and problem solving? How can computational fluency strategies improve student achievement as well as the current climate for learning? This test is administered twice a year, once in the fall and in the spring.

PROCEDURES

Interview Procedures

Data from the first participants were collected during an interview with three teachers. We started around 12:05 in the afternoon on Monday September 27, 2004 and concluded at 12:30. The second interview took place on Tuesday September 28, 2004 in the morning before school started. It started around 8:00 and concluded at 8:30. The third interview took place in the teacher's classroom on Thursday September 30, 2004 in the afternoon during her planning time. It started at 2:35 and concluded at 3:00 p.m.

The purpose of this classroom analysis is to determine if differentiated instruction will increase computational fluency through using a game format. To accomplish this, three learning goals and assessments were created to provoke fluency with thinking strategies. An instructional plan was designed to implement each goal.

Classroom Assessment Procedures

For each learning goal the students were divided into two groups. The students who were proficient were the blue group. The students who scored close to proficiency were the purple group. The first learning goal was for students to able to count on and count back to find the sum or differences. To reach their goal, Blue group played a game called "Follow the Rule." Each student has an index card and draws a two-column function table with 4 or 5 rows. Students need to identify the rule at the top of the card. For each row the student writes the numbers on the left, applies the rule, and ends up with the number on the right. Students are to leave the right hand column blank and switch cards.

Purple group played a game called "Face Off." They divided a deck of cards evenly between two players. Students turn the top two cards over to find the sum or differences. The player with the greatest sum or smallest differences keeps all four cards. Students continued playing until they have turned over all the cards in the deck. The player with the most cards wins. Goal two, states students will be able to use doubles facts to find the sum and the differences.

Both groups played the game "Hit the target." Students used a deck of cards and chose one card. The number on that card is their target. They place the remaining cards face up on the playing surface. With their partner, they take turns looking for two cards that they could add or subtract to get the number on their target card. Students keep playing until they cannot find any more combinations that equal the target number. They then repeat the game with a different target number. The last goal is for students to be able to understand that an addition fact can be used to find the differences in a related subtraction fact.

The blue group had to create story problems and write the related number sentences. If their story problem was a subtraction problem, then they wrote a related addition fact (e.g., 13-6=7 7+6=13). The purple group wrote six subtraction problems on a sheet of colored paper. Then they wrote the related addition problems on another sheet of paper. For more advanced work the students would then write a problem like 9+1=6+4. When the students were finished they worked in pairs and played a matching game with their cards. (To see a list of the games refer to Appendix B.)

Standardized Achievement Procedures

The second grade test is done in a booklet. Everything is ready to them. Each ITBS test has a reading, math, language vocabulary, science and social studies. The students are given a certain amount of times to complete each section. The whole testing process takes one week to complete. Afterwards, the tests are analyzed, and the results are received in late January.

Results

Introduction

In the following sections are the findings from the three sources of data: interview, classroom analysis and standardized achievement analysis. This section also includes the interview findings explains the teacher's viewpoint of the computational curriculum. The classroom findings describe instructional strategies that can help improve meaningful mastery of number combinations. These findings promote the development of computational fluency by provoking thinking strategies and using math games through differentiated instruction. The standardized test findings determine if there is a correlation between problem solving and computation.

Interview Findings

Each teacher was asked seven structured and unstructured questions about the additional math curriculum. The Computational Fluency curriculum consists of 150 mini-lessons that are to be taught in five minutes. Their response is presented in this section. During the interviews the teachers were asked how they felt about the curriculum. They responded by saying, how they are able to see the mini lessons increase computational fluency, by improving the students thinking strategies. The teacher's feel this approach is an efficient way to learn basic numbers combinations, because it focuses on students developing strategies that can be used to solve a wide-range of computational problems.

