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# Ambigrams: The Art and the Math

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# AMBIGRAMS:

# THE ART AND THE MATH

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What makes a symbol mean something? All words are made up of letters, which are symbols used to represent sounds in language. The art of creating ambigrams takes these symbols and contorts them using properties of symmetry. The result often challenges one's understanding of the symbols. Standing alone, an ambigram can be a beautiful work of art, depicting a curious relationship when rotated, or reflected. However, these word representations also raise many geometrical questions. There are nine different types of ambigrams, some can be viewed as the same word from multiple directions, others can be viewed as two words in one depending on the viewer's perspective. The curiosity behind the words draws one in, but the symmetries and complexity of creating something so simple is what maintains the viewers' interest. Ambigrams rely on viewers' senses of symmetry and draws on their knowledge of words and letter forms.

Ambigrams are a relatively new form of art so the history of ambigrams is brief. There are two leading ambigram artists; both began their work in the 1980's. John Langdon, the artist who created the designs for novel Angels & Demons, one of which is seen on the right [7], focuses on the geometrical classification and methodology of creating an ambigram. He names each style of ambigram based on the mathematical symmetries present in the design. Langdon

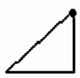
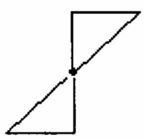
discusses the forms and methodology of creating ambigrams on his website, "Wordplay" [2]. He also gives tips and advice to aspiring ambigram artists on this page.

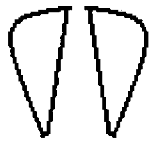
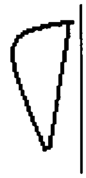
Scott Kim focuses heavily on the artistic aspects in his designs. He classifies his ambigrams, which he refers to as "word inversions" on the basis of not only geometrical symmetries, but also perceptual mechanisms and other organizing principles [3]. Kim's website "Scott Kim Puzzlemaster," [8] includes a gallery of his designs, as the one seen here, and also puzzles and activities for educators to use within their classrooms. Overall, ambigrams rely on aspects from mathematics, art, and even a little from language to be properly interpreted and understood.

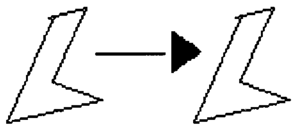
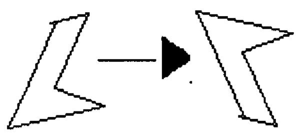
Because ambigrams are a relatively new concept, I found little mathematical research on them. Throughout this paper, I will address the role of ambigrams as a mathematical entity, an artistic outlet, and even an educational tool. Along with ways to use ambigrams, there are also descriptions of the nine different types of ambigrams, and a chart that can be used to create two of the most common types of ambigrams. Mathematically, ambigrams contain many symmetrical aspects, which are important to the study of Geometry. Within this, I have researched mathematical aspects of three of the ambigram types and found some of their specific properties. The symmetries of the designs are also closely related to art. Scott Kim lays out the artistic aspects of ambigrams, but I also did some designing of my own to create the letter chart. While working with ambigrams, I also discovered ways these designs could be used within an educational classroom. Because ambigrams can be elements of several different fields, it is easy to make connections between these fields for students working with ambigrams.

## Mathematics of Ambigrams

Ambigrams contain many geometric properties. On John Langdon's website he shows nine different types of ambigrams that are classified based on the type of symmetry they contain. In order to understand how an ambigram is created, classified, or viewed, one must have a basic understanding of mathematical symmetries or isometries.

In The Perceptive Eye: Art and Math, by Lillian F. Baker and Doris J. Schattschneider [1], definitions for the basics of symmetries are discussed. There are four basic types of symmetry: rotations, reflections, translations, and glide reflections. Rotations take an image and turn it around a fixed center point by a specific number of degrees. Point symmetry is a two-fold rotational symmetry, which rotates a figure 180 degrees about a fixed point, O.  Rotating this triangle 180 degrees around the top corner the image now looks like this:  where the top corner becomes the center point of the rotation. Point symmetry has two identical halves where the midpoints of the segments formed by two identical points in the image are all at point O [1]. Therefore, the midpoint of a line segment from one corner of the triangle to that corner's image under the rotation is at the center of the rotation.

