The impact of mathematics professional development on elementary teachers' mathematics content knowledge for teaching and implementation of innovative pedagogical practices

Vicki Oleson
University of Northern Iowa

Copyright ©2010 Vicki Oleson

Follow this and additional works at: https://scholarworks.uni.edu/etd

Part of the Science and Mathematics Education Commons

Let us know how access to this document benefits you

Recommended Citation
Oleson, Vicki, "The impact of mathematics professional development on elementary teachers’ mathematics content knowledge for teaching and implementation of innovative pedagogical practices" (2010). Dissertations and Theses @ UNI. 640. https://scholarworks.uni.edu/etd/640

This Open Access Dissertation is brought to you for free and open access by the Student Work at UNI ScholarWorks. It has been accepted for inclusion in Dissertations and Theses @ UNI by an authorized administrator of UNI ScholarWorks. For more information, please contact scholarworks@uni.edu.
THE IMPACT OF MATHEMATICS PROFESSIONAL DEVELOPMENT ON ELEMENTARY TEACHERS' MATHEMATICS CONTENT KNOWLEDGE FOR TEACHING AND IMPLEMENTATION OF INNOVATIVE PEDAGOGICAL PRACTICES

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree Doctor of Education

Approved:

Dr. Lynn E. Nielsen, Committee Chair

Dr. Glenn Nelson, Committee Member

Dr. Victoria Robinson, Committee Member

Dr. Jean Schneider, Committee Member

Dr. Jill Uhlenberg, Committee Member

Vicki Oleson
University of Northern Iowa

December 2010
ACKNOWLEDGEMENTS

My sincere appreciation is extended to the members of my dissertation committee: chairperson Dr. Lynn Nielsen, Dr. Glenn Nelson, Dr. Victoria Robinson, Dr. Jean Schneider, and Dr. Jill Uhlenberg. Their guidance and patience through the dissertation process will be long remembered. I would also like to extend special thanks to Dr. Glenn Nelson and Dr. Edward Rathmell for generously lending their expertise in mathematics education, and for providing direction and support throughout the course of my doctoral experience. A special thank you is also extended to Dr. Lynn Nielsen. His persistence, supportive supervision, and encouragement made completion of this dissertation possible. Finally, I am indebted to my colleagues, friends, and especially my husband and family for their essential support, tolerance, and understanding.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>LIST OF TABLES</th>
<th>ix</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td>xi</td>
</tr>
</tbody>
</table>

## CHAPTER I. INTRODUCTION

- Context of the Study ........................................ 5
- UNI's Contract with DoDEA .................................. 5
  - Iowa Core Curriculum .................................... 7
- *Making Sense* Courses ...................................... 8
  - *Making Sense* Course Development ...................... 8
  - *Making Sense of Numbers* ................................ 8
  - *Making Sense of Operations* ............................. 9
- Research Questions ........................................... 14
- Significance of the Study .................................. 14

## CHAPTER II. REVIEW OF LITERATURE

- Introduction .................................................. 16
- Teacher Change ............................................... 17
  - Teacher Change Theoretical Framework .................. 17
  - Teacher Change Defined .................................... 18
- Professional Development ................................... 20
  - Professional Development Theoretical Framework .... 20
  - Professional Development Defined ....................... 22
Data collection: Mathematical content knowledge test ........................................... 51
Data analysis: Mathematical content knowledge test ........................................... 51
Data Source #2: Stages of Concerns Questionnaire .................................................. 52
Data collection: Stages of Concern Questionnaires ............................................. 52
Data analysis: Stages of Concern Questionnaires ............................................... 53
Data Source #3: Levels of Use ............................................................................. 54
Data collection: Levels of Use ............................................................................. 55
Data analysis: Levels of Use ............................................................................. 56
Data Source #4: Innovation Configuration Map ....................................................... 56
Data collection: Innovation Configuration Map .................................................... 56
Data analysis: Innovation Configuration Map .................................................... 57
Summary .................................................................................................... 57

CHAPTER IV. RESULTS ......................................................................................... 59
Findings .................................................................................................................. 59

Jane ...................................................................................................................... 60
Jane: Change in mathematics content knowledge .............................................. 60
Jane: Change in stages of concern ................................................................. 60
Jane: Change in level of use .............................................................................. 62
Jane: Lesson implementation ............................................................................ 65

Deb ...................................................................................................................... 66
Deb: Change in mathematics content knowledge ........................................... 67
Deb: Change in stages of concern .................................................................... 68
Deb: Change in level of use ................................................................. 68
Deb: Lesson implementation ............................................................... 71
Mary ........................................................................................................ 72
Mary: Change in mathematics content knowledge .................................. 73
Mary: Change in stages of concern ......................................................... 73
Mary: Change in level of use ................................................................. 74
Mary: Lesson implementation ................................................................. 77
Ann .......................................................................................................... 78
Ann: Change in mathematics content knowledge .................................. 79
Ann: Change in stages of concern ......................................................... 79
Ann: Change in level of use ................................................................. 80
Ann: Lesson implementation ................................................................. 82
Sue .......................................................................................................... 82
Sue: Change in mathematics content knowledge .................................. 83
Sue: Change in stages of concern ......................................................... 84
Sue: Change in level of use ................................................................. 85
Sue: Lesson implementation ................................................................. 87
Pat .......................................................................................................... 89
Pat: Change in mathematics content knowledge ................................. 89
Pat: Change in stages of concern ......................................................... 89
Pat: Change in level of use ................................................................. 91
Pat: Lesson implementation ................................................................. 92
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discussions</td>
<td>118</td>
</tr>
<tr>
<td>Implications for Practice</td>
<td>120</td>
</tr>
<tr>
<td>Recommendations for Further Research</td>
<td>123</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>124</td>
</tr>
<tr>
<td>APPENDIX A: INSTRUCTIONAL SESSION SUMMARY</td>
<td>127</td>
</tr>
<tr>
<td>APPENDIX B: IMPLEMENTATION SESSION SUMMARY</td>
<td>129</td>
</tr>
<tr>
<td>APPENDIX C: ONLINE DISCUSSION BOARD SAMPLE</td>
<td>131</td>
</tr>
<tr>
<td>APPENDIX D: MDI:PSP INNOVATION CONFIGURATION MAP</td>
<td>144</td>
</tr>
<tr>
<td>APPENDIX E: STAGES OF CONCERN QUESTIONNAIRE</td>
<td>145</td>
</tr>
<tr>
<td>APPENDIX F: LEVEL OF USE BRANCHING CHART</td>
<td>148</td>
</tr>
<tr>
<td>APPENDIX G: CONTENT KNOWLEDGE TEST</td>
<td>149</td>
</tr>
<tr>
<td>APPENDIX H: SOCQ QUICK SCORING DEVICE</td>
<td>153</td>
</tr>
<tr>
<td>APPENDIX I: SOC DESCRIPTION OF THE STAGES</td>
<td>154</td>
</tr>
<tr>
<td>APPENDIX J: SOCQ INTERPRETATION CHART</td>
<td>155</td>
</tr>
<tr>
<td>APPENDIX K: SUMMARY OF TOOLS</td>
<td>156</td>
</tr>
</tbody>
</table>
### LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Professional Development within a Vygotskian Theoretical Framework</td>
</tr>
<tr>
<td>2</td>
<td>Stages of Concern</td>
</tr>
<tr>
<td>3</td>
<td>Statements by Participants (SoC)</td>
</tr>
<tr>
<td>4</td>
<td>Levels of Use</td>
</tr>
<tr>
<td>5</td>
<td>Statements by Participants (LoU)</td>
</tr>
<tr>
<td>6</td>
<td>Scoring of Pretest for Selected Participants</td>
</tr>
<tr>
<td>7</td>
<td>Pseudonyms for Selected Participants</td>
</tr>
<tr>
<td>8</td>
<td>Example of Concerns Statements Stage 3 Management</td>
</tr>
<tr>
<td>9</td>
<td>Components and Acceptable Variations MDI:PSP</td>
</tr>
<tr>
<td>10</td>
<td>Data Source: Teacher Change</td>
</tr>
<tr>
<td>11</td>
<td>Jane’s Lesson Analysis</td>
</tr>
<tr>
<td>12</td>
<td>Deb’s Lesson Analysis</td>
</tr>
<tr>
<td>13</td>
<td>Mary’s Lesson Analysis</td>
</tr>
<tr>
<td>14</td>
<td>Ann’s Second Lesson Analysis</td>
</tr>
<tr>
<td>15</td>
<td>Sue’s Lesson Analysis</td>
</tr>
<tr>
<td>16</td>
<td>Pat’s Lesson Analysis</td>
</tr>
<tr>
<td>17</td>
<td>Grade Band Group Content Knowledge</td>
</tr>
<tr>
<td>18</td>
<td>Highest Level of Use by Grade Band</td>
</tr>
<tr>
<td>19</td>
<td>Fidelity to the Model by Grade Band</td>
</tr>
<tr>
<td>20</td>
<td>High Pretest Scorers/Low Pretest Scorers Content Knowledge</td>
</tr>
<tr>
<td></td>
<td>Topic</td>
</tr>
<tr>
<td>---</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>21</td>
<td>Highest Level of Use by High/Low Pretest Scorers</td>
</tr>
<tr>
<td>22</td>
<td>Fidelity to the Model by High/Low Pretest Scorers</td>
</tr>
<tr>
<td>23</td>
<td>Level of Use by Whole Group</td>
</tr>
<tr>
<td>24</td>
<td>Fidelity to the Model by Whole Group</td>
</tr>
<tr>
<td>25</td>
<td>Change in Content Knowledge</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>Description</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Guskey's Model of the process of teacher change (1986)</td>
<td>4 and 119</td>
</tr>
<tr>
<td>2</td>
<td>Guskey's Model of the process of teacher change (2000)</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>Schematic of mathematical knowledge for teaching</td>
<td>28</td>
</tr>
<tr>
<td>4</td>
<td>Change in Jane's mathematics content knowledge January to June</td>
<td>61</td>
</tr>
<tr>
<td>5</td>
<td>Change in Jane's concerns January to June</td>
<td>62</td>
</tr>
<tr>
<td>6</td>
<td>Change in Deb's mathematics content knowledge January to June</td>
<td>67</td>
</tr>
<tr>
<td>7</td>
<td>Change in Deb's concerns January to June</td>
<td>69</td>
</tr>
<tr>
<td>8</td>
<td>Change in Mary's mathematics content knowledge January to June</td>
<td>74</td>
</tr>
<tr>
<td>9</td>
<td>Change in Mary's concerns January to June</td>
<td>75</td>
</tr>
<tr>
<td>10</td>
<td>Change in Ann's mathematics content knowledge January to June</td>
<td>80</td>
</tr>
<tr>
<td>11</td>
<td>Change in Ann's concerns January to June</td>
<td>81</td>
</tr>
<tr>
<td>12</td>
<td>Change in Sue's mathematics content knowledge January to June</td>
<td>84</td>
</tr>
<tr>
<td>13</td>
<td>Change in Sue's concerns January to June</td>
<td>85</td>
</tr>
<tr>
<td>14</td>
<td>Change in Pat's mathematics content knowledge January to June</td>
<td>90</td>
</tr>
<tr>
<td>15</td>
<td>Change in Pat's concerns January to June</td>
<td>91</td>
</tr>
<tr>
<td>16</td>
<td>Change in each grade band's content knowledge January to June</td>
<td>95</td>
</tr>
<tr>
<td>17</td>
<td>Each grade band's concerns in January</td>
<td>97</td>
</tr>
<tr>
<td>18</td>
<td>Each grade band's concerns in June</td>
<td>97</td>
</tr>
<tr>
<td>19</td>
<td>Change in high/low scorers' content knowledge January to June</td>
<td>101</td>
</tr>
<tr>
<td>20</td>
<td>Change in the high group's concerns January to June</td>
<td>102</td>
</tr>
</tbody>
</table>
21 Change in the low group’s concerns January to June ........................................102
22 Change in the whole group’s content knowledge January to June ...............105
23 Change in the whole group’s concerns January to June ...............................106
THE IMPACT OF MATHEMATICS PROFESSIONAL DEVELOPMENT ON ELEMENTARY TEACHERS' MATHEMATICS CONTENT KNOWLEDGE FOR TEACHING AND IMPLEMENTATION OF INNOVATIVE PEDAGOGICAL PRACTICES