All three of the teacher's felt that five minutes was not enough time to teach the concepts; they often spend more than the five minutes. Another big concern is that substitute teachers are not allowed to teach the mini-lessons. Only the teachers who received the Computational Fluency training are allowed to teach the curriculum. If a teacher is absent, then one of the trained teachers is required to come into the classroom to teach the lesson. The teachers were not very happy with that, due to scheduling conflicts. They felt the substitute was more than qualified to teach the lesson.

The teacher's expressed concerned with the pacing of the regular math curriculum and the additional Computational Fluency curriculum as a whole. The regular math curriculum started with chapter one and two, reviewing basic addition then subtraction. The Computational Fluency curriculum started with chapter three, which focused on Place Value. Students need to be reinforced on addition and subtraction skills at the beginning of the year, not place value, which is taught in the middle of the year. There were also concerns about teaching certain concepts in the correct order.

Teacher one and two are enjoying the curriculum. They are only able to see the benefits of the curriculum. Teacher three feels there is a lack of direct instruction for the lowest student and sees this as a problem. She also feels that their needs to be an overview of the curriculum, because there isn't a scope and sequence to teach the different objectives. One could look ahead and see what skills are coming up, but teachers really need an objective to teach each mini lesson. Overall, the teachers are receiving positive feedback from their students. They are still concerned about the lower achieving students compared to the high achieving students.

Classroom Assessment Findings

The purpose of this classroom analysis is to determine if computational fluency can increase student achievement through using a variety of games and differentiated instruction. There are two classes with 21 students. Each class has students with a wide range of scores. In order to be proficient the students need to score 80% or higher. Based on the data from the chapter two pretest, twelve students scored 80% or above, nine students scored in the mid range, between 60%-70%, and twenty-one students scored below 50%.

The classrooms were divided into groups based on the needs of the students. Classroom A had students who scored between 60% or higher, and classroom B had students who scored below 50%. Out of the 42 students, four of them are ELL students and one blinded student. Nine out of the twenty-one students did not score 80% or above on their pretest, but were really close. Classroom A was divided into two groups, which were blue group and purple group.

The data showed that the students who were in Classroom A were proficient and able to perform computations in multiple ways, including mental calculations, estimation, and the adaptation of a variety of appropriate strategies to solve problems. Purple group were very close to proficiency, but need additional strategies to use. This particular group worked on hands on games that allowed them to use different strategy. Some of the games focused on a certain strategy. After reviewing the concepts, and implementing the games, they were able to pass the assessments for that strategy. These students benefited from the differentiated instruction and using games to enhance their strategies. Within a three week time period, these nine students improved 85% or higher on their posttest.

Standardized Achievement Test Findings

This analysis in computation and problem solving reveals minimal change. The data shows a correlation between computation and problem solving. The data indicates 48% of the student's scored above the 40^{th} percentile and 51% scored below. (See Appendix C) The percentage of students scored the same in computation and problems solving. The data did not show an increase on student achievement based. This may be due to the fact, that students were only introduced to Computational Fluency a few weeks before the ITBS test. To actually see the impact on computational fluency will take at least 5 months.

DISCUSSION

Introduction

Computational understanding and proficiency are in many ways one of the essential goals in elementary mathematics. During this section, we will discuss the three methods that were used to increase student achievement through computational fluency. The interview section focuses on the Computational Fluency Curriculum, and the Classroom Assessment describes the different strategies and games that were used. The Standardized Analysis discusses the correlation between computational Fluency and problem and the results of the ITB'S test. The last part is a summary that gives an over view of this sections.

Interview Analysis

The Computational Fluency Curriculum mini lessons are focused on three main instructional goals: developing thinking strategies, provoking transition to new thinking strategies and provoking fluency with thinking strategies. These types of mini-lessons are carefully sequenced to encourage all students to develop computational fluency at a reasonable pace. With frequent opportunities to solve computational problems and supportive conditions to try out more sophisticated strategies, students are showing they are able to solve basic fact problems accurately and quickly, often by simply "retrieving" an answer.

