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Statistical Analysis of Second Order Drainage Basins on Four Pleistocene Surfaces

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KARSTEN, RICHARD A. (University of Wisconsin Extension, 9722 West Watertown Plank Road, Milwaukee, Wisconsin 53226.) Statistical Analyses of Second Order Drainage Basins on Four Pleistocene Surfaces. *Proc. Iowa Acad. Sci.* 80(4):192-197, 1973. This study statistically compares certain morphometric characteristics of second order drainage basins developed on Pleistocene glacial surfaces of different ages and compositions. The morphometry of the basins was found to be partially controlled by the type and age of unconsolidated materials into which the streams are cutting. The analysis also suggests that the Iowan Erosional Surface is closely approximating a time-independent or equilibrium condition, while the other surfaces are in comparative disequilibrium.

INDEX DESCRIPTORS: Morphometry; Pleistocene; Tazewell; Kansan; Iowan Erosional Surface; Glacial Materials; Loess; Equilibrium; Time-Independent; Principal Components Analysis; Stepwise Multiple Discriminant Analysis; Posterior Probabilities.

NATURE OF THE ANALYSIS

Such workers as Leopold, Wolman and Miller (1964) and Ruhe (1952) have noted that the evolution of a drainage basin and its resultant topographic expression are constrained by time and lithology. In addition, Brush (1961) analyzed the hydraulic, geologic and basin characteristics of 16 streams in the Ridge and Valley province in central Pennsylvania and found that lithology controls certain aspects of watershed morphometry, e.g., type of bedrock affects the stream gradients and the valley-side slopes. Although Brush's study related geologic factors to drainage basin characteristics in the Paleozoic rocks of the Appalachian Mountains, he succinctly states that "the general process is no doubt similar in most areas. In large alluvial valleys bedrock cannot be much of an influencing factor; yet large, thick alluvial deposits and terraces are in a sense *bedrock* materials upon which the stream works to form the landscapes."

This study concerns itself with the role of differences in unconsolidated materials and statistically compares certain morphometric characteristics of second order drainage basins developed on Pleistocene glacial surfaces.[2] The format is such that it should provide the basis for more incisive investigations about the development of drainage basins on comparatively young unconsolidated materials.[3]

STUDY AREA

Iowa, a classic area of Pleistocene continental glaciation, affords an opportunity to assess the age (length of time exposed to the operating processes) and type of materials with regard to differences in drainage basin morphometry.

Previous Investigations

Gordon (1960), employing correlation techniques, suggested that drainage texture is related to the length of time the erosional agents have operated on the glacial drift surfaces of Iowa. McConnell (1966) analyzed slope characteristics of erosional terrain of the glaciated Upper Mississippi Valley and found that slope increases as fineness of texture increases (in keeping with van Burkalow, 1945) and that relief assumes less importance in explaining steepness of slope as time of exposure to the fluvial processes increases. Tuttle, Milling and Rusnak (1966) investigated valley sizes and shapes in Iowa and found that valley size varies directly with time and inversely with resistance of materials. They also noted that the valleys developed on Wisconsin surfaces are in greater disequilibrium than those valleys developed on pre-Wisconsin drift. Thomas and Tuttle (1967) investigated average slope characteristics on glacial materials in Iowa and found them related to the age of the drift. Knox (1970) studied channel adjustment as related to lithologic factors in small drainage basins in Iowa and found that the type and age of materials exert some influence on the hydraulic geometry of the streams.

The surfaces considered here include the Tazewell, the Kansan with two thicknesses of eolian deposits, and the Iowan Erosional Surface developed on drift of either late Nebraskan or early Kansan age whose overall topographic expression was sculptured prior to the thin to discontinuous mantling of Wisconsin loess in the large, central area of this erosional complex (Ruhe and others, 1968).

Description of Drift Surfaces

The Tazewell surface is an early Wisconsin till plain radiocarbon dated at 22,000 years B.P. (Ruhe, 1969). It is overlain by ten to 20 feet of Wisconsin loess, radiocarbon dated at > 14,000 years B.P. (Ruhe, 1969). This surface may be described as constructional topography which has been modified by fluvial activity and is level to gently undulating with an integrated drainage system. The only hilly land occurs along the major drainage ways (Ruhe, 1965). Ruhe (1969) contends that the integrated drainage net was well established at the time of the Cary glaciation (14,000 years B.P.) and has since been only slightly modified. If this is the case, the hydrophysical agents that sculptured the landscape on

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[2] The second order drainage basins were delimited on the basis of crenulations in the contour lines and major slope reversals.