An Abstract of a Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree Doctor of Education

Approved:

Dr. Lynn E. Nielsen, Committee Chair

Dr. Michael J. Licari
Dean of the Graduate College

Vicki Oleson
University of Northern Iowa

December 2010
ABSTRACT

This study sought to understand the process of change elementary teachers experienced as they participated in mathematics professional development. This investigation explored the impact of mathematics professional development on teachers' content knowledge for teaching, their ability to implement innovative pedagogical practices, and the relationship between these two components of teacher change. Data collected from a semester-long professional development course involving 20 teachers from rural southwest Iowa was analyzed for this study.

Data was collected from six teachers involved in the professional development course. Data collected during the professional development course included a pre and post test of teachers' content knowledge, a questionnaire designed to determine teachers' feelings about implementing innovative practice, online discussion posts, and observation notes. Concerns Based Adoption Model tools and techniques were used to analyze the data.

The results of the study indicated that the professional development experiences increased five of the six teachers' content knowledge. More dramatic increases occurred with teachers who measured lower levels of content knowledge at the beginning of the course. As a result of the professional development, teachers' concerns about implementing innovative pedagogical practices were altered. Concerns shifted from the need for information, personal concerns, and concerns about management of the innovative pedagogical practice to concerns about collaborating with colleagues and adjusting the innovation to achieve greater impact on students. Five of the six teachers
achieved basic use of the innovation while four of the six teachers implemented the innovation with fidelity to the model.

Teachers who entered the professional development experience with higher mathematical content knowledge tended to shift their concerns towards students with greater immediacy. These teachers more readily shifted from concerns for self to concerns about the impact of the innovation on students. Teachers who entered the professional development experience with lower mathematical content knowledge tended to maintain higher levels of concern. Higher levels of use of the innovation were achieved for teachers who entered the professional development experience with higher mathematical content knowledge. The ability to implement the innovation with fidelity to the model was not impacted by the teachers' level of mathematical content knowledge.

Three patterns emerged from the data related to the research questions. Teachers who possessed higher content knowledge tended to adapt to change more easily. These teachers tended to reflect behaviors that indicate higher levels of use of innovation. Higher levels of content knowledge did not tend to impact teachers' ability to implement innovative teaching practices with fidelity.
CHAPTER I

INTRODUCTION

A strong nation is linked to a mathematically competent citizenry. "The eminence, safety, and well-being of nations have been entwined for centuries with the ability of their people to deal with sophisticated quantitative ideas" (National Mathematics Advisory Panel, 2008).

The National Mathematics Advisory Panel (2008) describes the United States' historical position of leadership in mathematics as follows:

During most of the 20th century, the United States possessed peerless mathematical prowess – not just as measured by the depth and number of the mathematical specialists who practiced here but also by the scale and quality of its engineering, science, and financial leadership, and even by the extent of mathematical education in its broad population. (p. xi)

The United States' position of leadership in the area of mathematics is at risk. State, national, and international assessments indicate that the United States is not rising to the challenge of providing quality mathematics education. While U.S. students perform competitively on straightforward computation, these tests indicate that they lack conceptual understanding of mathematics (National Research Council [NRC], 2001).

The National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics (2000) describes a vision for school mathematics:

*Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals.* (p. 3)
The ability to enact this vision depends upon well-prepared, competent mathematics teachers. The NCTM *Principles and Standards* (2000) suggests, “Effective teaching requires knowing and understanding mathematics, students as learners, and pedagogical strategies” (p. 17). Understanding students, understanding content, and understanding the act of teaching are the key factors to instruction that translates into student achievement.

