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## Groundwater Flow Patterns of the Ames Aquifer

R. R. van der Ploeg,<sup>1</sup> Don Kirkham,<sup>2</sup> and L. V. A. Sendlein<sup>3</sup>

R. R. VAN DER PLOEG, DON KIRKHAM, AND L. V. A. SENDLEIN. Groundwater Flow Patterns of the Ames Aquifer. *Proc. Iowa Acad. Sci.* 80(2): 103-110; 1973.

**SYNOPSIS:** Extensive field studies have indicated that the Ames aquifer (at Ames, Iowa) approximates a confined horizontal aquifer of uniform thickness and height. The aquifer is partly fed through the channel beds of two surface streams, the Skunk River and Squaw Creek. Part of the boundary of the Ames Aquifer is formed by a till

layer that can be considered impervious. Pumping tests have indicated that the Skunk River and Squaw Creek maintain nearly a constant head distribution along the boundaries of the aquifer while the aquifer is pumped. A theoretical solution is presented that yields the well discharge, the hydraulic head at every point in the flow region, and the stream function when the aquifer is pumped by one completely penetrating well.

A mathematical procedure has been developed to calculate a flow net for the Ames aquifer. The method uses potential flow theory. A solution is sought that, besides satisfying Laplace's equation for two-dimensional radial flow and the boundary conditions as observed in the field, will give the hydraulic head distribution throughout the aquifer while the aquifer is pumped. The solution sought will also give the well discharge and the streamlines that show how water is flowing to the well in the aquifer. The flow net that we calculate for the Ames aquifer is of practical importance because it indicates how pollution of the well water by a phenol source, seeping into the aquifer from a waste pit of an old gas plant, can be prevented or decreased by a proper choice of location of the pumped well, or by a pumping program for established wells.

an elliptically shaped aquifer where the well is located in the center of the ellipse. Van der Ploeg *et al.* (1971) provided a solution for an elliptically shaped, confined aquifer for any arbitrary location of the well in the aquifer. They made use of a modified Gram-Schmidt process as developed by Powers *et al.* (1967) and described further in Kirkham and Powers (1971). Later on Kirkham and Van der Ploeg (1971) extended the solution for elliptically shaped aquifers to irregularly shaped confined aquifers pumped by a well. Kirkham and Van der Ploeg also showed how the problem of confined horizontal flow to a well in a circular aquifer, of which part of the outer boundary is impervious, could be solved. In all the works cited so far, the hydraulic head at the outer boundary of the aquifer is taken, at the boundary inflow and outflow surfaces, to be a constant.

### LITERATURE REVIEW

Analytic solutions for steady-state confined horizontal flow to a well are in the groundwater literature for a limited number of flow configurations. A most simple problem of confined flow to a well, considered in most textbooks on groundwater hydrology, is that of flow to a well located in the center of a circular aquifer (Todd, 1967, p. 82). In other books, as in Muskat (1946, p. 172) or Harr (1962, pp. 253-255), one can also find solutions for horizontal confined well flow in a circular aquifer where the well is not located in the center of the circular aquifer. In Polubarinova-Kochina (1962, pp. 366-368) one can find a solution for the problem of confined horizontal flow to a well in

### THE AMES AQUIFER

The City of Ames, Iowa obtains its water supply from a sand and gravel body of glacial outwash origin. The Ames aquifer has been studied quite extensively in recent years by Sendlein and his students (Ver Steeg, 1968; Akhavi, 1970). These and other studies have indicated that the Ames aquifer is approximately a horizontal, confined aquifer of rather uniform thickness and permeability, consisting of sand and gravel and about 60 ft. thick. The Ames aquifer is hydraulically connected to the Skunk River and Squaw Creek, through the channel beds of these streams. Fig. 1 (from Akhavi, 1970) shows the surface geography of the Ames aquifer area. The Ames aquifer is located between Squaw Creek and the Skunk River at about 100 ft depth. The figure also shows the elevation above sea level of the piezometric surface in the Ames aquifer when it is not pumped.

Fig. 2 (Akhavi, 1970) shows a cross section of the aquifer as prepared by Akhavi from field data.