Students have improved their computation skills based on their weekly addition and subtraction time test. Over half the second graders are scoring between 75%-100%. The data shows that there is an increase in student achievement. The students need to score

80% to be proficient. Each of these mini-lessons is framed around the solution of a related set of word problems and targets two or three basic number combinations. These lessons follow a six-step format. (See Appendix D)

The computational fluency curriculum activities deliberately focused on helping students make the transition to using a new thinking strategy. The mini-lessons are designed to make it difficult for students to successfully use their informal more familiar strategies, thus offering some natural incentive for students to try-out other more sophisticated and efficient strategies.

Classroom Assessment

The Computational Fluency games were designed for the students to think like mathematicians. These activities allow students to work individually or in small groups. A solution wasn't suggested, so that students would be able to look to themselves for mathematical justification and authority, thereby developing confidence to validate their work. During the Classroom Analysis findings, many modifications were made. There were brailed materials for a student who is blind. That student enjoyed the hands on games. Some time the materials took three weeks to get brailed, so the teacher just used regular manipulatives. These students did have some manipulatives brailed. Students also had a chance to work with brailed math manipulative.

The students really used a variety of strategies. For example, children in the lower elementary grades can learn that numbers can be decomposed and thought about in many different ways--that 24 is 2 tens and 4 ones and also two sets of 12. A chart was developed to monitor each student's growth. All of the student's names were listed along with the strategies, and every time they used the strategy they received a plus. This was a way for the teacher to know what strategies the students knew and did not know.

Standardized Achievement

After examining the ITB'S results, it is difficult to make a correlation between the current strategies and student achievement, due to when the test was taken and the strategies were assessed. The test was taken Spring 04, and the strategies were introduced and assessed in the Fall 04. Once the scores are reported for this year, analysis will be able to be made. This year Computational Fluency is apart of our Building Improvement Plan. Research has indicated that beginning with problem situations yields great problem-solving competence and equal or better computational competence. Children who start with problem situations directly model solutions to these problems. They later move to more advance mathematical approaches as they progress through levels of solutions and problem difficulty. Thus, their development of computational fluency and their acquisitions of problem-solving skills are intertwined as both develop with understanding.

Summary

For many years, researchers have contrasted conceptual and procedural aspects of learning mathematics, debating which aspect should come first. Recent research, however, portrays a more complex relationship between these conceptual and procedural aspects, concluding that they are continually intertwined and potentially facilitate each other. Number pervades all areas of mathematics. According to the Number and Operations Standard, students with number sense naturally decompose numbers. They use particular numbers as references; solve problems using the relationships among operations and knowledge about the base-ten system. Students also estimate a reasonable result for a problem, and have a disposition to make sense of numbers, problems, and results.

Problem solving is an integral part of all mathematics learning. In everyday life and in the workplace, being able to solve problems can lead to great advantages. However, solving problems is not only a goal of learning mathematics but also a major means of doing so. Problem solving should not be an isolated part of the curriculum but should involve all content standards.

Problem solving means engaging in a task for which the solution is not known in advance. Good problem solvers have a "mathematical disposition." They analyze situations carefully in mathematical terms and naturally come to pose problems based on situations they see. For example, a young child might wonder, how long would it take to count to a million? Good problems give students the chance to solidify and extend their knowledge and to stimulate new learning. Most mathematical concepts can be introduced through problems based on familiar experiences coming from students' lives or from mathematical contexts. For example, middle-grades students might investigate which of several recipes for punch giving various amounts of water and juice is "fruitier." As students try different ideas, the teacher can help them to converge on using proportions, thus providing a meaningful introduction to a difficult concept. Each teacher would need to have resources available to assist them with teaching Computation, problems solving or number sense.