The 2009 report on teacher professional development entitled, *Professional Learning in the Learning Profession: A Status Report on Teacher Development in the United States and Abroad* (Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009) found that attempts to develop competent teachers through professional development were failing. Four findings that indicate inadequate attempts to provide effective professional development for teachers in the United States are as follows:

- More than 9 out of 10 U.S. teachers have participated in professional learning consisting primarily of short-term conferences or workshops.
- While teachers typically need substantial professional development in a given area (close to 50 hours) to improve their skills and their students’ learning, most professional development opportunities in the U.S are much shorter.
- American teachers say that much of the professional development available to them is not useful.
- American teachers spend much more time teaching students and have significantly less time to plan and learn together, and to develop high quality curriculum and instruction than teachers in other nations. (p. 5–6)

How do teachers develop an understanding of the students, the content, and the act of teaching whilst increasing competency? Hammerness et al. (2005) suggest three learning principles for facilitating teacher development. The first principle is based upon the idea that teachers come to the classroom with preconceptions about how the world, and teaching, works. Often, in order to develop their craft, teachers must relearn
concepts and information through a new paradigm. Existing attitudes and beliefs must be addressed so that teachers can see through a new paradigm. Second, teachers must develop competence in their ability to implement strategies so that they can "enact" what they know. Teachers must develop a deep knowledge of content and theory, and must understand how to contextualize and organize knowledge in a way that enables them to retrieve what they have learned and enact it in their classrooms. Third, teachers must develop a "metacognitive" approach to instruction and receive the tools needed to enable them to reflect on their teaching in order to help them understand and handle the complexities of life in the classroom.

The learning principles Hammerness et al. (2005) suggest apply to the development of both pre-service teachers and in-service teachers. The capacity of in-service teachers must be addressed. Professional development provides the opportunity to further expand teachers' understanding of students, the content, and the act of teaching. The purpose of professional development is to increase student achievement.

To be effective, professional development must lead to teacher learning and improved student outcomes. According to Garet, Porter, Desimone, Birman, and Kwang (2001), effective professional development involves three core features and three structural features. Core features that lead to effective professional development include a focus on content knowledge, opportunities for active learning, and coherence with other learning activities. Structural features that affect teacher learning include the form of the professional development activity, collective participation, and the duration of the
activity. A more in-depth discussion of effective professional development can be found in the review of literature.

The focus of this study is professional development of teachers already in the field of education and the change they experience through professional development. The theoretical underpinning for this study is based on research devoted to understanding and measuring teacher change. Guskey's model of the process of teacher change (1986) provides a suitable framework. Guskey suggests staff development that leads to change in teachers' classroom practices, leads to change in student learning outcomes, which results in change in teachers' beliefs and attitudes as shown in Figure 1:

![Figure 1. Guskey's Model of the process of teacher change adapted from a figure in an article entitled “Staff Development and the Process of Teacher Change,” by T.R. Guskey, 1986, Educational Researcher, 15(5), 7.](image)

Each step in Guskey's model is dependent upon change. This study seeks to understand the process of change elementary teachers experience as they participate in mathematics professional development that is designed to increase mathematical content knowledge for teaching and improve teachers' ability to implement innovative pedagogical practices. The impact teachers' mathematical content knowledge for teaching has on their ability to implement innovative pedagogical practices will also be examined.
Context of the Study

Prior to this study, another course, *Making Sense of Numbers* (UNI, 2009), was the focus of a pilot study. *Making Sense of Numbers* is the first in a series of six professional development courses designed to increase mathematical content knowledge and improve implementation of research-based teaching strategies. *Making Sense of Operations* (UNI, 2009), the second course in the six-course series, is the professional development focus of this study. *Making Sense of Numbers* and *Making Sense of Operations*, from the six-part professional development series *Making Sense* resulted from the convergence of two projects that occurred from 2005–2009. The first project involved professional development provided to teachers in the Department of Defense Education Activity (DoDEA) system through a contract with the University of Northern Iowa (UNI). The second project involved development of a common core curriculum for the state of Iowa. This project was undertaken by the Iowa Department of Education (IDoE). The convergence of these two projects provides the context for this study.

**UNI's Contract with DoDEA**

From 2005–2008, the researcher led a project for which the University of Northern Iowa (UNI) was under contract with the Department of Defense Education Activity (DoDEA). The purpose of this project was to provide high-quality professional development for elementary school teachers in the area of mathematics. The DoDEA selected *Developing Mathematical Ideas* (Schifter, Bastable, & Russell, 1999–2008), a mathematics professional development curriculum developed over several years that includes casebooks focused on mathematics content areas of number and operation,
geometry, measurement, data, and algebraic thinking. Developing Mathematical Ideas (DMI) cases highlight kindergarten through sixth grade teachers and students. The Education Development Center (EDC) produced DMI through a National Science Foundation (NSF) teacher enhancement project entitled Teaching to the Big Ideas. UNI was contracted by DoDEA to train facilitators to teach DMI workshops, facilitate DMI workshops for elementary teachers, and to move the DMI workshops from a face-to-face format that involved 10 half-day sessions to an online format that involved 10 online sessions.

The possibility of facilitating additional workshops emerged when the face-to-face workshops were moved into an online format. From 2005 to 2007, the DoDEA increased the number of workshops offered to their teachers. During a two-week period in the summer of 2005, six face-to-face workshops were facilitated. In the 2006–2007 contract year nearly twenty online workshops were added to the six face-to-face workshops bringing the total number of workshops facilitated to approximately twenty-six. This presented a need for additional DMI workshop facilitators, particularly for online courses.

To meet the need for additional DMI workshop facilitators, UNI recruited approximately 25 mathematics educators from across the state of Iowa to become workshop facilitators. This group of Iowa mathematics educators all possessed a minimum of a master’s degree in mathematics education. The group included Area Education Agency (AEA) mathematics consultants, university mathematics education faculty, and mathematics teachers. This group of mathematics educators attended a
weeklong DMI online facilitator training. An experienced facilitator mentored each newly trained facilitator when they taught their first workshop.

When Iowa educators became aware of the DMI professional development curriculum through their training, they recognized clear alignment between the philosophical approach and research base of the DMI workshops and current mathematics education initiatives in the state of Iowa and particularly the newly drafted *Iowa Core Curriculum* (Iowa Department of Education, 2010a).

**Iowa Core Curriculum**

The AEA mathematics consultants trained to facilitate DMI courses also received training on implementation of the Iowa Core Curriculum (ICC; IDoE, 2010a). These AEA mathematics consultants recognized common threads between the ICC and DMI.

The Iowa Core Curriculum provides a guide to delivering challenging and meaningful content to students that prepares them for success in life. The Iowa Core Curriculum identifies essential concepts and skills for kindergarten through 12th grade in literacy, mathematics, science, social studies, and 21st century skills. It also includes direction for teachers regarding effective instruction and assessment. It takes learning to a deeper level by moving students beyond superficial knowledge to deep conceptual and procedural knowledge. It also enhances student engagement by emphasizing interesting, robust, and relevant learning experiences. The 2008 legislative session, through Senate File 2216, requires all school districts and accredited nonpublic schools to implement the Iowa Core Curriculum (July 1, 2012 for grades 9 through 12 and 2014-15 for kindergarten through 8th grade). (IDoE, 2010a)

"The Iowa Core Curriculum for K–12 Mathematics identifies the essential characteristics, skills, and content of the world-class mathematics curriculum that Iowa needs" (IDoE, 2010b). In mathematics, the ICC focuses on providing deep understanding of important mathematics. As noted above, the ICC will be fully implemented for grades kindergarten through 8th by academic year 2014–2015. AEA
mathematics consultants are currently in the process of implementation of the ICC in the state of Iowa.

Making Sense Courses

Making Sense Course Development

Area Education Agency mathematics consultants trained to teach the DMI courses and involved with implementation of the ICC requested that the DMI courses be taught in Iowa. Through a grant secured from the Iowa Mathematics and Science Education Partnership (IMSEP), a pilot course was developed entitled Making Sense of Numbers (UNI, 2009). Making Sense of Numbers utilizes a DMI course called Building a System of Tens (Schifter et al., 1999a) for the mathematics content knowledge for teaching and the ICC essential characteristics and skills for implementation of innovative pedagogical practices. Two Area Education Agency mathematics consultants piloted Making Sense of Numbers for 25 teachers from northwest Iowa during spring 2009.

Making Sense of Numbers

The following overview is taken from the introduction in the facilitator guide of Making Sense of Numbers (UNI, 2009):

Making Sense of Numbers is a course developed for elementary teachers to engage in mathematics through exploration of mathematical content and research-based pedagogical practices. The ultimate goal is for students to be engaged in effective learning opportunities, which result in improved student achievement. The course is aligned to the Iowa Teaching Standards, and embeds the components of the Iowa Core Curriculum (IDoE, 2010a) and Iowa Professional Development Model (IDoE, 2010c). Resources for this course include Building a System of Tens: Casebook (Schifter, Bastable, & Russell, 1999a); Iowa Core Curriculum K–12 Mathematics (IDoE, 2010b) essential characteristics, concepts, and skills; and articles from mathematics education journals. (p. 1)
The mathematical content of *Making Sense of Numbers* focuses on the base-ten system. Participants in the course learn that the "base-ten system is a complex concept for students to grasp and is core to understanding the mathematical system" (p. 1).