[3] Other investigators, such as Horton (1945), Strahler (1952 and 1954), Schumm (1956), Hack (1957), Melton (1956a), Melton (1956b), Strahler (1958), Wolman and Miller (1960), Morisawa (1962) and Schumm (1966) related landscape development to lithologic and time controls.

the Tazewell surface were active at the same time that the eolian materials were being deposited.

The Kansan till plain is a dissected, loess-mantled, drift surface with a virtually integrated surface drainage network. Thwaites (1963) described this surface in Davisian terms as being in the stage of *advanced youth* because of the loess-capped, broad, tabular uplands that function as drainage divides. The age of the Kansan till is estimated as varying from 300 to 500 thousand years B.P. Ruhe (1969) states that the proglacial Missouri River valley was the source area for the overlying Wisconsin loess (formerly identified as Tazewell). The loess is thickest near the source area and thins to the east, the northeast and southeast, ranging from $>$ 100 feet in the west to $>$ 2 feet in the east (Thorpe, *et al.*, 1952; Ruhe, 1969; Iowa Geological Survey, 1970). It ranges in age from 16,500 to 29,000 years B.P. at its base (Ruhe, 1969). The broad, loess-mantled, tabular divides are apparently still in too immature a stage of development for an upland drainage system to have been established.[4]

The Iowan Erosional Surface (formerly known as the Iowan [Early Wisconsin] Drift Plain) is a multileveled plain or a descending undulating to rolling surface. Wisconsin loess, ranging in age from 22,000 to 40,000 years B.P., forms a constructional surface over eroded till of either late Nebraskan or early Kansan age. The loess ranges in thickness from 32 feet near the Cary surface in the west to less than eight feet to the east, northeast and southeast. The central part of the Iowan Erosional Surface is essentially loess-free. It has an integrated regional drainage system but does contain undrained uplands which are capped with varying thicknesses of eolian materials (Ruhe and others, 1968; Ruhe, 1970).

SCALE OF RESEARCH AND DATA COLLECTION

Ten map second order basins were selected on each of the four Pleistocene surfaces by a stratified random sampling method. Measurements were collected from 7.5 minute U.S. Geological Survey topographic quadrangles (Figure 1) for the following variables:

- X_1 basin relief, measured from the highest point on the basin divide to the confluence of the two second order streams; dimension L;
- X_2 available relief, measured from the highest point on the basin divide to the outlet of the third order stream; dimension L;
- X_3 angle of bifurcation in degrees;
- X_4 gradient of the first order streams, a sine measurement which is indicative of potential degradation G_1; $L/L = 0$;
- X_5 gradient of all streams in the basin, a sine measurement which is indicative of potential degradation as controlled by the second order stream G_2; $L/L = 0$;
- X_6 drainage density, a measurement derived by dividing total stream length by the basin area, which is indicative of the degree of drainage integration within the basin; $\Sigma L/A = L/L^2 = L^{-1}$;
- X_7 slope in percent, a sine measurement, measured orthogonally at six random points in each basin from basin divide to the stream; $S = v/h$, $L/L = 0$;
- X_8 shape ratio, which is the axial length of the basin divided by basin area; $L/L^2 = L^{-1}$;
- X_9 length of overland flow; $LOF = 1/2DD$ or $LOF = 1/L^{-1} = L$;
- X_{10} number of confluences;
- X_{11} basin orientation (azimuthal transformation).

[4] Shrader and Hussey (1953) imply that the drainage network of the Kansan surface was established early, then later re-established, and that the topography was dissected by short periods of great erosion and long periods of little erosion, which may explain the comparative youthfulness of the uplands.