Single Reflections, or line symmetry, have two identical halves, which are mirror images across the reflection line. In this picture:  the original shape looked like this:  This shape was then reflected across a given vertical line  creating the two mirror image halves of the final picture.

Translation symmetry moves an image in a set direction and length. In the following illustration the figure is moved in the direction the arrow points and the same distance as the length of the arrow.  Finally, glide reflection symmetry combines a translation and a reflection of a figure; however, the line of reflection and the direction of the translation must be parallel. Taking the translation example from above, it can be turned into a glide reflection by reflecting the figure along the same line as the arrow:  . Now the translation has become a glide reflection, with the image reflected and translated. When working with ambigrams, the most important isometries will be rotations, specifically point symmetry, and reflections.

Ambigrams fall into two different categories: symmetrical and asymmetrical [2]. Symmetrical ambigrams display the same word, and view it from two different perspectives; asymmetrical ambigrams can be read as a word from one direction and as another word from a different direction or perspective. The symmetries described above are used individually to create three different forms of symmetrical ambigrams. The first and most recognized symmetrical ambigram is the rotational ambigram. This design takes a single word, like MOW, and rotates it  a full 180 degrees around the center point in the middle of the O, MOW and reads as the exact same word. The "M" and "W" are rotationally symmetrical so "mow" can still be read when rotated even though the first letter is actually an upside down "W" and the third is now an upside down "M."

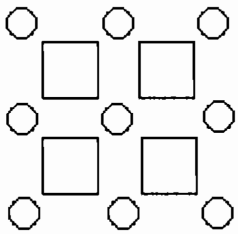
Two other symmetrical ambigrams that use just one of the isometries are the bilateral or "mirror" ambigrams and totem ambigrams. Both of these designs have vertical reflection symmetry. The Mirror Ambigrams have a line of symmetry midway through a word written horizontally. Words such as "TOT" easily qualify as bilateral ambigrams. The line through the middle TOT shows where the line of reflection is, and the letters on the right side will match up with the letters on the left if the word is folded across that line. Totem ambigrams are words written vertically with a line of symmetry down the middle and are read from top to bottom. This is easily seen in words like "WHAT" when written vertically:

W
H
A
T

Both the right and left side are mirror images of the other. Many letters of the alphabet are symmetrical thus forming many words that could be totem ambigrams.

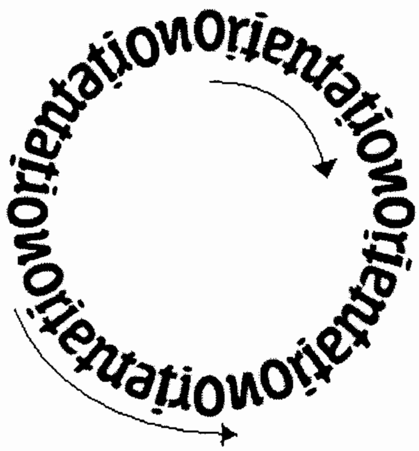
The Perceptive Eye: Art and Math [1], the authors describe two periodic design types that are often found in art, Periodic Border Designs and Periodic Allover Designs [1]. Periodic means the pattern repeats indefinitely to create a continuous border or covering

of a plane. The border designs, also known as frieze patterns, have translational symmetry, and are contained between two edges to make a continuous strip of design. Borders can have any of the following elements with the translation: a half-turn about a point at the center of the strip, a reflection perpendicular to the edges of the strip, a reflection parallel to the center of the strip, or a glide reflection parallel to the center of the strip. This example contains two perpendicular reflections. Overall, there are seven different combinations of these aspects, creating seven different basic border patterns. The Periodic Allover Design, also known as wallpaper or planar design, is translational in two directions and covers an entire plane with the pattern. This example contains six lines of reflection to continue the entire pattern endlessly. There are seventeen different basic patterns for a planar design using the different Isometries. [1]