*Making Sense of Numbers* focuses on student interviews (listening to the mathematical thinking of children) and problem-based instructional tasks. The course was revised in the fall of 2009. As a result of the revision, the mathematical content focus and the emphasis on student interviews remained the same, but the focus on problem-based instructional tasks was replaced with Meaningful Distributed Instruction: Preview for Number Sense.

*Making Sense of Operations*

The second course in the series of *Making Sense* courses and the course that is the focus of this study, *Making Sense of Operations*, was developed at UNI in Fall 2009. The mathematical content of *Making Sense of Operations* focuses on number operations of whole numbers and fractions: addition, subtraction, multiplication, and division. The innovative pedagogical practice of *Making Sense of Operations* focuses on Meaningful Distributed Instruction: Preview for Symbolic Procedure.

*Making Sense of Operations* was taught during the months January through June spring 2010. Full–day face–to–face Instructional Sessions were held January 19th, March 2nd, April 27th, June 9th and June 10th. Between face–to–face Instructional Sessions, teachers participated in Online Implementation Sessions.
A complete description of the objectives of the face-to-face Instructional Sessions, taken from the course facilitator guide, can be found in Appendix A. In summary, face-to-face Instructional Sessions focused on the following objectives:

- **Session 1: Making Sense of Operations** introduced participants to the course, the technology needed to complete assignments, and to other participants in the course.

- **Session 2: Counting Up, Counting Back, and Counting By** involved participants in reading case studies and viewing video to help them consider the ways students use counting to solve problems and explored Distributed Practice that is Purposeful and Meaningful.

- **Session 3: Addition and Subtraction as Models** highlighted cases in which children demonstrating more sophisticated understandings of operations than what was demonstrated by the children in Sessions 1 and 2.

- **Session 4: What is Multiplication? What is Division?** explored what multiplication and division are, and how all four operations are related to each other.

- **Session 5: When Dividing Gives an Answer Less than One** engaged in a mathematics activity to stretch their ideas about fractions.

- **Session 6: Combining Shares, or Adding Fractions** explored adding fractions with unlike denominators from both the perspective of children’s understanding and their own understanding of fractions.
• **Session 7: Taking Portions of Portions or Multiplying Fractions** looked at multiplication of fractions with factors less than one whole using an area model and a number line model for multiplying fractions.

• **Session 8: Multiplying Mixed Numbers** continued to examine multiplying fractions emphasizing mixed fractions and teachers solved multiplication of mixed fractions problems using the area model.

• **Session 9: Expanding Ideas about Division in the Context of Fractions** asked teachers to try to make sense of dividing fractions and complete a mathematics activity on solving a division problem involving fractions.

• **Session 10: Highlights of Related Research** highlighted the research that supports instructional strategies throughout the course and the Iowa Core Curriculum.

A complete description of the objectives of the Online Implementation Sessions, taken from the course facilitator guide, can be found in Appendix B. In summary, Online Implementation Sessions focused on the following objectives:

• **Implementation 1: What is Distributed Practice that is Meaningful and Purposeful?** broadened and deepened participants understanding of Meaningful Distributed Instruction with new learning and asked them to reflect upon their experience as a learner and as a teacher regarding distributed instruction.

• **Implementation 2: Representing** focused on Distributed Practice that is Meaningful and Purposeful in the form of a preview, specifically, a preview that uses representations of algorithms prior to learning algorithms. Participants
analyzed a standard algorithm or procedure in mathematics and explained why the
algorithm or procedure works using connected representations.

• Implementation 3: Connecting Deep Procedural and Conceptual Knowledge
explored what it means to build both procedural and conceptual knowledge.
Teachers created the first of ten MDI Previews for Symbolic Procedures for a
mathematical concept and receive feedback.

• Implementation 4: Understanding reviewed the Iowa Core Curriculum definition
of Teaching for Understanding and discussed implications for classrooms and
created the remaining nine previews for symbolic procedures.

• Implementation 5: Reflecting on Your Practice explored the power of teachers’
own internal voice as well as the external voices of collaborative colleagues to
refine the journey of becoming a reflective practitioner. Teachers developed a
reflective response describing learning that occurred as a result of implementing
Meaningful Distributed Instruction Previews for Symbolic Procedures into
classroom routines.

• Implementation 6: Course Evaluation completed by teachers.

Making Sense of Operations was co-facilitated in Creston, Iowa, by an Area
Education Agency 14 mathematics consultant and a middle school mathematics teacher
from the same Area Education Agency. The mathematics consultant and another
mathematics consultant enrolled in the course made classroom visits to the 20 teacher
participant classrooms to observe and document them implementing of Meaningful
Distributed Instruction: Preview for Symbolic Procedure. During the observations, the
mathematics consultants also provided coaching in the form of encouragement and suggestions for refinement.

Two main delivery methods for Making Sense of Operations were the face-to-face Instructional Sessions and the Online Implementation Sessions. The course co-facilitators both attended each of the face-to-face Instructional Sessions. The two Area Education Agency mathematics consultants monitored the online postings and responded to teacher comments and questions during the Online Implementation Sessions. Teachers responded to two sets of questions on an online discussion board during each of the six Online Implementation Sessions. Additionally, during each of the six Online Implementation Sessions, teachers responded to two other teachers' posts in their grade band. The Online Implementation Sessions provided teachers the opportunity to reflect and collaborate with other teachers. Online Implementation Session 4 and a sample of the posts from the second through fourth grade band can be found in Appendix C.

The pilot course development and facilitation of Making Sense of Numbers during the spring of 2009 prompted the research questions posed in this study. Making Sense of Operations is the professional development focus of this study and was facilitated during the spring of 2010. This study, focused on Making Sense of Operations, examines mathematical content knowledge for teaching, in this case referring to number operations. The study also examines innovative pedagogical practice, referred to as Meaningful Distributed Practice: Preview for Symbolic Procedure.
Research Questions

This study seeks to understand the process of change elementary teachers experience as they participate in mathematics professional development. The focus of this professional development is twofold. First, it is designed to deepen participating elementary teachers' mathematical content knowledge for teaching. Second, it is designed to improve participating elementary teachers' ability to implement innovative pedagogical practices. This study seeks to answer the following questions:

1. What is the impact of mathematics professional development on six elementary teachers' mathematical content knowledge for teaching?
2. What is the impact of mathematics professional development on six teachers' implementation of innovative pedagogical practices?
3. What impact does teachers' content knowledge for teaching have on their ability to implement innovative pedagogical practices?

Significance of the Study

The purpose of teacher professional development is to increase student achievement. Thomas Guskey (2002) states that:

High-quality professional development is a central component in nearly every modern proposal for improving education...Professional development programs are systematic efforts to bring about change in the classroom practices of teachers, in their attitudes and beliefs, and in the learning outcomes of students.” (p. 381)

Recent efforts have been launched to improve education by creating a fundamental shift in what children learn and how they are taught...however, relatively little systemic research has been conducted on the effects of professional development on improvements in teaching or on student outcomes (Garet et al., 2001). Educational
reform and improved student achievement depend on proficient teachers. Quality professional development increases teacher proficiency but requires change. Guskey’s (1986) model provides a framework for change and presents a variety of opportunities for future research. Guskey (2002) envisions future research based on the model of teacher change to “stimulate renewed interest in the various components of the change process, the nature of the relationship between components, and the transition from one component to the next” (p. 338). This study contributes to the research on teacher change that Guskey envisions through a better understanding of how teachers change as a result of professional development.
CHAPTER II
REVIEW OF LITERATURE

Introduction

This study seeks to understand the process of change elementary teachers experience as they participate in mathematics professional development. The goal of this professional development is to deepen participating elementary teachers' mathematical content knowledge for teaching and improve their ability to implement innovative pedagogical practices, thus increasing student achievement. To understand this process, the review of literature:

- Examines research on teacher change and professional development.
- Examines mathematics teaching.
- Describes the Making Sense courses and the specific mathematics content knowledge for teaching and the innovative pedagogical practice contained in the courses.
- Examines measurement tools used to measure the teacher change, focus of this study.