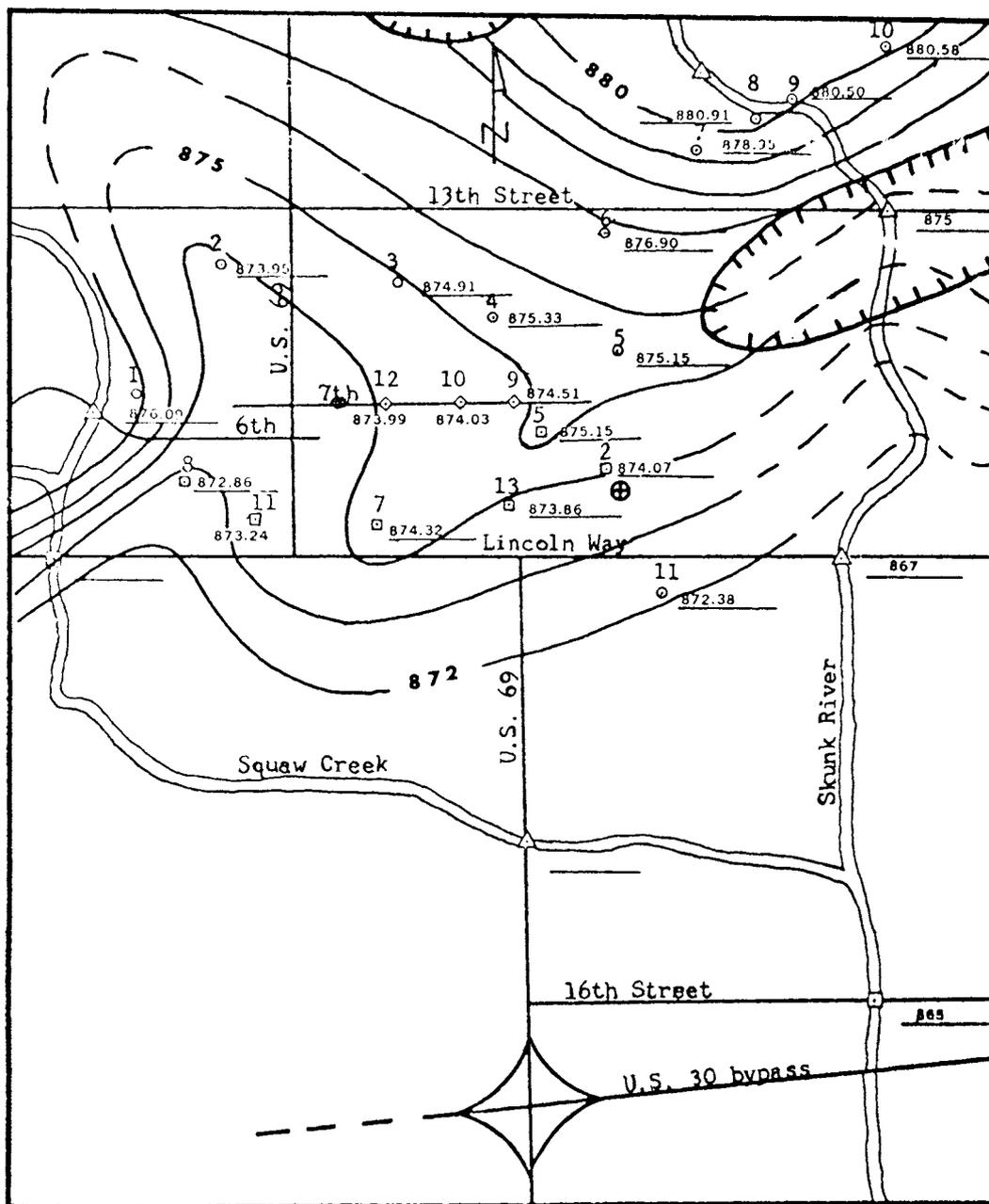
Fig. 3 is a combination of Figs. 1 and 2 and shows a schematic cross section of the Ames aquifer in east-west direction. One can see that the upper confining layer is the upper till and that the lower confining layer is Mississippian bedrock. One can see also that the hydraulic head in the channel beds of either the Skunk River or Squaw Creek is well above the upper confining layer of the Ames aquifer.

Fig. 4 (Akhavi, 1970), which is also prepared from field data, shows the rather uniform transmissibility of the Ames aquifer. In our theoretical analysis later on, we use a transmissibility of 200,000 gpm/ft.

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⊕ Source of Phenol  
 --- Till Boundary  
 △ Staff Gage  
 ○ Observation Well  
 □ USGS Recorder  
 ◇ City Well, pumping  
 ■ City Well, stand-by

PIEZOMETRIC SURFACE MAP,  
 BEFORE PUMPING

Scale: 1" = 1500'  
 Contour Interval 1 Foot

Figure 1. Location of the Ames aquifer, and piezometric surface map of the aquifer, before pumping. (Akhavi, 1970).

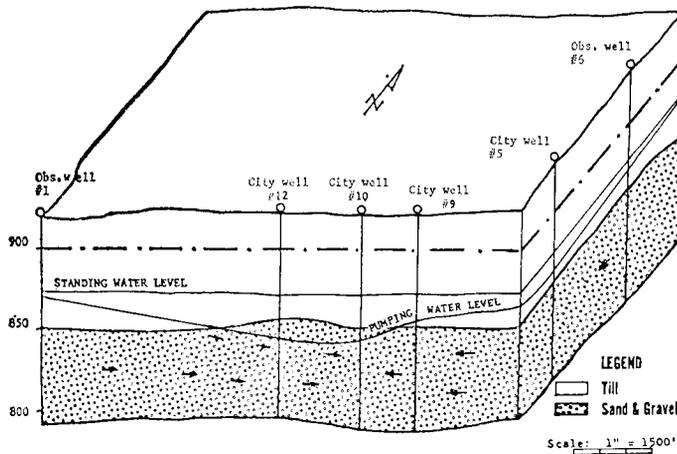


Figure 2. A block diagram through the Ames aquifer, as prepared from field data. (Akhavi, 1970).

THEORETICAL ANALYSIS

the head distribution along the boundary of the Ames aquifer, whether pumped or not, is such that, at the north, the hydraulic head is 880 ft (except at the impervious boundary), and that, at the south, the hydraulic head is 870 ft. Along Squaw Creek and the Skunk River, the head drops linearly from 880 ft to 870 ft. With the limits of the aquifer and the head distribution along the limits thus established, we decided to analyze a pumping test, as described by Ver Steeg (1968, p. 67), on the Ames city well number 9. The drawdown in the pumped well at equilibrium was 9.2 ft. The pumping rate was 1280 gpm. and the well radius, including gravel pack, is 2.5 ft. The drawdown is such that the water level in the well is above the upper confining layer of the aquifer such that confined radial flow is assured.