Recommendations

There are a couple of resources that will help increase student achievement in math. Each one focuses on a different mathematical area. The series Math Matters is written to help primary students make the connection between the mathematics they are learning in school and everyday life. Each book is designed on a single math concept and how mathematics is used in everyday life. Math Matters will give students a chance to model good problem- solving techniques and reasoning skills for determining solution. It also has a teacher and parent section that gives suggestions for connecting concepts. *Nimble Numeracy* by Phyllis E. Fischer, deals with direct instruction in counting and basic arithmetic to develop fluency with numbers. Teachers will find this book useful for developing number concepts and the fluency that children need to build a strong sense of numbers. *Count on Math* is a collection of three wonderful videos, which is produced by Disney Educational Products. The three videos focus on *Exploring Geometry*, *Organizing Data*, and *Solving Equations*. They contain good visual representation of two and three dimensional, function boxes to solve problems, and how to organize data. These videos are upbeat and exciting that will keep the students attention.

Also each video has a teacher's guide along with follow up activities. *Facts to 20: Addition and Subtraction* is a collection of math puzzle books, brainteasers and games. Each book contains seventy-five challenging activities suitable for young learners to problem solve and think logically. Ivan Moscovich created Mind Games, which contains interactive highly motivated puzzles inside the books. These games would work well at a learning station, after school program or the whole class. All of the above are recommendations that can improve learning in the classroom.

Future Steps

This study suggests that students need to develop a range of strategies for solving problems, such as using diagrams, looking for patterns, or trying special values or cases. These strategies need instructional attention if students are to learn them. However, exposure to problem-solving strategies should be embedded across the curriculum. Students also need to learn to monitor and adjust the strategies they are using as they solve a problem. After talking with the teachers, there are a few things that the teachers need to do in order to have a successful learning climate. Each teacher plays an important role in developing students' problem-solving dispositions. They must choose problems that engage students. They need to create an environment that encourages students to explore, take risks, share failures and successes, and question one another. In such supportive environments, students develop the confidence they need to explore problems and the ability to make adjustments in their problem-solving strategies.

Teachers should provide activities and experiences that develop a conceptual understanding of number and operations, instead of focusing on the memorization of rules and procedures. Meaningful mathematical learning then can occur. When left to use strategies that are natural for them, children are wonderful problem solvers and are able to make sense of numbers in their world. Computational fluency means being able to solve problems accurately, efficiently, and in more than one-way. When a student achieves computational fluency, he or she is able to do the following things: look at a problem and decide the best method for solving it, solve the problem following a logical and sequential order of steps, calculate the correct answer, and most importantly, be able to explain how he or she arrived at that answer.

Computational fluency requires that students have a solid understanding of the operations and their relationships to each other, a large knowledge base of how numbers relate to each other, and a good understanding of the base-ten and the place value system. There are questions to consider to you as you assess computational fluency among your students. (See Appendix E) With hard work and dedications from the teachers, parents and students, computation will increase student achievement.

References

Bass, H. (2003). Computational fluency algorithms, and mathematical proficiency: One mathematician's perspective. *Teaching Children Mathematics*, *9* (6), 322-327.

Baroody, A. (2003). *Addition and subtraction in the primary grades: Early childhood mathematics, 9* (6), 72-102

Dowker, A. (1992). Computational strategies of professional mathematicians. *Journal for Research in Mathematics Education, 23* (1), 45-55.

Gabriele, A. (2004). Computational Fluency Curriculum Project. University of Northern Iowa. Cedar Falls, Iowa

Griffin, S. (2003). Early childhood corner: Laying the foundation for computational fluency in early childhood. *Teaching Children Mathematics, 9*(6), 306- 309.

Flowers, J. (2003). Developing teacher's computational fluency: Examples in subtraction. *Teaching Children Mathematics, 9* (6), 330-334.

Hiebert, J. (1999). Relationships between research and the nctm standards. *Journal for Research in Mathematics Education, 30* (1), 3-19

Hiebert, J. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.

Huinker, D., Freekman, L.J., & Stenmeyer, B.M. (2003). Subtraction strategies from children's thinking: Moving toward fluency with greater numbers. *Teaching Children Mathematics, 9*, (6), 347-353.