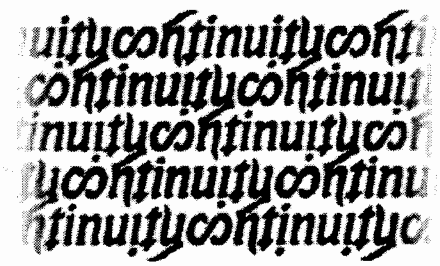


The next three symmetrical ambigrams use periodic symmetries within their design. Bilateral chain ambigrams take one word and connect it end to end in a line because the word cannot be an ambigram on its own. The word "Future" can be read from the following design in a way similar to the previously mentioned bilateral ambigrams. The first line of reflection is through the middle of the "T" but the "E" does not have a reflection to the other side of the word so the symmetry is not exact. Since "E" did not have a mirror image at the beginning of the word, it would not be completely symmetrical with standing alone which is why it needs to be connected in a chain:  [2]. When put as a chain the "E" is now part of the symmetry and contains another line of reflection which incorporates the ambigram feature. These designs are like frieze patterns with reflection and can be read forwards and in a mirror.

Rotational chain ambigrams are similar to bilateral chains but instead of having reflection symmetry they have rotational symmetry and are often displayed in a circle not a straight line. Rotational chains can be read from either the outside or the inside of the circle, as shown with the arrows in the image. In this example, from John Langdon's webpage [2], the word "Orientation" forms a rotational ambigram. However, it has more than one point of rotation, one in the middle of the "ono" pattern, and one within the "nt" combination, which is why it must be linked together.



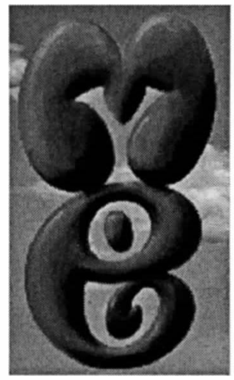
The last symmetrical ambigram is the web ambigram. The web design is similar to the Periodic Allover Design discussed in The Perceptive Eye[1] and covers an entire plane if extended. The web is made up of multiple horizontal rotational chain ambigrams connected vertically by some feature of the word. [2] In this example, "continuity" has a rotation between the "n" and "u" and between the "c" and "o" in each strip of the image. There is also a rotation connecting the strips between the tail of the "y" connected to the "n" below it.



The chain ambigrams and the web ambigrams also have a special mathematical feature since they are all periodic patterns. Throughout my research, I found that each of these chain types of ambigram contains an orbifold, which is the mathematical term for the smallest part of a pattern that is repeated to create the complete design. Using orbifold notation there is a way to classify and name each of those ambigram types [9]. A rotational chain when written in a straight line has an orbifold notation of 22∞ where each two represents a separate rotation in the design and the infinity symbol shows that the pattern can extend endlessly. A bilateral chain has orbifold notation of $*\infty\infty$. The asterisk represents reflections and each infinity symbol shows that there are an infinite number of two different reflections when the pattern continues. Lastly, the web ambigram has an orbifold notation of 2222 in which each two corresponds to a different center of rotation all having 180 degree rotational symmetry.

There are also three asymmetrical ambigram designs, which take two different words and combine them into one ambigram. These designs, symbiotograms, figure/ground, and oscillations [2], do not follow geometric properties but form more of an optical illusion depending on how they are viewed. Symbiotograms are most closely related to the symmetrical ambigrams, and are written to be viewed as one word, then rotated 180 degrees to be read as another word. Looking at "Wow" when rotated it can read as "Mom," WOW rotates around WOW to become MOM. When creating symbiotograms, the words used are most often opposites or complements of each other, so when viewed the words have a connection. The example

of "true" and "false" is a symbiotogram from John Langdon's website and shows opposite words in one image, showing "true" in one direction and the displaying "false" when rotated.



Figure/ground relationships depict one word normally, and another word can be found in the white space of the first. In this example found in Wordplay [4] the word "Me" can be read vertically in the bubble shaped letters, while in the spaces formed by these letters the word "You" appears. Thirdly, oscillations are viewed as two words (teach/learn) in one: tearn [4] where the figures in the design can represent different letters at the same time.