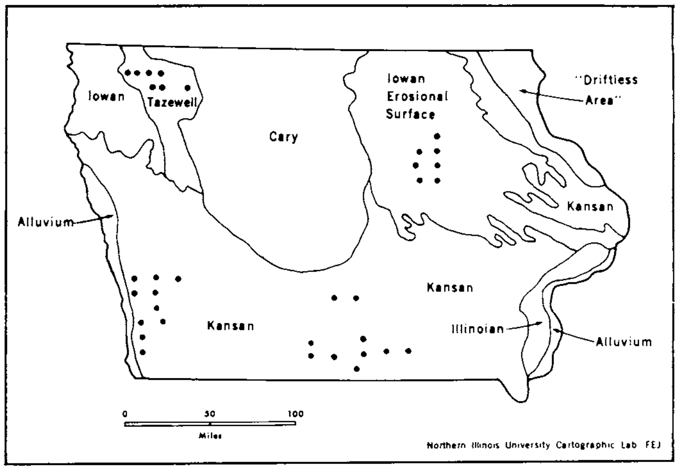

Figure 1. Locations of sample quadrangles and major glacial boundaries in Iowa (adapted from the Iowa Geological Survey, 1955).

STATISTICAL TECHNIQUES

Fractile diagrams were constructed to appraise the statistical normality of the variables at the 95 percent confidence level. Slope in percent required a logarithmic transformation while number of confluences required a square root transformation. All other variables are normally distributed.

A correlation analysis (Table 2) revealed certain redundancies among the eleven indices. Principal components analysis was employed in order to collapse this set of intercorrelated variables into a smaller set of mathematically orthogonal attributes of the variables.

The ability of the scores on the attributes to discriminate the age and type of Pleistocene glacial surfaces was assessed with stepwise multiple discriminant analysis.

HYPOTHESES

The overall hypothesis assessed in this analysis was that gross differences in age and type of materials on the four Pleistocene glacial surfaces account for, in part, morphometric differences and certain stream characteristics of the second order drainage basins. The hypothesis is justified in the following discussion in which several more specific hypotheses are stated.

The glacial materials of Iowa are being sculptured by the hydrophysical agents and processes. The results of this sculpturing are reflected in the topographic expression of the landscape. By rendering the variables into either dimensional, dimensionless or ratio forms (Strahler, 1958), and by controlling for scale differences by treating basins of but a single order, it is possible to evaluate the operation of the degradational processes as they are affected by age and type of materials.

It seems that currently developing second order basins with their constituent first order streams which are incising themselves into Kansan materials should have steeper stream gradients and valley-side slopes and more relief than those developing on younger materials. This becomes more apparent when one considers that the outlets of these basins in the older materials into higher order basins are all part of an integrated drainage net.

Those basins which are evolving on thick eolian deposits should also have steeper stream gradients and valley-side slopes and more relief than those incised into a thin mantle of loess overlying heterogeneous glacial drift.

Valley development is exaggerated in basins of thick loessial cover in spite of the high rates of infiltration normally associated with loessial materials. Loess is highly susceptible to abrasion by flowing water, and hence channel development proceeds more rapidly than in basins underlain by other types of glacial materials.

After the recession of the Kansan ice sheet, the hydrophysical agents began operating on the materials deposited by it. Some 16,500 to 29,000 years B.P., the Wisconsin loess was deposited on this constructional-erosional surface (a polygenetic surface [Milling and Tuttle, 1965]); it was at about the same time that the Wisconsin ice retreated and exposed the Tazewell surface. The Tazewell surface, while being modified by the hydrophysical processes, was mantled at the same time with Wisconsin loess (somewhat more than 14,000 years B.P.). The Iowan Erosional Surface, being either late Nebraskan or early Kansan in age and mantled with thin to discontinuous Wisconsin loess, has been subject to modification since sometime after the retreat of the Kansan ice sheet. It is reasonable that the second order drainage basins on the older surfaces should have a more mature topography, or be in a more advanced stage of development, than those on the younger surfaces. This suggests that those basins developed on older materials should display a greater tendency toward an equilibrium condition (Hack, 1960) and that these basins may be approaching their time-independent state (Chorley, 1962).

With regard to the variables, slope, basin elongation, basin relief, available relief, angle of bifurcation, stream gradient, drainage density and number of confluences should vary directly with one another. Furthermore, these parameters should display greater amplitude in the more mature basins and on the thicker, more stable eolian deposits.

TESTS OF THE HYPOTHESES

Table 1 enumerates the descriptive statistics for all 40 basins and contains descriptive statistics for each of the four Pleistocene surfaces.

The Tazewell surface has the smallest basin relief, X_1, and the thick loess Kansan surface the greatest amount.