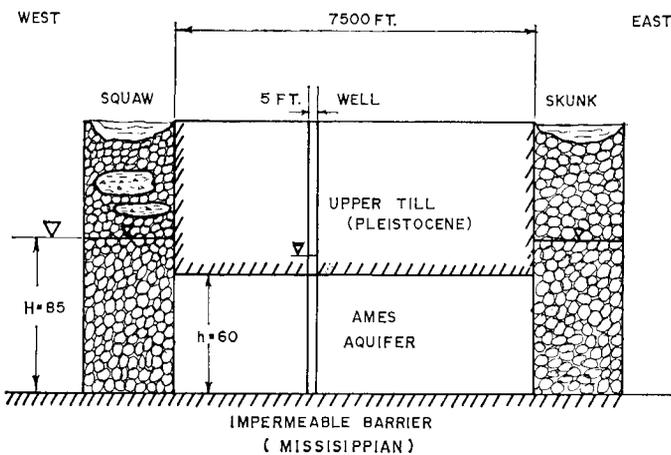


Figure 3. A schematic east-west cross section through the Ames aquifer.

Fig. 5 (Akhavi, 1970) shows the elevation of the piezometric surface in the Ames aquifer after a pumping test. During this pumping test, water was pumped from the Ames city wells numbers 9, 10, and 12 at a capacity of 3000 gpm. From this pumping test, from other pumping tests (Akhavi, 1970; Backsen, 1963), and from Fig. 1, it appears that the boundary of the confined portion of the Ames aquifer corresponds to the dashed line shown in Fig. 6. At the south and east, the boundary of the confined portion of aquifer is formed by the floodplains of Squaw Creek and Skunk River. An impervious till nose, shown on Figs. 1 and 5 at the east, thus falls just outside the aquifer limits. A till nose of large extent on the north of the aquifer, however, is included as an impervious boundary of the aquifer. From Figs. 1 and 5 and other field evidence, we conclude that

The analysis is much the same as used by Van der Ploeg *et al.* (1971) and Kirkham and Van der Ploeg (1971). That is, we use a modified Gram-Schmidt method as in Powers *et al.* (1967), or as in Kirkham and Powers (1971).

An expression is sought that enables one to calculate the hydraulic head at every point in the flow region. From the potential function, the stream function (that is, the flow lines to the well) can be calculated.

As in Van der Ploeg *et al.* (1971), or as in Kirkham and Van der Ploeg (1971) a system of polar coordinates ( $r, \theta$ ) is chosen with the origin in the center of the well; here city well number 9. The angle  $\theta$  is measured as shown in Fig. 6. The reference level for the hydraulic head is the equilibrium drawdown water level in the well. In the pumping test by Ver Steeg (1968), the drawdown in the well was 9.2 ft. Using this 9.2-ft well drawdown and the piezometric surface of Fig. 1, the reference level thus is at 874.5 ft - 9.2 ft = 865.3 ft. The hydraulic head difference across the aquifer to the well, hence, varies from 14.7 ft in the north to 4.7 ft in the south. The head distribution is indicated in Fig. 6. A line of longest distance from the well to the aquifer boundary is denoted by the symbol  $a$ . This line makes an angle of  $155^\circ$  with the line  $\theta = 0$  in Fig. 6. The length  $a$  is taken for convenience as the unit length. In reality, however,  $a = 4846$  ft. Since the well radius  $r_w$ , which includes the gravel pack, is 2.5 ft, the ratio  $r_w/a$ , which is used in the analysis, is equal to  $r_w/a = 0.00052$ . The location of the impervious till nose, sticking into the aquifer from the north, can now be determined. Field data indicate that the nose is nearly circular to the south. Taking the nose as aquifer boundary between  $\theta = 82.5^\circ$  and  $\theta = 117.5^\circ$  (as shown in Fig. 6), the aquifer boundary here can be described as a circle. The equation of this circle referred to an origin at city well number 9 can be given as

$$(r \cos \theta + 0.1a)^2 + (r \sin \theta - 1.03a)^2 - (0.36a)^2 = 0 \quad (1)$$

where  $r$  is the distance from the city well to a point on the nose and  $\theta$  the angle between  $r$  and the line  $OA$  in Fig. 6. (The line  $OA$ , when extended to the left, passes through city wells numbers 10 and 12). The problem now can be stated as: a hydraulic head function  $\phi$ , has to be found, such that it satisfies Laplace's equation in polar coordinates for radial flow, namely