Art of Ambigrams

Taking examples of the golden ratio, and M.C. Escher's tessellations and other works of art, one can see that art and mathematics often coincide. Ambigrams are another of the many places where this fusing of subjects happens. Many ambigrams begin as an artistic undertaking but in the end, it is difficult to deny the mathematical relation the project has. Art and mathematics are both necessary to create an ambigram, and a knowledge of both is important to be successful in creating any ambigram.

In his book Inversions [3], Scott Kim describes the three aspects his "word inversions," or ambigrams, are classified by: geometric symmetries, perceptual mechanisms, and other organizing principles. The geometric symmetries Kim discusses include reflections, rotations, translations, and glide reflections as a combination of reflections and translations, but also include scaling as a symmetry. Scaling means an image is shrunk down or enlarged but the proportions remain equal to those of the original image. When combining scaling with the other symmetries, a recursion is formed; the shape multiplies in number, as it gets smaller. Tessellations can also be formed when a pattern repeats in two dimensions filling a space.

Kim has four important perceptual mechanisms to classify inversions by: closure, figure/ground, filtering, and containment [3]. In closure, an image can be opened or closed depending on the context; a given example is a circle with a break in one side can be seen as an "O" when closed or a "C" when open. Figure/ground is defined as when the borderlines of a figure are ambiguous, either side can be in or out; this is often seen in black and white cutouts, the black can be the figure or the white space can be the figure. Filtering is done when the eye filters a figure out of a larger image to see something it can make sense of. Containment is a type of filtering using color to define different parts of a figure, the figure seen can be the whole picture or just the piece that is in a different color.

Kim also mentions four other organizing principles: dissection, symbolism, regrouping, and context [3]. Dissection cuts a figure into multiple pieces moves them around, once again making the meaning of the boundary lines less concrete. Symbolism takes the letters' meanings and makes them more visual with pictures; an example given in the reading is forming the letter P out of something that looks like a bent pencil. Regrouping takes the parts of letters and shows that one can rearrange them to look like a different letter; if the vertical line on the letter "p" is switched to the other side of the circle it becomes the letter "q" easily. The context of a figure is also very important to inversions; the meaning of a symbol can be interpreted based on the meaning of the surrounding symbols and what one would normally expect to see in that given position.

## Creating Ambigram Letter Chart

When making an ambigram the easiest way I found to start is by using one word at first to make a symmetrical rotational ambigram, then going on to more difficult ones by using two words of the same length to create a symbiotogram. The attached chart (see appendix) gives a list of combinations of all letters needed to make either a 180-degree rotational ambigram or a symbiotogram.

The chart of the ambigram alphabet that I have created includes a symbol for each pairing of two letters, A through Z. In order to begin creating this project, I first paired up the letters beginning with A and working my way through the alphabet. For each pair, I wrote the first letter, then flipped the paper and wrote the second letter directly below the first. This way, one letter was upside down and the other was properly oriented; giving me a chance to see what the letters had in common as a place to start combining them into one shape. I brainstormed and sketched out different ways to combine their characteristics while trying to make sure each letter was still legible within the new shape. I wanted the symbol to be identifiable as each separate letter depending on which way it was flipped so I had to make sure I was not stretching the qualities of each letter beyond recognition.

Once I had a rough idea of what all three hundred fifty-one characters would look like I wanted to make them more usable. To do this I needed to take my hand-drawn shapes and recreate them on a computer. This task was challenging because I needed to find the proper program and learn how to use the program efficiently. Once I learned how to create things using Adobe Illustrator, a graphic design program, I had to take my sketches and break them into parts that I would accurately reconstruct in the program.

Through this process, I learned a lot about the differences between handwriting and drawing to computer illustration and fonts. Once I finished creating the graphical interpretations of my drawings, I placed each letter one by one into a spreadsheet to better organize the letters. This way, to find a combination of two particular letters one just has to find where the two meet on the spreadsheet. Using the chart as a reference to letter combinations, anyone can easily make either a symmetrical rotational ambigram, or an asymmetrical symbiotogram with words of the same length.

## Using Ambigrams in the Classroom

Ambigrams can make an excellent teaching tool to introduce many different topics and use different ways to get students involved in their learning. These designs can be used across several different fields of study including art, math and language arts. When using ambigrams in the classroom they can address different learning styles using art, language, and visual cues. Creating ambigrams can be a way to interest students and use some of their creativity on different topics and new ideas.