The Iowan Erosional Surface contains the least available relief, X_2, followed closely by the Tazewell surface, while the thick loess Kansan surface has the greatest amount, suggesting that the Iowan Erosional Surface is more closely approaching an equilibrium state, while the Tazewell surface is only beginning to adjust itself to the hydrophysical agents. The thick loess Kansan surface appears to be in an intermediate stage of adjustment.

The Tazewell surface has the largest angle of bifurcation, X_3, followed closely by the thin loess Kansan surface. Theoretically, a larger bifurcation angle is indicative of a more mature topography; the older surfaces have evidently not reached that stage of development.

Stream gradients, X_4 and X_5, are smallest on the Tazewell and steepest on the thick loess Kansan surface. This suggests that the rate of downcutting on the Kansan surface is greater, as is the amount of material entrained. Conversely, the Tazewell surface's drainage is so poorly integrated that the rate of downcutting by the streams and the amount of materials entrained by them are quite small.

Drainage density, X_6, is largest on the thick loess Kansan surface and smallest on the Tazewell surface. Rather than suggesting that this index is indicative of a more mature, better integrated drainage system on the Kansan surface, it seems more realistic to consider it a function of slope, stream gradient and the presence of eolian materials. The large value for the Iowan Erosional Surface is indicative of a maturely dissected topography despite the low relief of the surface.

Slope in percent, X_7, is steepest on the thick loess Kansan surface and gentlest on the Tazewell surface. This appears to reflect directly the amount of basin relief and available relief, degree of drainage integration and fineness of materials. The intermediate slope value for the Iowan Erosional Surface is further suggestive of a mature surface that is approaching an equilibrium condition.

The shape ratio index, X_8, shows that the basins on the Tazewell surface are comparatively compact or circular and that those on the thick loess Kansan surface and the Iowan Erosional Surface are more elongated. This suggests two possibilities, (1) that the Kansan and Iowan surfaces are more mature and are approaching an equilibrium condition, and (2) that the Tazewell surface drainage is less efficient, supporting points made earlier regarding this surface.

Length of overland flow, X_9, is shortest on the Iowan Erosional Surface and the thick loess Kansan surface, and is greatest on the Tazewell. A comparatively short length of overland flow is indicative of an efficient, well integrated drainage network where surface runoff and resultant erosion are of greater intensity than where LOF is large.

The number of confluences, X_{10}, is largest for the Tazewell surface and the thin loess Kansan surface. An index value of one applies to a basin containing only two first order streams and the second order stream. Indices larger than one indicate that these basins contain *wasted* first order streams (Scheiddegger, 1970). Rather than assuming that the more streams a basin contains, the more efficient and integrated the drainage, the opposite appears more plausible. The most apparent reasons for this seem to be the relief, slope, gradient and materials parameters.

The basin orientation index, X_{11}, is approximately 45 degrees or a northeast-southwest trend for all 40 basins observed. Thus there is apparently little difference in the azimuthal orientation.

TABLE 1. SUMMARY STATISTICS FOR ELEVEN VARIABLES*

	Variable	All Basins		Tazewell		Iowan		Kansan (thin loess)		Kansan (thick loess)	
		μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
X_1	Basin Relief (feet)	127.75	51.71	87.00	25.84	104.00	16.47	119.00	26.85	201.00	37.55
X_2	Available Relief (feet)	165.75	57.37	134.00	40.33	132.00	24.86	154.00	32.73	243.00	42.18
X_3	Angle of Bifurcation (degrees)	79.40	25.74	97.60	24.15	71.40	16.03	92.10	17.79	56.50	22.66
X_4	Gradient of the First Order Streams (sine)	2.68	0.92	1.74	0.19	2.13	0.25	2.96	0.44	3.90	0.60
X_5	Gradient of All Streams (sine)	2.30	0.88	1.54	0.16	1.87	0.31	2.56	0.70	3.24	0.91
X_6	Drainage Density (miles)	1.84	0.42	1.57	0.40	1.92	0.36	1.78	0.45	2.08	0.35
X_7	Slope in Percent (sine)	0.64	0.26	2.41	0.84	3.66	0.95	5.42	1.79	9.44	2.63
X_8	Shape Ratio	1.30	0.41	0.97	0.35	1.40	0.39	1.31	0.28	1.49	0.45
X_9	Length of Overland Flow (ratio)	0.29	0.07	0.34	0.07	0.27	0.06	0.30	0.07	0.25	0.04
X_{10}	Number of Confluences	1.37	0.45	2.40	1.51	1.80	0.92	2.30	2.75	1.80	1.03
X_{11}	Basin Orientation (azimuth transform)	41.30	27.92	35.00	27.22	38.70	22.62	46.90	3.12	44.60	30.47