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} = 0 \quad (2)$$

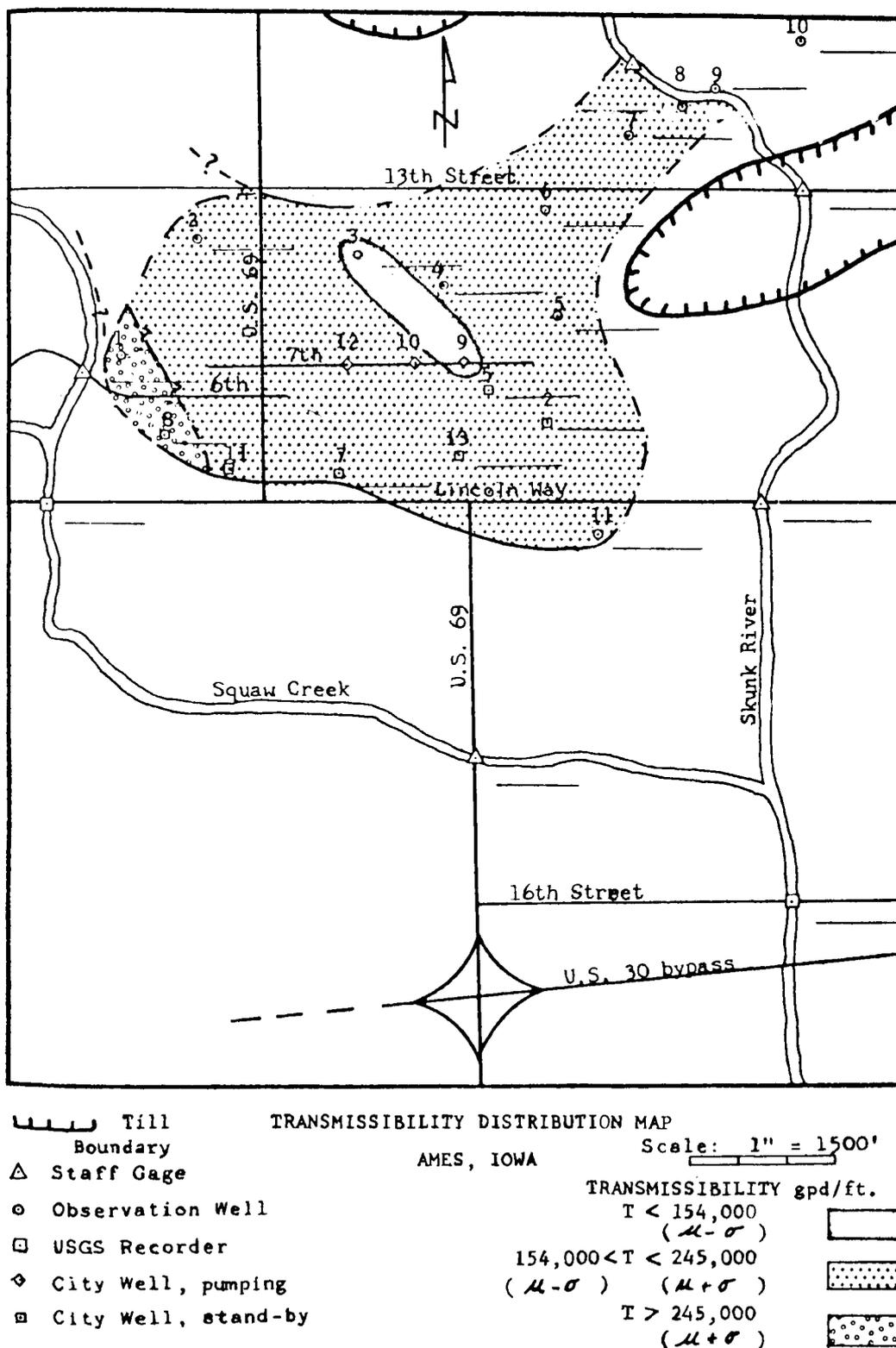
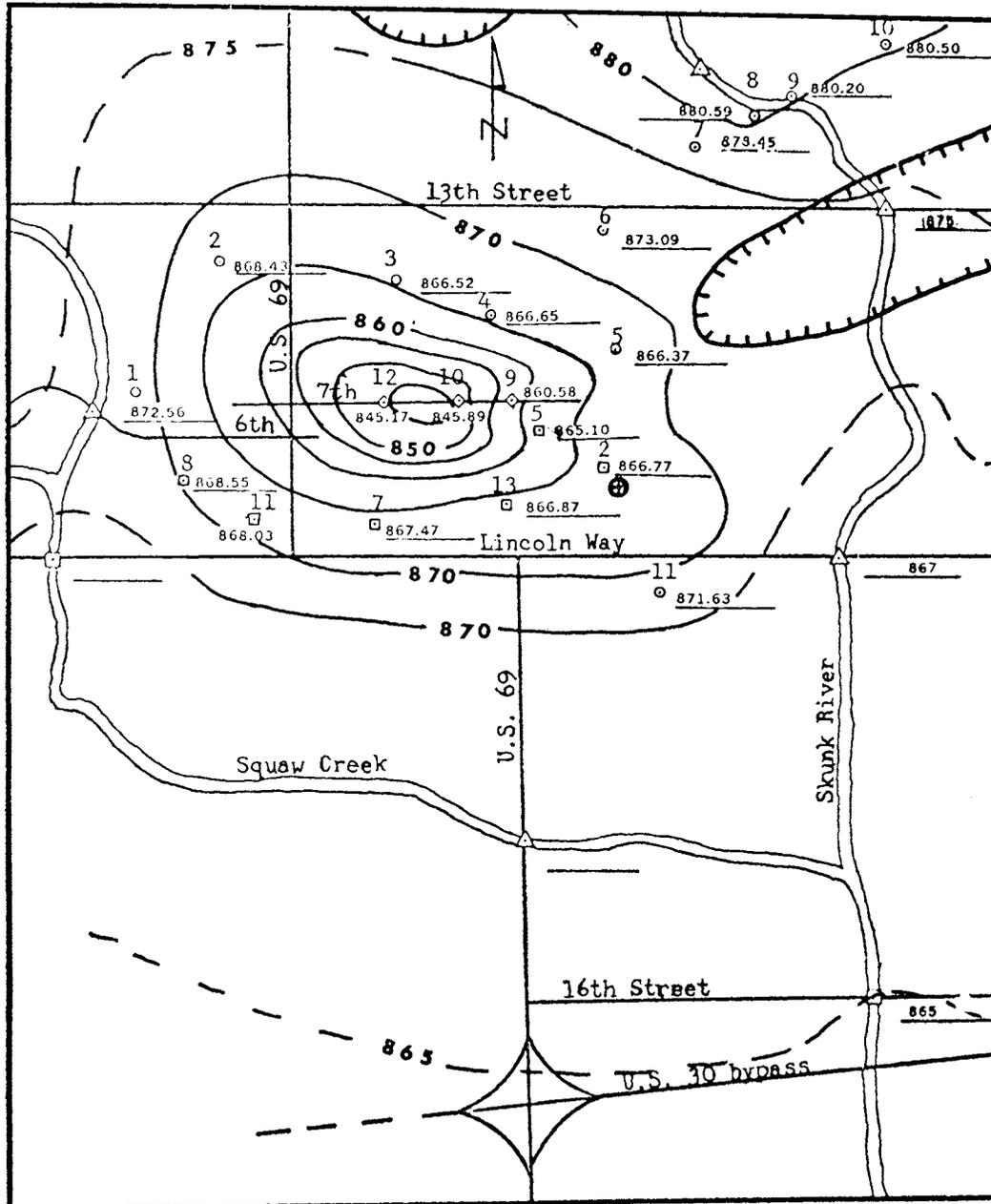