This review of literature examines each topic from the perspective of theoretical framework and definition. The relationship between the theoretical framework and definition of each topic and this study is identified.
Teacher Change

Teacher Change Theoretical Framework

According to Guskey (1986), “...staff [professional] development programs are a systematic attempt to bring about change—change in the classroom practices of teachers, change in their beliefs and attitudes, and change in the learning outcomes of students” (p. 5). Guskey (2000) extended the work of Lewin (1935); an early change theorist who suggested that in the process of change, change in attitudes and beliefs comes first in a progression. Based on Guskey’s (2000) research, he suggests a different progression:

...significant change in teachers’ attitudes and beliefs occurs primarily after they gain evidence of improvement in student learning. These improvements typically result from changes that teachers have made in their classroom practices.... The crucial point is that it is not the professional development per se, but the experience of successful implementation that changes their attitudes and beliefs. They believe that it works because they have seen it work, and that experience shapes their attitudes and beliefs. (p. 139)

Guskey’s model described in Evaluating Professional Development (2000) and described earlier in the introduction as a theoretical underpinning of this study is again highlighted in Figure 2.

Figure 2. Guskey’s Model of the process of teacher change adapted from a figure in a book entitled “Evaluating Professional Development,” by T.R. Guskey, 2000, 139. Copyright 2000 by Corwin Press, Inc.
Guskey's model of teacher change informs the design of the professional development described in this study and the process used to collect evidence of teacher change as they experience professional development.

Teacher Change Defined

How is teacher change defined? Franke, Carpenter, Fennema, Ansell, and Behrend (1998) studied teacher change in the context of professional development and conceptualized teacher change as a basis for continued growth and problem solving rather than a process of acquiring a fixed set of teaching skills or learning how to use a particular program of instruction. These researchers termed this type of change as "self-sustaining, generative change" (p. 67). Self-sustaining, generative change requires that teachers change their beliefs, their knowledge of what it means to learn, and their conceptions of classroom practice.

Another theorist, Korthagen (2008), further differentiates the meaning of teacher change in Linking Practice and Theory,

...there is a world of difference between two ways in which we can use the word change as a verb. The first is the transitive use of the word, for example, in the sentence "I wish to change this teacher." The second is the intransitive use, as in "teacher X changes." The former use of the verb to change implies that there is an external pressure, however subtle, put on the teacher. The latter sentence refers to change directed by the teacher him-or herself.... A major mistake when implementing innovations in education has been made by outsiders who wish to change things but who do not take into account the needs and concerns of the teachers and the circumstances in which they work. (p. 6)

As suggested by these theorists, in this study, teacher change refers to transformative change. The teacher directs this type of change but it is guided through
professional development that respects the needs and concerns of the teachers and the circumstances in which they work.

Hord, Rutherford, Hulling, and Hall (2006) further clarify this type of teacher change. In *Taking Charge of Change* they identify six key concepts of teacher change:

1. Change is a process, not an event. Change is a process occurring over time, usually several years.

2. Change is accomplished by individuals. The role of the individual is of utmost importance in the change process.

3. Change is a highly personal experience. Each person responds to change differently, and paying attention to each person's progress can enhance the change process.

4. Change involves developmental growth. Individuals express or demonstrate growth in terms of their feelings and skills.

5. Change is best understood in operational terms. Teachers naturally relate to change in terms of what it means to them and how it will affect their current classroom practice.

6. The focus of facilitation should be on individuals, innovations, and the context. The real meaning of any changes lies within people. The focus of change facilitators must be on the individuals involved. (p. 5–7)

Hord, Rutherford, Hulling, and Hall (2006) use the term "client-centered" (p. 7) to further clarify teacher change. The concept of client-centered change is consistent with self-sustaining, generative change described by Franke et al. (1998) and self-directed
change described by Korthagen (2008). This study seeks to understand the type of teacher change founded upon these concepts.

Professional Development

Professional Development Theoretical Framework

This study examines teacher change as a result of professional development. The professional development that is the focus of this study is grounded in a theoretical framework that aligns with Vygotsky's theories. Barohny Eun (2008) laid out an alignment between Vygotsky's theories and professional development that describes the theoretical framework for the Making Sense of Numbers and Making Sense of Operations professional development courses.

Barohny Eun (2008) suggests that while key elements and models have been identified related to the mechanism of how teachers acquire knowledge and skills to effectively teach, these elements and models have not been grounded in a unified theory. “Grounding professional development in a theoretical framework is not only important in revealing the process of development itself but also for devising plans that contribute to the effectiveness of professional development programs” (p. 135).

Eun (2008) grounds a theoretical framework for professional development in aspects of Vygotsky's theories of development relevant to effective professional development. The Making Sense of Numbers and Making Sense of Operations are based upon this theoretical framework.

Vygotsky's sociocultural theory of development is built upon the social origin and cultural mechanisms of development. Eun relied on four interrelated concepts
fundamental to understanding Vygotsky's developmental theories.

The first concept, social origin of mental functions, explains how the individual mental functions arise from specific social interactions and retain a social nature even in the most private spheres of human consciousness. Underlying Vygotsky's insistence on the social nature of psychological development is the second central concept, which is the unity of behavior and consciousness. The third concept, mediation, explains the specific mechanisms involved in the transition between social interaction and individual mental functioning, as well as the integratedness of behavior and consciousness. Finally, the fourth concept, psychological systems, serves as evidence of development. (p. 136)

Eun (2008) situates professional development within a Vygotskian theoretical framework, described in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Key theoretical concepts</th>
<th>Related professional development practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social interaction</td>
<td>Workshops, colloquia, seminars, mentoring, study groups</td>
</tr>
<tr>
<td>Internalization</td>
<td>Individually guided activities (video self-assessments, journal writing)</td>
</tr>
<tr>
<td>Mediation</td>
<td>Continuous follow-up support that includes the three types of mediators: tolls (material resources); signs (newsletters and journals); and other humans (professional networks)</td>
</tr>
<tr>
<td>Psychological systems</td>
<td>Development of professional development programs that focus on changing teachers' attitudes as well as instructional practices</td>
</tr>
</tbody>
</table>


Making Sense professional development courses are grounded within a Vygotskian theoretical framework. Teachers experience social interaction through face-
to-face workshops and interaction in online discussions. Teachers internalize learning through guided individual activities designed to increase their pedagogical content knowledge for teaching mathematics and they individually implement innovative pedagogical practices in their classrooms. Teacher efforts to increase their mathematical knowledge for teaching and their ability to implement innovative practices are mediated through online resources and observation in their classrooms by facilitators. *Making Sense* courses seek to impact psychological systems through changing teacher beliefs and practice.

**Professional Development Defined**

Guskey (2000) defines professional development as those “processes and activities designed to enhance the professional knowledge, skills, and attitudes of educators so that they might, in turn, improve the learning of students” (p. 16). Guskey further outlines three defining characteristics of professional development.

1. It is an intentional process. Steps to ensure the intentionality of professional development include: (a) begin with a clear statement of goals and purposes; (b) ensure that the goals are worthwhile; (c) determine how to assess the goals.

2. It is an ongoing process. To remain abreast of new knowledge and understanding, educators at all levels must be continuous learners throughout the span of their professional career.
3. It is a systemic process. A systemic approach to professional development involves both individual and organizational development to ensure improvement.

Garet et al. (2001) conducted a national study that sampled 1,027 mathematics and science teachers to determine the effect of different characteristics of professional development on teachers’ learning. This research identified three core features of professional development activities that have significant, positive effects on teachers’ self-reported increases in knowledge and skills and change in classroom practice. Three core features of effective professional development include:

- Effective professional development focuses on content knowledge.
- Effective professional development provides opportunities for active learning.
- Effective professional development maintains coherence with other learning activities in which teachers are engaged.