Kanter, P. (1998). *Every day counts partner games: Games to develop number sense.* Great Source Education Group

Kaye, P. (1998). *Games for math: Playful ways to help your child learn from kindergarten to third grade*. Random House USA Inc

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: NCTM

Postlewait, K., Adams, R.M., & Shih, C. (2003). Promoting meaningful mastery of addition and subtraction. *Teaching Children Mathematics, 9*, (6), 354-357.

Robertson, L. (1999). Number power: A cooperative approach to mathematics and social development. Addison Wesley Publishing Company

Russell, S. J. (2000). Developing computational fluency with whole numbers in the elementary grades. *New England Mathematics Journal, 32*, (2), 40-54.

Russell, S. J. (1999). Mathematical reasoning in the elementary grades. In *Developing mathematical reasoning in grades K-12*. Yearbook of the National Council of Teachers of Mathematics, Reston, VA

Russell, S. J., Smith, D., Storeygard, J., & Murray, M. (1999). *Relearning to teach arithmetic: Multiplication and division*. Parsippany, NJ: Dale Seymour Publications.

Wickett, M. (2003). Discussion as a vehicle for demonstrating computational fluency in multiplication. *Teaching Children Mathematics, 9*, (6), 318-321.

Appendix A

Each interviewee was asked seven questions:

1.) What are your thoughts about the additional Computational Fluency Curriculum?

2.) Describe how this curriculum will increase computational fluency?

3.) What barriers do you encounter while teaching this curriculum?

4.) How would you assess student performance since teaching this curriculum?

5.) Explain how this curriculum will develop our students into life long learners?

6.) Describe what skills students are expected to obtain while teaching this curriculum?

7.) Are there any concerns or questions you have about this curriculum?

Follow up questions

1.) How do you feel about this computational fluency curriculum?

2.) Explain how you see this curriculum increasing computation?

3.) What obstacles do you face while teaching this curriculum?

4.) Describe how students are assessed while teaching this curriculum?

5.) Explain how this curriculum will develop our students into life long learners?

6.) What skills do you expect students to have while teaching this curriculum?

7.) What questions or concerns do you have about this curriculum?

Appendix B

Number Ladder. Each partner creates a "ladder" with a number from 1–10 written in mixed order on each rung. (The ladders do not have to be alike.) Use a number cube and a bean or small marker. Put the marker on the bottom rung, then roll the number cube and add that number to the number on the rung where the marker is placed. If the correct sum is given, the student can move to the next rung and continue. If a mistake is made, the student must go back to the bottom of the ladder. When one partner finishes or makes an error, the other partner takes a turn. One way to use this game for subtraction practice is to roll two number cubes, find the sum, then subtract the number on the ladder from that sum. *Adaptation for playing at home with an adult:* The adult purposely makes at least one error, and the child partner must try to catch it; otherwise, the adult most likely will make it to the top of his or her ladder first. (Kaye 1987)

Pyramid. Using the number cards in a deck of playing cards, set up a pyramid with one card at the top, two cards overlapping the bottom edge of that card, three cards overlapping the edges of the two cards, and so on, until there are six cards at the bottom of the pyramid. Pick up cards with number combinations that equal 10. Only cards that are fully uncovered can be used. At first, students use only two cards at a time that represent addends such as 3 and 7. Once students understand how to play, encourage them to use more flexible thinking and as many cards as necessary to make combinations such as $9 + 3 - 2$ or $2 \times 3 + 4$ for their solutions. (Kaye 1987)

Double War. Two players each have a deck of playing cards with the face cards removed. Each player turns up two cards at a time. The players add the values of their cards to see who has the greatest sum. That person wins and takes all four cards. If the sums match, each player deals out three cards from his or her pile and then turns the top or third card over for a playoff. Whoever has the greatest number from the two cards turned up during the playoff wins all the cards from that play. *Adaptation: Double War/Difference:* Use the same procedure as Double War, but this time subtract the value of the cards turned up. The least difference wins. (This is simply an adaptation of the game of War played by children with a single deck of cards when the players decide which of the upturned cards is greater to win that round. Double War also can be played with products to practice multiplication facts.)