* μ—estimated mean
σ—estimated standard deviation

TABLE 2. PRINCIPAL COMPONENTS ANALYSIS INPUT MATRIX OF SIMPLE CORRELATIONS

r		BR X_1	AR X_2	AB X_3	GIS X_4	GAS X_5	DD X_6	SP X_7	SR X_8	LOF X_9	NC X_{10}	BO X_{11}
BR	X_1	1.000	.920*	–.489*	.793*	.639*	.265	.719*	.181	–.335*	–.034	.249
AR	X_2		1.000	–.400*	.728*	.593*	.233	.663*	.148	–.291	–.001	.164
AB	X_3			1.000	–.453*	–.465*	–.520*	–.499*	–.464*	.477*	.202	–.265
GIS	X_4				1.000	.890*	.425*	.886*	.425*	–.483*	–.124	.112
GAS	X_5					1.000	.442*	.840*	.562*	–.454*	–.400*	.056
DD	X_6						1.000	.499*	.637*	–.924*	–.174	.214
SP	X_7							1.000	.515*	–.570*	–.252	.053
SR	X_8								1.000	–.628*	–.684*	.070
LOF	X_9									1.000	.128	–.126
NC	X_{10}										1.000	.100
BO	X_{11}											1.000

* $r_{.95\ df\ (38)} > .312$

Principal Components Analysis

An examination of the simple correlation matrix (Table 2) suggests that the mix of variables has a high degree of colinearity which may be alleviated by a principal components analysis. The results of the principal components analysis are presented in Table 3. The components (P_1, P_2, P_3) are the mathematically orthogonal dimensions or fundamental attributes of the correlation matrix; the P grouping is little systems broken out of the main system which maximize the correlations and relationships. These mutually independent clusters of interrelated indices resolve 79.10 percent of the total variance in the correlation matrix. Based upon examination of the varimax rotated loadings, P_1 may be considered a potential erosion or gradient component, P_2 a texture component, and P_3 a shape component (Table 3).

TABLE 3. PRINCIPAL COMPONENTS ANALYSIS

	P_1	P_2	P_3
Component	Gradient	Texture	Shape
Eigenvalue	5.522	1.872	1.307
Varimax Solution (Rotated Loadings)			
Basin Relief	.927	.143	.160
Available Relief	.905	.071	.150
Angle of Bifurcation	–.408	–.604	–.018
Gradient First Order Streams	.895	.279	–.121
Gradient All Streams	.781	.316	–.383
Drainage Density	.165	.923	–.049
Slope in Percent	.815	.372	–.236
Shape Ratio	.175	.695	–.606
Length of Overland Flow	–.241	–.869	.061
Number of Confluences	–.056	–.214	.834
Basin Orientation	.085	.413	.554

Discriminant Analysis

The utility of appraising the morphometry of second order drainage basins with regard to the age and type of Pleistocene glacial materials that they are evolving on was assessed by several statistical procedures which comprise a stepwise multiple discriminant analysis. All tests are conducted at the 95 percent level of confidence.

These tests examine (1) the ability of each set of component scores to discriminate between ages and types of Pleistocene surfaces, (2) the significance of differences between vectors of the means on the principal components for all possible combinations of surfaces, and (3) the overall significance of discrimination. The results and the function for each class are given in Tables 4 and 5.

Table 4 shows that two of the three composite variables successfully discriminate between surfaces, holding the others constant, and that the linear combinations or functions have not evolved by chance.