Through these core features, structural features significantly affect teacher learning. These structural features that impact the effectiveness of professional development include the form of the learning activity, the collective participation of teachers from the same school, grade, or subject, and the duration of the activity. This study found it more important to focus on collective participation and duration and the core features listed above than on the type of activity. Effective professional development included collective participation of groups of teachers over longer periods of time.
Darling-Hammond et al. (2009) of Stanford University prepared a more recent report, *Professional Learning in the Learning Profession: A Status Report on Teacher Development in the United States and Abroad*. The purpose of this report was to provide policymakers, researchers, and school leaders with a teacher-development research base that can lead to powerful professional learning, instructional improvement, and student learning. According to this report, the research shows the following:

...sustained and intensive professional learning for teachers is related to student-achievement gains. An analysis of well-designed experimental studies found that a set of programs which offered substantial contact hours of professional development (ranging from 30 to 100 hours in total) spread over six to 12 months showed a positive significant effect on student achievement gains. According to the research, these intensive professional development efforts that offered an average of 29 hours in a year boosted student achievement by approximately 21 percentile points. Other efforts that involved a limited amount of professional development (ranging from 5 to 14 hours in total) showed no statistically significant effect on student learning. (p. 9)

Other key findings of this report include the following:

1. Professional development should be intensive, ongoing and connected to practice.
2. Professional development should focus on student learning and address the teaching of specific curriculum content.
3. Professional development should align with school improvement priorities and goals.
4. Professional development should build strong working relationships among teachers.

This research on professional development provides a foundation for the *Making Sense* courses. The purpose of the *Making Sense* courses is to improve instruction to improve student learning. The developers of *Making Sense* courses recognize that quality professional development is an ongoing process of defining worthwhile goals and attempting to meet and measure those goals.
Understanding effective mathematics instruction is critical to improving student outcomes. *Preparing teachers for a changing world: What teachers should learn and be able to do* (Hamerness et al., 2005) highlights three general areas of knowledge, skills, and dispositions that are important for any teacher to acquire:

- Knowledge of learners and how they learn and develop within social contexts,
- Conceptions of curriculum content and goals: an understanding of the subject matter and skills to be taught in light of the social purposes of education, and
- An understanding of teaching in light of the content and learners to be taught, as informed by assessment and supported by classroom environment. (p. 5)

This review of literature on effective mathematics teaching focuses on what teachers must know and be able to do to be effective teachers of mathematics. This important section of the literature review defines and examines three theoretical frameworks for effective mathematics teaching.

**Mathematics Teaching Theoretical Frameworks**

The first theoretical framework is based upon the synthesis of research reported in *Adding it Up* [National Research Council (NRC), 2001]. The authors of *Adding it Up* (NRC, 2001) describe what it means to be mathematically proficient. The second theoretical framework is based upon the seminal work of Lee Shulman (1986) in which he identified the knowledge that grows in the minds of teachers. The third theoretical framework, based upon the work of Hill and Ball (2009), which extends Shulman's work and further defines mathematical content knowledge for teaching.
First theoretical framework for mathematics teaching. What does it mean to be mathematically proficient? This question could be posed in relation to students’ mathematical proficiency or teachers’ mathematical proficiency. The National Research Council (2001) used the term mathematical proficiency to capture what it means for anyone to learn mathematics successfully. Mathematical proficiency has five intertwined strands:

- Conceptual understanding—comprehension of mathematical concepts, operations, and relations.
- Procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
- Strategic competence—ability to formulate, represent, and solve mathematical problems.
- Adaptive reasoning—capacity for logical thought, reflection, explanation, and justification.
- Productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy. (NRC, 2001)

The theoretical framework for mathematical proficiency provides a basis for what teachers should know and be able to do and what should be their goal for their students’ understanding of mathematics.

Second theoretical framework for mathematics teaching. Shulman’s work during the 1980s provides a theoretical framework for understanding the specific knowledge that teachers need. Shulman (1986) suggested three categories of content knowledge: (a) subject matter content knowledge, (b) pedagogical content knowledge, and (c) curricular knowledge. Content knowledge refers to the subject matter content a teacher must understand. Teachers must not only understand that something is so, the teacher must further understand why it is so. Pedagogical content knowledge goes beyond knowledge
of the subject to the dimension of subject matter knowledge for teaching, which includes ways of representing ideas and understanding what makes a topic easy or difficult. The third type of knowledge Shulman (1986) suggests teachers must possess is curricular knowledge, which includes the programs designed for teaching a subject, a variety of instructional materials available and an understanding of when to use particular programs or materials.

**Third theoretical framework for mathematics teaching.** Hill and Ball (2009) suggest a theoretical framework to explain an extension of Shulman’s work that described pedagogical content knowledge. Hill and Ball coined the term mathematical knowledge for teaching and a schema is found in Figure 3.

Hill and Ball’s (2009) framework is similar to Shulman’s in that mathematical knowledge for teaching includes both subject matter knowledge and pedagogical content knowledge. It differentiates common content knowledge from specialized content knowledge in that teachers must be able to, for example, model or represent a mathematical concept. Mathematical knowledge for teaching is a finer-grained category of what Shulman termed pedagogical content knowledge. Knowledge at the mathematical horizon refers to the kind of mathematical peripheral vision needed in teaching when a view of the larger mathematical landscape is required.

This study seeks to understand the process of change elementary teachers experience as they participate in mathematics professional development designed to
deepen participating elementary teachers' mathematical content knowledge for teaching.

Effective mathematics teaching requires deep understanding of mathematics. These three theoretical frameworks describe the complex understanding of mathematics required to effectively teach mathematics.

Mathematics Teaching Defined

The National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* (2000) suggests that to be effective teachers must:

- Know and understand deeply the mathematics they are teaching.
- Understand and be committed to their students as learners of mathematics.
• Reflect and make continual efforts to seek improvement.
• Have frequent and ample opportunities and resources to enhance and refresh.
  their knowledge. (p. 17)

To effectively teach mathematics, Hill, Rowan, and Ball (2005) suggest that teachers must possess mathematical content knowledge for teaching:

By “mathematical knowledge for teaching,” we mean the mathematical knowledge used to carry out the work of teaching mathematics. Examples of this “work of teaching” include explaining terms and concepts to students, interpreting students’ statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematical concepts, algorithms, or proofs. (p. 373)

The examples listed in this definition paint a picture of mathematical content knowledge for teaching, a crucial component of effective teaching.

In a pamphlet authored by Douglas A. Grouws and Kristin J. Cebulla originally prepared for the Handbook of Research on Improving Student Achievement (2000), research findings suggest 10 key ideas about mathematics teaching that leads to improvement of student achievement. These key ideas about mathematics teaching include:

1. Provide students with the opportunity to learn. The extent of the students’ opportunity to learn mathematics content bears directly and decisively on student mathematics achievement.
2. Focus on meaning. Focusing instruction on the meaningful development of important mathematical ideas increases the level of student learning.
3. Students can learn both concepts and skills by solving problems.
4. Giving students both an opportunity to discover and invent new knowledge and an opportunity to practice what they have learned improves student achievement.
5. Teaching that incorporates students’ intuitive solution methods can increase student learning, especially when combined with opportunities for student interaction and discussion.
6. Using small groups of students to work on activities, problems and assignments can increase student mathematics achievement.
7. Whole-class discussion following individual and group work improves student achievement.
8. Teaching mathematics with a focus on number sense encourages students to become problem solvers in a wide variety of situations and to view mathematics as a discipline in which thinking is important.
9. Long-term use of concrete materials is positively related to increases in student mathematics achievement and improved attitudes towards mathematics.
10. Use of calculators in the learning of mathematics can result in increased achievement and improved student attitudes.

These key ideas about effective mathematics teaching identify what effective teachers of mathematics should do, and these key ideas also apply to how teachers should learn mathematics through professional development. As is outlined in the next section of the review of literature, the principles of effective mathematics instruction apply to both the way children learn mathematics, and the way teachers grow and develop their own understanding of mathematics.

**Meaningful Distributed Instruction**

The mathematical content of *Making Sense of Operations* (UNI, 2009) focuses on number operations of whole numbers and fractions: addition, subtraction, multiplication, and division. The innovative pedagogical practice of *Making Sense of Operations* focuses on Meaningful Distributed Instruction: Preview for Symbolic Procedure. Daniel Willingham (2002), a cognitive scientist who studies how people learn—and particularly how people remember—wrote on how massed versus distributed practice influences students’ long-term retention of factual knowledge. Willingham (2002) found that research evidence indicates “distributing study over time over several sessions generally leads to better memory of the information than conducting a single study session. This phenomenon is called spacing” (¶ 2). Distributing study over time and over several
sessions is fundamental to Meaningful Distributed Instruction: Preview for Symbolic Procedure.