Figure 4. Transmissibility distribution map of the Ames aquifer (Akhavi, 1970).



- ⊕ Source of Phenol
- Till Boundary
- ▲ Staff Gage
- Observation Well
- USGS Recorder
- ◇ City Well, pumping
- City Well, stand-by

PIEZOMETRIC SURFACE MAP,  
 DURING PUMPING

Scale: 1" = 1500'  
 Contour Interval 5 Feet

Figure 5. Piezometric surface map in the Ames aquifer during an extensive pumping test (Akhavi, 1970).

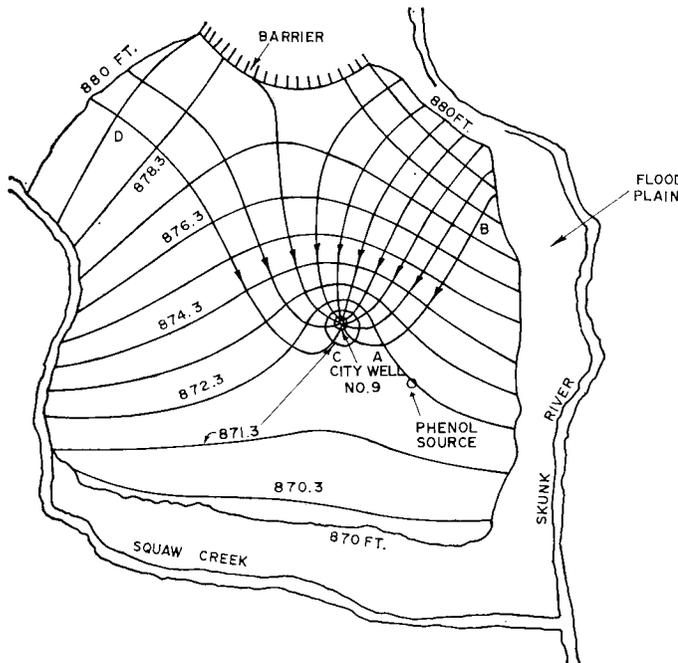


Figure 6. Boundaries of the Ames aquifer as determined from pumping tests, and distribution of the hydraulic head on the pervious part of the boundary.

subject to the boundary conditions (BC's)

BC 1:  $\phi = 0$  for  $r = r_w$ ,  $0^\circ \leq \theta < 360^\circ$

BC 2a:  $\phi = a$  constant value, as measured or determined, ranging from 4.7 ft to 14.7 ft, and shown in Fig. 6 along the outer boundary of the aquifer, for  $0^\circ \leq \theta \leq 82.5^\circ$ , and  $117.5^\circ \leq \theta < 360^\circ$

BC 2b:  $\partial\phi/\partial n = 0$  at the impervious boundary in the north of the aquifer for  $82.5^\circ < \theta < 117.5^\circ$

Boundary condition 2b states that no flow occurs across the impervious boundary in a direction normal to that boundary;  $\partial\phi/\partial n$  is the normal derivative.

As expression for the hydraulic head function  $\phi$ , the same expression is used as was used by Kirkham and Van der Ploeg (1971) for an irregularly shaped confined aquifer. The expression is

$$\begin{aligned}
 (\phi/\phi_0) = & A_{N0} \frac{1n(r/r_w)}{1n(a/r_w)} \\
 & + \sum_{m=1,3,\dots}^N A_{Nm} \left\{ \frac{(r/a)^{(m+1)/2} - [r_w^2/(ar)]^{(m+1)/2}}{1 - (r_w^2/a^2)^{(m+1)/2}} \right\} \sin \frac{m+1}{2} \theta \\
 & + \sum_{m=2,4,\dots}^N A_{Nm} \left\{ \frac{(r/a)^{m/2} - [r_w^2/(ar)]^{m/2}}{1 - (r_w^2/a^2)^{m/2}} \right\} \cos \frac{m}{2} \theta
 \end{aligned} \tag{3}$$

in which  $\phi_0$  is a unit hydraulic head,  $N$  is an integer, which goes to infinity for an exact solution, and in which the coefficients  $A_{Nm}$  have to be determined. In our analyses, we have taken  $N = 15$ . The way the coefficients  $A_{Nm}$  have to be determined is described in Van der Ploeg *et al.* (1971) or Kirkham and Powers

(1971), and will be discussed here only where the method is different from the procedure as used by Van der Ploeg *et al.* (1971). The coefficients  $A_{Nm}$  have to be determined by use of boundary conditions 2a and 2b. This can be done by considering the hydraulic head on the outer boundary of the aquifer as a function of the angle  $\theta$  and by considering the value of the normal derivative ( $=0$ ) on the impervious boundary also as a function of  $\theta$ .