TABLE 4. DISCRIMINANT FUNCTIONS FOR CLASSES OF SECOND ORDER DRAINAGE BASINS

Component		F
P_1 Gradient (potential erosion)		45.79*
P_2 Texture		9.07*
P_3 Shape		1.13
Tazewell	$f(\hat{T}_i) = -4.66910P_{i1} + 1.42767P_{i2} - 2.45978$	
Iowan Surface	$f(\hat{I}_i) = -2.08583P_{i1} + 1.48559P_{i2} - 0.78840$	
Kansan thin loess	$f(\hat{K}_i) = 0.11004P_{i1} - 0.09992P_{i2} - 0.00321$	
Kansan thick loess	$f(\hat{K}_i) = 6.64499P_{i1} - 2.81336P_{i2} - 5.21233$	

* $F_{.05\ df\ (3,34)} > 2.65$

Table 5 shows that there is a significant difference between the surfaces, or that the drainage basin morphometry of one surface is different from another, and that on the whole, the surfaces differ one from another (Wilks' Lambda).

TABLE 5. SIGNIFICANCE OF DIFFERENCE BETWEEN CLASSES

	F – Matrix			
	Tazewell	Iowan	Kansan (thin loess)	Kansan (thick loess)
Tazewell	X	5.224*	12.527*	71.221*
Iowan		X	4.327*	46.921*
Kansan (thin loess)			X	24.351*
Kansan (thick loess)				X

Wilks' Lambda = 0.157, F = 17.797**

* $F_{.05\ df\ (2,35)} > 3.27$

** $F_{.05\ df\ (6,70)} > 2.23$

The procedure also provides for the reconstitution of the classes, based upon the largest posterior probability of membership. The results are shown in Table 6. It is evident that differences in the scores on the independent dimensions of drainage basin morphometry are attributable to the age and type of Pleistocene glacial materials.

Perusal of the apparently anomalous basins indicates that the single Tazewell basin which has the characteristics of the Iowan Erosional Surface contains first order streams which have steeper gradients than the average Tazewell basin and is also more elongated than the other Tazewell basins. The two basins on the Iowan Erosional Surface that have the morphometric characteristics of the Tazewell surface have smaller stream gradients and drainage densities and longer lengths of overland flow than the other basins on the Iowan Erosional Surface. The basins developing on the thin loess Kansan surface displayed the greatest variability. This may be attributable to two factors: (1) the large loess-capped, tabular divides; and (2) the thin loess mantle as it relates to the loess-till contact. The three thin loess Kansan basins that have characteristics typical of the Iowan Erosional Surface have smaller basin and available relief than the other Kansan basins. The thin loess Kansan basin that is similar to the Tazewell basins has smaller basin and available relief, gentler stream gradients, less drainage density, and a longer length of overland flow than the typical thin loess Kansan basin. The thick loess Kansan basin which displays morphometric characteristics of the thin loess Kansan surface has a smaller gradient for all streams, smaller drainage density, a longer length of overland flow, and is more compact than the other basins in its class.

TABLE 6. NUMBER OF CASES CLASSIFIED INTO GROUPS

		TO			
		Tazewell	Iowan	Kansan (thin loess)	Kansan (thick loess)
FROM	Tazewell	9	1	0	0
	Iowan	2	8	0	0
	Kansan (thin loess)	1	3	6	0
	Kansan (thick loess)	0	0	1	9

The class means for the standardized composite variables are presented in Table 7. They suggest that the Iowan Erosional Surface more closely approximates a time-independent or equilibrium condition, which is compatible with Ruhe's theory (Ruhe and others, 1968). Furthermore, both Kansan surfaces, especially the thick loess surface, are being strongly incised by their surface streams and are in comparative disequilibrium. The Tazewell surface is only beginning to be modified by the hydrophysical agents and is also in disequilibrium.

TABLE 7. CLASS MEANS FOR THE COMPOSITE VARIABLES

	Tazewell	Iowan	Kansan (thin loess)	Kansan (thick loess)
P_1	–1.162	–.0247	0.007	1.302
P_2	–0.029	0.715	–0.056	–0.630
P_3	0.007	0.326	–0.198	–0.134

CONCLUSIONS

The morphometry of second order drainage basins developed on Pleistocene glacial surfaces (and certain characteristics of the streams flowing in them) is controlled, in part, by the type and age of unconsolidated materials that the streams are incising themselves into and that surface runoff is operating on.

As noted earlier, the format of this study is such that it should lead to expanded or complementary investigations about the development of drainage basins on Pleistocene glacial materials. However, more precise consideration (with objective measurement) will ultimately have to be given to the type and age of Pleistocene glacial materials when appraising the development or evolution of drainage systems on this type of surface, and in particular, their hydraulic geometry.

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