Edward Rathmell (2009) described the specific type of Meaningful Distributed Instruction (MDI) that is the focus of the study, MDI Preview for Symbolic procedure in the facilitator guide for Making Sense of Operations:

Understanding a symbolic procedure means far more than “getting the right answer.” A mathematical symbolic procedure or written skill involves step-by-step thinking that leads from a computational problem to a solution. Memorizing this step-by-step procedure may enable a student to answer the problem, even answer it correctly. Yes, that is important, but understanding means much more. **To understand a symbolic procedure or skill, students must**

1. **Understand concepts** underlying the skill (refer to Meaningful Distributed Instruction: Developing Number Sense) and
2. **Connect**
   a. the **step-by-step actions** that can be used with models to solve problems to
   b. the corresponding thinking procedures that can be used to record those steps with paper and pencil.

A student with this deep understanding will understand the meaning of each of the symbols that is recorded in a symbolic procedure or algorithm, be able to illustrate the step-by-step process with manipulatives or diagrams, and be able to explain how the step-by-step actions with manipulatives or diagrams are connected to the symbols that get recorded in the algorithm. This deep understanding enables students to use the computational procedure flexibly in everyday situations. For example, if the numbers involved are nice, they may recognize that they can solve the same problem mentally and not even have to write the symbolic procedure on paper. (p. 8)

This section of the review of literature examined research on teacher change, professional development, mathematics teaching, and Meaningful Distributed Instruction. Next, this review of literature examines measurement tools used to measure teacher change that is the focus of this study.
Tools that Measure Change

Measuring Teacher Change

In addition to describing teacher change, Hall and Hord (2006) authored the Concerns Based Adoption Model (CBAM). CBAM provides tools and techniques for understanding change. When attempting to understand change, it is crucial to indentify what “it” (the change) is.

An important step for change success is to develop consensus about what full implementation should look like. One way to do this is to develop an Innovation Configuration Map (ICM), which is similar to a rubric for assessing innovation implementation. (p. 109)

An ICM was developed for the innovative pedagogical instructional practice highlighted in the Making Sense of Operations course (see Appendix D). This ICM serves as a guide for facilitators and administrators as they make classroom observations and it serves as a reflective tool for teachers as they implement MDI: Preview for Symbolic Procedure in their classroom.

A second CBAM tool measures the personal side of change. “The people who are involved in a change effort have personal reactions and feelings about the innovation and about their involvement in the change process” (p. 109). The personal side of change, people’s feelings and reactions to change, can impede change if not understood. The Stages of Concern Questionnaire (SoCQ) is a tool to measure the personal side of change.

A third CBAM tool focuses on the “behaviors of each person as he or she gradually learns about and becomes a competent user of an innovation” (p. 109). Levels of Use (LoU) is an observational and interview tool that measures how teachers use an innovation across time.
In this study, Guskey’s model provides the roadmap to observe teacher change. According to Guskey, professional development will lead to change in teacher practice, which will lead to change in student outcomes, and finally to change in teachers’ beliefs. This study will focus mainly on the first two steps of Guskey’s model, professional development leading to change in teacher practice. Hall and Hord (2006) provide a lens through which teacher change can be defined and examined. CBAM is a tool and technique for examining and measuring teacher change. These two theoretical frameworks each uniquely ground this study.
CHAPTER III

METHODOLOGY

This study seeks to understand the process of change elementary teachers experience as they participate in mathematics professional development. The focus of this professional development is twofold. First, it is designed to deepen participating elementary teachers’ mathematical content knowledge for teaching. Second, it is designed to improve participating elementary teachers’ ability to implement innovative pedagogical practices. This study seeks to answer the following questions:

1. What is the impact of mathematics professional development on six elementary teachers’ mathematical content knowledge for teaching?

2. What is the impact of mathematics professional development on six teachers’ implementation of innovative pedagogical practices?

3. What impact does teachers’ content knowledge for teaching have on six teachers’ ability to implement innovative pedagogical practices?

The methodology section of this study consists of a description of the theoretical framework of the methodology; a summary of the pilot study based upon a professional development course, Making Sense of Numbers (UNI, 2009); the research approach for this study; and a summary.

Theoretical Framework of Methodology

Lichtman (2006) describes a “generic approach” (p. 78) to qualitative research as a combination of several qualitative approaches rather than one approach. While this qualitative study relies heavily on case study research methods that combine participant
observation and review of documents, this study also relies on phenomenology as a method (Bogdan & Biklen, 2007). This study is designed to understand change in teachers as a result of mathematics professional development and the impact of a professional development course focused on increasing teachers' mathematics content knowledge for teaching and helping teachers implement innovative pedagogical practice. A phenomenological approach is evident in this study as this study seeks to understand the lived experience of teachers and how they change as they learn mathematics content knowledge for teaching and implement innovative pedagogical practices (Lichtman, 2006).

This study combines case study research methods with a phenomenological approach to form a generic approach in order to understand how teachers change as a result of mathematic professional development. This case study will focus on the change 20 teachers experienced as they participated in a professional development course consisting of 10 half-day instructional sessions and six online implementation sessions occurring over 20 weeks. Data on change will be gathered through pre and post testing of content knowledge, surveys, review of documents collected throughout the course, and classroom observations.

During spring 2009, a pilot study was conducted on facilitation of the Making Sense of Numbers course that prompted the incorporation of specific data collection into the Making Sense of Operations course. Following is a description of the pilot study process and the results of the pilot study.
Pilot Study *Making Sense of Numbers*

The professional development course that is the focus of this study, *Making Sense of Operations* (UNI, 2009), is the second in a series of courses designed to increase teachers’ content knowledge for teaching and to help them implement innovative teaching practices. The pilot course in the series, *Making Sense of Numbers* (UNI, 2009), was developed through a grant received by the researcher from the Iowa Mathematics and Science Education Partnership (IMSEP) and was first taught during spring 2009 to a group of 25 teachers in northwest Iowa. The online implementation sessions of both courses include an online discussion board designed to encourage teachers to reflect on their understanding of mathematics content knowledge for teaching and implementation of innovative pedagogical practices.

The researcher observed changes in teachers’ understanding of mathematical content knowledge for teaching and their ability to implement innovative pedagogical practices while analyzing the online discussion board postings of teachers participating in the pilot course *Making Sense of Numbers*. The online implementation sessions of *Making Sense of Numbers* include a discussion board on which teachers post responses to questions posed by the facilitator. Teachers post six to seven responses over the 16 weeks of implementation during the course.

**Stages of Concern**

While reading teachers’ posts on the online discussion board while participating in *Making Sense of Numbers*, it became apparent to the researcher that teachers’ feelings and perceptions about change were following the predictable Stages of Concern.
identified originally by Fuller (1969) and later by Hall and Hord in *Implementing Change: Patterns, Principles, and Potholes* (2006). Hall and Hord (2006) listed the following Stages of Concern and how that stage may be expressed in Table 2.

Table 2

*Stages of Concern*

<table>
<thead>
<tr>
<th>Stages of Concern</th>
<th>Expression of Concern</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refocusing</td>
<td>I have some ideas about something that would work even better.</td>
</tr>
<tr>
<td>Collaboration</td>
<td>I am concerned about relating what I am doing with what my co-workers are doing.</td>
</tr>
<tr>
<td>Consequence</td>
<td>How is my use affecting my clients (students)?</td>
</tr>
<tr>
<td>Management</td>
<td>I seem to be spending all of my time getting materials ready.</td>
</tr>
<tr>
<td>Personal</td>
<td>How will using it affect me?</td>
</tr>
<tr>
<td>Informational</td>
<td>I would like to know more about it.</td>
</tr>
<tr>
<td>Awareness</td>
<td>I am not concerned about it.</td>
</tr>
</tbody>
</table>


The progression predicted by Hall and Hord (2006) was evident through analysis of the *Making Sense of Numbers* course the online discussion posts. The posts revealed that the progression of teachers' feelings and perceptions about implementing innovative pedagogical practices aligned with Hall and Hord's Stages of Concern (SoC). The
progression through the Stages of Concern could be identified through the online
discussion board posts of a sixth grade teacher in Table 3.