Let  $f(\theta)$  be the value of the dimensionless hydraulic head as we are given it by boundary condition 2a. That is, we may write

$$f(\theta) = \text{value of } \phi/\phi_0$$

as given over the pervious part of the boundary.

If we let the subscript  $p$  stand for pervious boundary, we can write the last expression as

$$f(\theta) = [\phi/\phi_0]_p, \quad \begin{aligned} & 0^\circ \leq \theta \leq 82.5^\circ \\ & 117.5^\circ \leq \theta < 360^\circ \end{aligned} \tag{4a}$$

In the same way, if we let the subscript  $i$  stand for the impervious boundary, we can write

$$f(\theta) = \left[ \frac{\partial(\phi/\phi_0)}{\partial(n/a)} \right]_i, \quad 82.5^\circ < \theta < 117.5^\circ \tag{4b}$$

where we know physically that the left side of Eq. 4b is 0.

Eqs. 4a and 4b considered together define a step function for the range  $0^\circ \leq \theta < 360^\circ$ . This step function can be developed into a polynomial of the form

$$f(\theta) = \sum_{m=0}^N A_{Nm} u_m(\theta), \quad (m = 0, 1, 2, \dots) \tag{5}$$

by the modified Gram-schmidt method of Powers *et al.* (1967) as is shown in Kirkham and Van der Ploeg (1971). The  $A_{Nm}$  in Eq. 5 are the  $A_{Nm}$  of Eq. 3. Each  $u_m(\theta)$  of Eq. 5 is a 3-step function. The value of each  $u_m(\theta)$  for two of its steps is indicated by the  $\theta$  ranges and the  $f(\theta)$  value of Eq. 4a. The third step in each  $u_m(\theta)$  is for the range of  $\theta$  given at the right of Eq. 4b, and the value of each  $u_m(\theta)$  for this range ( $82.5^\circ < \theta < 117.5^\circ$ ) is (seen from Eq. 4b) to be given by the coefficient of  $A_{Nm}$  of Eq. 3 after its dimensionless derivative [namely,  $\partial(\phi/\phi_0)/\partial(n/a)$ ] has been taken and evaluated on the impervious boundary.

The part of  $u_m(\theta)$  for the two ranges of Eq. 4a is easily obtained. In Eq. 3 we substitute for  $r$  the measured distance  $R$  from the well to the outer aquifer boundary where  $R$  varies as  $\theta$  changes. Then for  $r = R$ , and  $\theta$  in the range,  $0^\circ \leq \theta \leq 82.5^\circ$  and  $117.5^\circ \leq \theta < 360^\circ$ , we obtain  $u_0(\theta)$  and  $u_m(\theta)$  from Eq. 3 (see Kirkham and Van der Ploeg, 1971) as

$$u_0(\theta) = \frac{1n(R/r_w)}{1n(a/r_w)} \tag{6a}$$

$$\begin{aligned}
 u_m(\theta) = & \frac{(R/a)^{(m+1)/2} - [r_w^2/(aR)]^{(m+1)/2}}{1 - (r_w^2/a^2)^{(m+1)/2}} \sin \frac{m+1}{2} \theta \\
 u_m(\theta) = & \frac{(R/a)^{m/2} - [r_w^2/(aR)]^{m/2}}{1 - (r_w^2/a^2)^{m/2}} \cos \frac{m}{2} \theta, \quad m = 1, 3, \dots, \text{ and } m = 2, 4, \dots
 \end{aligned} \tag{6b}$$

For Eqs. 6a and 6b values of  $R$  are determined graphically from a large drawing such as Fig. 6. The values of  $R$  were drawn for each  $2.5^\circ$ . For the impervious portion of the boundary (discussed in the next few paragraphs), values of  $R$  were obtained similarly.

The part of  $u_m(\theta)$  for the impervious part of the boundary is not as easy to obtain as Eqs. 6a and 6b. We shall now outline the steps for getting the value of  $u_m(\theta)$  for the range of  $\theta$  where the boundary is impervious.

At the impermeable boundary, for  $r = R$ , and  $\theta$  as  $82.5^\circ < \theta < 117.5^\circ$ , the function  $u_m(\theta)$  is governed by the dimensionless normal derivative

$$\frac{\partial(\phi/\phi_0)}{\partial(n/a)} = 0 \quad (7a)$$

which, for brevity, we may write as

$$\frac{\partial\phi}{\partial n} = 0 \quad (7b)$$

By vector notation Eq. 7b can be written as

$$\frac{\partial\phi}{\partial n} = \nabla\phi \cdot \bar{n} \quad (8)$$

In other words,  $\partial\phi/\partial n$  is equal to the dot product of the gradient of  $\phi$  and the unit normal vector to the circular impervious barrier. In polar coordinates the gradient of  $\phi$  may, from the geometry (or see Kaplan, 1959, p. 155), be written as

$$\nabla\phi = \left( \frac{\partial\phi}{\partial r}, \frac{1}{r} \frac{\partial\phi}{\partial\theta} \right) \quad (9)$$

To obtain the unit normal vector to a circular element of the impervious barrier, we may denote the left side of Eq. 1 as  $S(r, \theta)$ ; i.e.,

$$S(r, \theta) = (r \cos \theta + 0.1a)^2 + (r \sin \theta - 1.03a)^2 - (0.36a)^2 = 0 \quad (10)$$