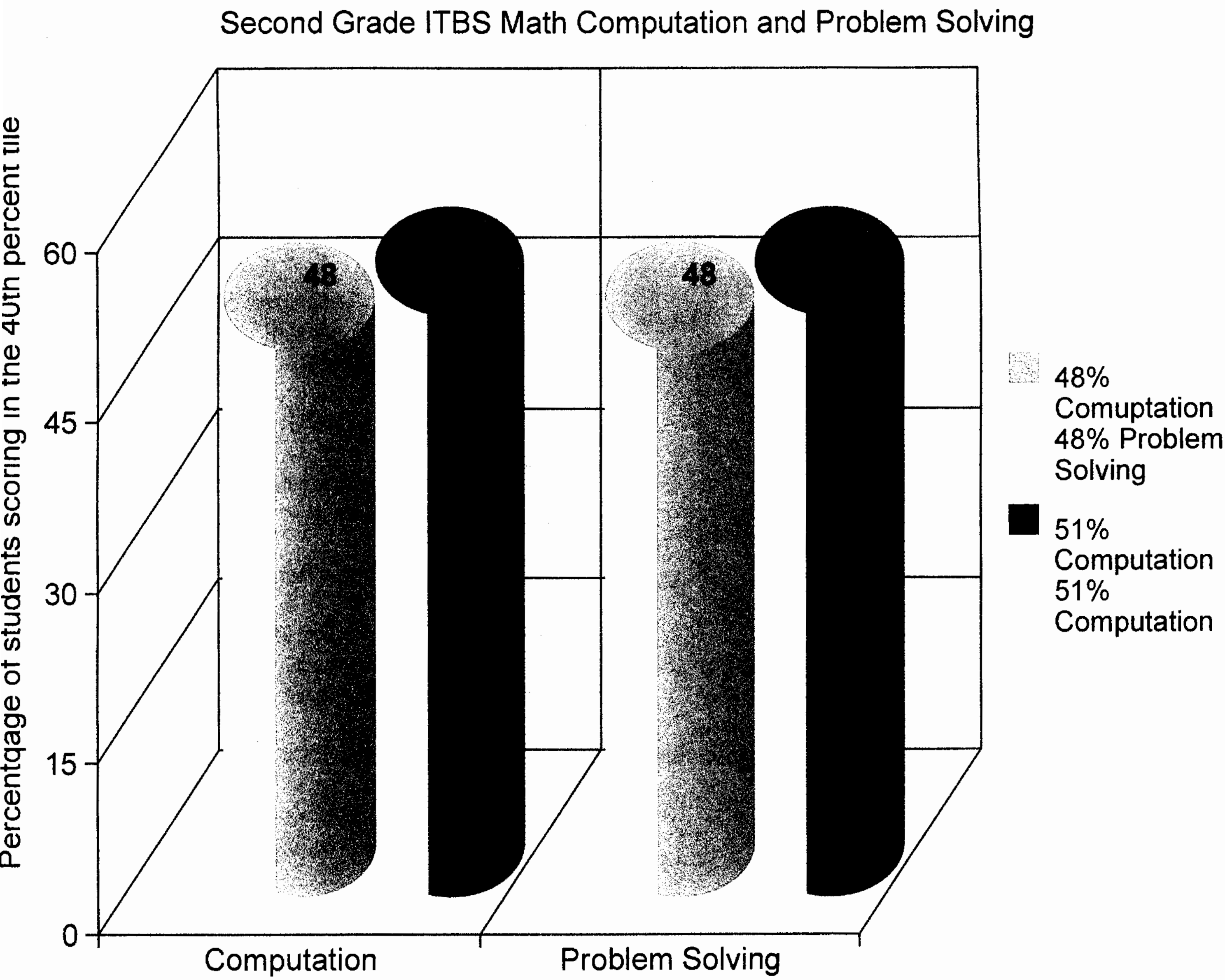
Five Makes Ten. Use a deck of playing cards with the face cards and tens removed. Deal each player five cards. Each player tries to make equations that equal 10. The player who makes the most equations wins the round. Recording the equations is a good idea. Players can use each card only once in the same equation. *Adaptation: "Six Makes Ten":* In this game, deal six cards to each player and follow the rules for Five Makes Ten. Notice how one more card increases the number of equation possibilities. (Kaye 1987)