Ambigrams can be used in art to illustrate many different types of symmetries and how they can be implemented within a design. The designs can also be part of an art unit on text or word art and designs, leading to discussion of different fonts and graphic designs. Even though ambigrams are not considered a direct optical illusion they can still be discussed in this context, particularly oscillations or figure/ground ambigrams because they show how things can change depending on how a person views the art and each person's original perspective. Ambigrams are often related to word puzzles. Students could create their own kinds of puzzles with original art designs.

In a language arts class, educators could use ambigrams to discuss the relationships of words. At an elementary level, students could use ambigrams to discuss opposites and synonyms and create asymmetrical ambigrams to display their learning. At the high school level, students could use ambigrams as a way to discuss different idioms and commonly used sayings. Each student could choose an idiom and create an ambigram with its meaning, such as "One Baker's Dozen" means thirteen things.

As seen throughout this paper, mathematics and ambigrams often go hand in hand. In an elementary or middle school mathematics class, some ambigrams can be used to display symmetries. Rotational and bilateral ambigrams are clear and interesting examples of rotation and reflection symmetries. Since some students may be more word oriented, using words with the math may help them better remember the experience and the meaning of the symmetries. In a high school or even college math class, ambigrams can open discussion to more combinations of symmetries and even orbifolds of the images. Chain ambigrams provide a good example of frieze patterns, and using a web ambigram, students can discuss the orbifolds and combination of symmetries needed to produce it.

## Conclusion

After working with ambigrams, I have gained insight on how interconnected different subject areas can be. Ambigrams have a large base in language arts, art and math, and after further study, I understand how these subjects can be studied together and can be related to each other. An ambigram may look like just an intriguing piece of art at first glance but since I have studied the details of creating the symmetries contained within each word I understand the effort and patience that is put into each small work.

The nine different types of ambigrams provide illustrations of various forms of symmetry. After finding their orbifold characteristics of the two different chain ambigrams and the web ambigram, I was better able to understand how they are each formed. While making the ambigram letter chart, I extended my learning into a new field of graphic design that I had never had the chance to work with before. I also gained more insight on how far I could stretch the figures before they became unrecognizable. After creating some of my own original designs, I found that it is easier to for people to decipher more common words, or words that do not have curious spellings.

Overall, working with ambigrams gave me a better insight on how people interpret different ideas. Some people understand things better through sight, others through words, and some others need to actually interact with something before they can fully understand it. Ambigrams are a great way to reach all those types of learners, and gives everyone a chance to internalize a new concept in an exciting way. As a future educator, I plan to use this project to inspire students in math classes and show how beautiful and creative one can be when it comes to mathematics.

Literature Cited

[1] Baker, Lillian F. The Perceptive Eye: Art and Math. Allantown, PA: Allantown Art Museum, 1979.

[2] Langdon, John. "Wordplay." 9/29/2008 <www.johnlangdon.net>.

[3] Kim, Scott. Inversions: a catalog of calligraphic cartwheels. Peterborough, NH: Byte Books, 1981.

[4] Langdon, John. Wordplay: The Philosophy, Art, and Science of Ambigrams. New York: Broadway Books, 2005.

[5] Gough, Jonathan. "Ambigram.net." 9/29/2008 <http://ambigram.net/about>.

[6] Langdon, John. "Typographic Doppelgangers." Step Inside Design May/June 2007: <http://www.stepinsidedesign.com/STEP/Article/28765>

[7] Brown, Dan. Angels & Demons. New York: Pocket Books, 2000.

[8] Kim, Scott. “Scott Kim Puzzlemaster.” 3/9/2009 <www.scottkim.com>.

[9] Conway, John H.; Heidi Burgiel; Chaim Goodman-Strauss. The Symmetries of Things. Wellesley, MA: A K Peters, Ltd., 2008.

## Examples

### Symbiotograms

Mannequin and Astronaut

Chaos and Order

### Rotational Ambigram

Callie

Ambigram Letter Chart