The normal to this circle is obtained (Kaplan, 1959, p. 103) by taking the gradient of  $S$ . That is,  $n$  is given (or defined by) the expression

$$n = \nabla S = \left( \frac{\partial S}{\partial r}, \frac{1}{r} \frac{\partial S}{\partial\theta} \right) \quad (11)$$

To get the unit normal vector defined as  $\bar{n}$  from  $n$  of Eq. 12, we must divide the vector  $n$  by its length  $|n|$ . The unit normal vector thus can be written as

$$\bar{n} = \left( \frac{\frac{\partial S}{\partial r}}{|n|}, \frac{\frac{1}{r} \frac{\partial S}{\partial\theta}}{|n|} \right) \quad (12)$$

where the first term at the right is the component of  $\bar{n}$  in the  $r$  direction, and the second term at the right is the component of  $\bar{n}$  in the  $\theta$  direction.

We can get  $|n|$  as follows. We find  $\partial S/\partial r$  from Eq. 10 as

$$\partial S/\partial r = 2r + 0.2a \cos \theta - 2.06a \sin \theta \quad (13)$$

And we find  $(1/r)\partial S/\partial\theta$  from Eq. 10 as

$$(1/r)\partial S/\partial\theta = -(0.2a \sin \theta + 2.06a \cos \theta) \quad (14)$$

The magnitude of the vector  $n$  of Eq. 11 is known (Kaplan, 1959, p. 109) to be given by

$$|n| = \{(\partial S/\partial r)^2 + [(1/r)\partial S/\partial\theta]^2\}^{1/2} \quad (15)$$

so that use of Eqs. 13 and 14 in Eq. 15 gives us immediately an expression for  $|n|$  that we need not write here.

Turning now to  $\partial\phi/\partial n$  of Eq. 8, we write, from vector analysis,  $\partial\phi/\partial n$  in view of Eqs. 8, 9, and 11 as

$$\frac{\partial\phi}{\partial n} = \frac{\frac{\partial\phi}{\partial r} \frac{\partial S}{\partial r}}{|n|} + \frac{\frac{1}{r} \frac{\partial\phi}{\partial\theta} \frac{1}{r} \frac{\partial S}{\partial\theta}}{|n|} \quad (16)$$

where we now know  $|n|$ ,  $\partial S/\partial r$  and  $(1/r)\partial S/\partial\theta$  from Eqs. 13, 14 and 15. We need  $\partial\phi/\partial r$  and  $(1/r)\partial\phi/\partial\theta$ . We find  $\partial\phi/\partial r$  in dimensionless form from Eq. 3 as

$$\begin{aligned} \frac{\partial(\phi/\phi_0)}{\partial(r/a)} &= A_{N0} \frac{a}{r} \frac{1}{1n(a/r_w)} \\ &+ \sum_{m=1,3,\dots}^N A_{Nm} \frac{m+1}{2} \frac{a}{r} \left| \frac{\left(\frac{r}{a}\right)^{(m+1)/2} + \left(\frac{r_w^2}{ar}\right)^{(m+1)/2}}{1 - (r_w^2/a^2)^{(m+1)/2}} \right| \sin \frac{m+1}{2} \theta \\ &+ \sum_{m=2,4,\dots}^N A_{Nm} \frac{m}{2} \frac{a}{r} \left| \frac{\left(\frac{r}{a}\right)^{m/2} + \left(\frac{r_w^2}{ar}\right)^{m/2}}{1 - (r_w^2/a^2)^{m/2}} \right| \cos \frac{m}{2} \theta \end{aligned} \quad (17)$$

And we find the dimensionless potential form of  $(1/r)\partial\phi/\partial r$  from Eq. 4 as

$$\begin{aligned} \frac{1}{r} \frac{\partial(\phi/\phi_0)}{\partial\theta} &= \sum_{m=1,3,\dots}^N A_{Nm} \frac{m+1}{2} \frac{1}{r} \left| \frac{\left(\frac{r}{a}\right)^{(m+1)/2} - \left(\frac{r_w^2}{ar}\right)^{(m+1)/2}}{1 - (r_w^2/a^2)^{(m+1)/2}} \right| \cos \frac{m+1}{2} \theta \\ &- \sum_{m=2,4,\dots}^N A_{Nm} \frac{m}{2} \frac{1}{r} \left| \frac{\left(\frac{r}{a}\right)^{m/2} - \left(\frac{r_w^2}{ar}\right)^{m/2}}{1 - (r_w^2/a^2)^{m/2}} \right| \sin \frac{m}{2} \theta \end{aligned} \quad (18)$$

We can now put values of  $\partial S/\partial r$ ,  $(1/r)\partial S/\partial\theta$ ,  $|n|$ ,  $\partial\phi/\partial r$ , and  $(1/r)\partial\phi/\partial\theta$  found from Eqs. 13, 14, 15, 17, and 18, in Eq. 16 and find an expression (which we do not need to write) for  $\partial\phi/\partial n$  from which by multiplication by  $(\phi_0/a)$  we immediately find the dimensionless normal derivative  $\partial(\phi/\phi_0)/\partial(n/a)$  in terms of  $A_{Nm}$ . The coefficients of the  $A_{Nm}$  in this expression are the  $u_m(\theta)$  for the impermeable boundary. We now know (in view of Eqs. 6a and 6b) the  $u_m(\theta)$  for the complete range  $0 \leq \theta < 360^\circ$ ; and the procedure (in which we must also use Eqs. 4a and 4b) to get the  $A_{Nm}$  of Eq. 5 is now straightforward as described in Powers *et al.* (1967). With the  $A_{Nm}$  now known, we can substitute these  $A_{Nm}$  in Eq. 3 and determine the hydraulic head throughout the aquifer. We can then also (as in Kirkham and Van der Ploeg, 1971) determine the stream function  $\Psi$  and the well discharge  $Q$ .