Can You Make a Difference? Make a vertical list of numbers from 1 to 9. Using Ten Frame or Ten Grid cards, deal each player five cards and place them faceup. If a player can make an equation that has a difference from 1 through 9, he or she puts a check mark beside that number on his or her list. The player with the most checks wins. *Note:* Ten Frame or Ten Grid cards can be bought commercially, or they can be made by drawing a two-by-five array and placing dots inside the boxes to equal various numbers. For example, to represent the number 7, five of the boxes on one side would have a dot in them and two boxes on the other side would have a dot. The three remaining boxes would be blank. (Kanter, Gillespie, and Clark 1998)

The Game of Ten. Use a set of playing cards without the face cards. Place the deck facedown. The dealer picks up the top card and places it on the table faceup. The next player turns up the next card. If a ten is uncovered or can be made by combining the card with any uncovered, previously picked card(s), the player announces how he or she knows it is a ten and takes the cards used in that turn. If a player cannot make a ten, the card is placed faceup on top of the previous card and the turn goes to the next player. Cards can be used as they are uncovered. Continue until all the cards are used. (Baroody 1993)

Take 'em Out, Put 'em Back. Students play in pairs. Each pair needs one small paper bag labeled with the number of interlocking cubes placed (loose) inside. Note that the number of cubes inside the bag depends on the set of basic number facts the students are working with at the time. The first player takes out a handful of cubes, shows them to his or her partner, and asks how many are still in the bag. The partner answers, then checks the cubes in the bag. The partners return the cubes to the bag, switch roles, and play again. (Robertson et al. 1999)

Appendix C


Second Grade ITBS Math Computation and Problem Solving
Percentqage of students scoring in the 40th percent tile
60
45
30
15
0
48
48
Computation
Problem Solving
48% Comuptation 48% Problem Solving
51% Computation 51% Computation

Appendix D

Introduce the Activity- Come up with something to orient students to the 5-minute mini-lesson. (e.g., O.k. please remove everything off our desk we are going to do some top notch math).

Pose the Problem- remember to repeat or rephrase the problem

Time for thinking-(extended wait time)

Wait for the majority of the students to raise their hands.

Sharing Solutions-

Solicit answers, then strategies.
Validate all thinking.
Avoid correcting mistakes, let students self-correct

Highlight Thinking-Select a "good" strategy

Emphasize one student's strategy
Use model to help make the strategy more visible

Make a Connection-have student's try highlighted strategy on a new problem.

Solicit answers only
Select one student to share how they used the highlighted strategy.
Use model to help make the strategy more visible

Lesson Wrap –up- Summarize lesson goal and rehearse and rehearse selected basic facts.

Avoid calling on individual students
Use Choral Review

Appendix E

Here are some questions to consider as you assess computational fluency among your students:

- Does the student know and draw on basic facts and other number relationships?
- Does the student use and understand the structure of the base ten number system--for example, does the student know the result of adding 100 to 2340 or multiplying 40 x 500?
- Does the student recognize related problems that can help with the problem?
- Does the student use relationships among operations?
- Does the student know what each number and numeral in the problem means (including sub problems)?
- Can the student explain why the steps he or she uses works?
- Does the student have a clear way to record and keep track of his or her procedure?
- Does the student have a few approaches for each operation so that a procedure can be selected for the problem