## RESULTS AND DISCUSSION OF RESULTS

Fig. 7 shows the flow net for the Ames aquifer when the aquifer is pumped by city well number 9. The drawdown at the well is 9.2 ft, and the water in the well is standing at 865.3 ft (above sea level). Equipotentials are shown at 2-ft intervals. The streamlines shown are such that, between each adjacent pair, there flows 0.1 of the total well discharge. The figure shows that almost all the well discharge flows to the well from the north. This is a fortunate circumstance because a pollution source is present in the aquifer southeast of city well number 9. If this well were the only pumped well in the aquifer and if the drawdown would not exceed the present 9.2 ft, the quality of the well water would not be much affected by the phenol source. When city wells number 9, 10, and 12 (Fig. 5) are pumped all together, however, such that the drawdown at city well number 9 exceeds the present 9.2 ft considerably, then the quality of the water of well number 9 might well be and is found influenced by the polluting source. When city well number 9 is pumped alone, as in Fig. 7, we see that the phenol source is in a relatively stagnant area between the streamlines labeled *AB* and *CD*. The reason the area is stagnant is that 1/10 of the total flow enters the well from the area to the south of the lines *AB* and *CD*, and this is a large area compared with the area between, and north of, the lines *AB* and *CD* where 9/10 of the flow originates for the well.

If in the future, additional wells have to be drilled into the aquifer it seems desirable, both from a qualitative and quantitative standpoint, to locate such new wells north of the present ones (city wells numbers 9, 10 and 12).

A check on our method is readily made. When the test pumping of city well number 9 was actually performed by Ver Steeg (1968), there were 1280 gpm pumped from the well, with a

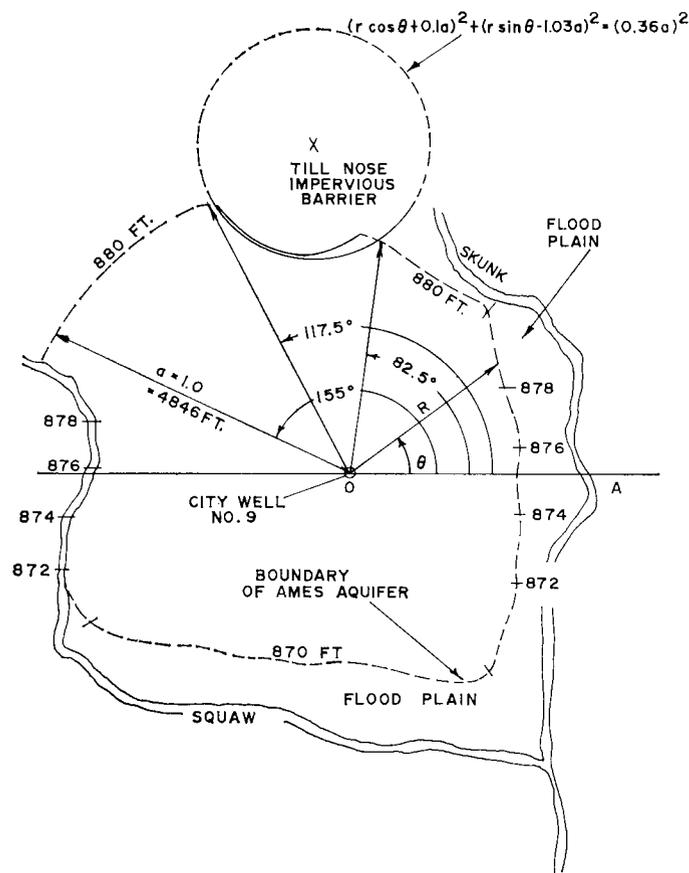


Figure 7. Flow net for the Ames aquifer, when the aquifer is pumped by city well number 9. Phenol source is shown southeast of the well.

drawdown of 9.2 ft. For this amount of drawdown, our theoretical analysis yields 1163 gpm, less than 10-percent difference, for engineering purposes acceptable.

The procedures we have worked out and field tested against the Ames aquifer are general, and therefore can be applied to the analysis of other confined aquifers. Work to take several wells into account by the methods described here is contemplated.

## CONCLUSIONS

Field data show that the Ames aquifer has many properties of an ideal horizontal confined aquifer of uniform thickness and permeability. A theoretical analysis of the flow patterns in the aquifer, when the aquifer is pumped by one completely penetrating well, has been made. A pumping test gave 1280 gallons per minute for 9.2 ft drawdown of city well number 9 of the aquifer; our theory gave 1163 gallons per minute. A flow net, prepared for a pumping test on city well number 9, shows that a 9.2-ft well drawdown, the quality of the well water is not much affected by a phenol source present in the aquifer. For a different location of the pumping well, or for a larger drawdown, the phenol source might be of concern. Because most of the drawdown of the piezometric surface is very near to the well, it may be possible to extend our method to 2 or more pumping wells if it is assumed that there is no interaction between the wells. For other confined horizontal aquifers with positive (source of flow) and negative (impermeable) boundaries, our method to calculate flow nets and well discharge can be used if enough field data are available.

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