Table 3

*Statements by Participants (SoC)*

<table>
<thead>
<tr>
<th>Response</th>
<th>Statement</th>
<th>Stage of Concern</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>“I thought that the majority of students would use the traditional algorithm...I was surprised with his process...and how he incorporated rounding...which leads me to implementing a change in how I carry out the mental math portion of the Saxon Math lessons.”</td>
<td>This statement indicates awareness and interest in more information. Level 0 Awareness Level 1 Informational</td>
</tr>
<tr>
<td>1b</td>
<td>“After having read the first chapter and the article by Joyce &amp; Showers then posing the mathematical problem to my class, I realized that it would make sense to change the way we do the mental math portion of our Saxon Math lesson...as I began to visualize how this implementation would work, I could foresee the same student being the ones to share their ideas. Though we are talking about doing mental math, I would like to incorporate my marker boards for students to show their work, which may help increase participation. For those I see having difficulty or for those repeatedly lacking participation, I may utilize our Math Completion time to do individual interviews.”</td>
<td>These statements indicate that the teacher understood how to implement after reading and began to personalize the process. She then moved into the phase of management, or how implementation would look and what implementation would require in the classroom. Level 2 Personal Level 3 Management</td>
</tr>
</tbody>
</table>

(table continues)
<table>
<thead>
<tr>
<th>Response</th>
<th>Statement</th>
<th>Stage of Concern</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>&quot;I was amazed at how this student communicated work she did to solve a</td>
<td>These statements indicate that the teacher moved into the consequence level. Her</td>
</tr>
<tr>
<td></td>
<td>problem lately and I was eager to work one-on-one with her again to check</td>
<td>concern was for how the change was affecting her student.</td>
</tr>
<tr>
<td></td>
<td>for more understanding...She has a good understanding of the concepts and</td>
<td>Level 4 Consequence</td>
</tr>
<tr>
<td></td>
<td>procedures, but her computation errors keep her from excelling!...I could</td>
<td></td>
</tr>
<tr>
<td></td>
<td>have asked her to find another way to solve the problem using addition.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Thus having her show the skill of adding three fractions instead of two at</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a time.&quot;</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>I have been trying to implement ways and activities to challenge my</td>
<td>These statements indicate that this teacher has moved into a phase of collaboration</td>
</tr>
<tr>
<td></td>
<td>students that use more effective questioning and more wait time and allow</td>
<td>and refocusing. She is asking how this change relates to what her co-workers and</td>
</tr>
<tr>
<td></td>
<td>them to develop and deepen their mathematical understanding. But it’s not</td>
<td>district are doing, and she is looking for ideas that may work better.</td>
</tr>
<tr>
<td></td>
<td>enough! The problem I’m dealing with lies in the ‘executive control’ and</td>
<td>Level 5 Collaboration</td>
</tr>
<tr>
<td></td>
<td>the environment we’ve created with the use of Saxon Math and the success</td>
<td>Level 6 Refocusing</td>
</tr>
<tr>
<td></td>
<td>that ‘appears’ to be happening. Hopefully the ICC (Iowa Core Curriculum)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>will help pave the way for needed change. If I understand correctly what</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Joyce &amp; Showers, the ICC, and the other related materials are saying, my</td>
<td></td>
</tr>
<tr>
<td></td>
<td>district has a lot of work to do in order to have full implementation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>accomplished.”</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>&quot;In order for me to continue to integrate problem-based instruction, there</td>
<td>These statements indicate a phase of collaboration and consequence. She is asking</td>
</tr>
<tr>
<td></td>
<td>are three key supports I will need...continued administrative support, to</td>
<td>for support and recognizing that the approach is beneficial to students.</td>
</tr>
<tr>
<td></td>
<td>build a network of colleagues interested in problem-based instruction, and</td>
<td>Level 4 Consequence</td>
</tr>
<tr>
<td></td>
<td>more professional development. I see the benefit in student learning!&quot;</td>
<td>Level 5 Collaboration</td>
</tr>
</tbody>
</table>

(table continues)
**Response Statement Stage of Concern**

<table>
<thead>
<tr>
<th>Response</th>
<th>Statement</th>
<th>Stage of Concern</th>
</tr>
</thead>
<tbody>
<tr>
<td>5a</td>
<td>“Problem-solving has been a collaborative focus for sixth grade teachers in my building. I hope to share and promote the instructional strategies I’ve learned from this class with this group as part of my network. From what I understand, CGI (Cognitively Guided Instruction) works hand-in-hand with Problem-Based Instruction.”</td>
<td>This statement indicates that the teacher recognizes the need to share with her colleagues. Level 5 Collaboration</td>
</tr>
</tbody>
</table>

*Note.* Adapted from posts from the online version of “Making Sense of Numbers,” by UNI (2009).

**Levels of Use**

Levels of Use, a second diagnostic dimension of the Concerns Based Adoption Model (Hall & Hord, 2006) was also evident upon examination of the online discussion board posts. Whereas the Stages of Concern addresses people’s reactions, feeling, perceptions, and attitudes, the Levels of Use focuses on behaviors and shows how users are acting with respect to specific change (Hall, Dirksen, & George, 2006, p. 1). The Levels of Use of an innovation listed in Table 4 indicate a level number and a description of the level.

Levels of Use (LoU) is measured through teacher interviews and observations, however, the researcher observed the progression of Levels of Use in the online discussion board posts of the *Making Sense of Numbers* course. The progression of statements made by a kindergarten through third-grade special education teacher, listed in Table 5, demonstrate an increased Level of Use of the innovative pedagogical practice of
student-centered instruction, which focuses on instruction based upon the thinking of the students.

Table 4

*Levels of Use*

<table>
<thead>
<tr>
<th>Levels of Use of the Innovation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 Nonuse</td>
<td>State in which the user has little or no knowledge of the innovation, has no involvement with the innovation, and is doing nothing toward becoming involved.</td>
</tr>
<tr>
<td>I Orientation</td>
<td>State in which the user has acquired or is acquiring information about the innovation and/or has explored or is exploring its value orientation and its demands upon the user and the user system.</td>
</tr>
<tr>
<td>II Preparation</td>
<td>State in which the user is preparing for first use of the innovation.</td>
</tr>
<tr>
<td>III Mechanical Use</td>
<td>State in which the user focuses most effort on the short-term, day-to-day use of the innovation with little time for reflection. Changes in use are made more to meet user needs than client needs. The user is primarily engaged in a stepwise attempt to master the tasks required to use the innovation, often resulting in disjointed and superficial use.</td>
</tr>
<tr>
<td>IVA Routine</td>
<td>Use of the innovation is stabilized. Few if any changes are being made in ongoing use. Little preparation or thought is being given to improving innovation use or its consequences.</td>
</tr>
<tr>
<td>IVB Refinement</td>
<td>State in which the user varies the use of the innovation to increase the impact on clients within immediate sphere of influence. Variations are based on knowledge of both short- and long-term consequences.</td>
</tr>
<tr>
<td>V Integration</td>
<td>State in which the user is combining own efforts to use the innovation with the related activities of colleagues to achieve a collective effect on clients within their common sphere of influence.</td>
</tr>
<tr>
<td>VI Renewal</td>
<td>State in which the user reevaluates the quality of use of the innovation, seeks major modifications or alternatives to the present innovation to achieve increased impact on clients, examines new developments in the field, and explores new goals for self and the system.</td>
</tr>
</tbody>
</table>

*Note.* Adapted from “Measuring Implementation in Schools: Levels of Use,” by G. E. Hall, D. J. Dirksen and A. A. George, 2006, p. 5. Copyright 2006 by SEDL
<table>
<thead>
<tr>
<th>Response</th>
<th>Statement</th>
<th>Level of Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&quot;My students need more modeling of thinking. When I do ask them to do a problem on their own, most students just sit there.&quot;</td>
<td>This statement indicates that the teacher was at a level of nonuse. Level of Use: 0 Nonuse</td>
</tr>
<tr>
<td>2</td>
<td>&quot;One thing that I learned from my interview is that I need to do more questioning in my classroom. I need to get these kids to explain their thinking without feeling like they are in trouble.&quot;</td>
<td>This statement indicates that the teacher explored student-centered instruction and started to think about what she might need to do to implement. Level of Use: I Orientation Level of Use: II Preparation</td>
</tr>
<tr>
<td>3</td>
<td>&quot;I have started questioning more, only to really get responses from two out of my three (students)...I am not sure if students are developing their understanding, but I have found that I am developing my understanding of their thinking by asking questions. I think that I will question more and try to get some of my kids to think more outside the box.&quot;</td>
<td>This statement indicates that the teacher is making an effort to implement on the day-to-day basis, and the use is meeting her needs more so than the students'. Level of Use: III Mechanical Use</td>
</tr>
<tr>
<td>4</td>
<td>&quot;The focus of my lesson was on fact families and their relationships. I learned that my students are very visual, and that some things need to be written differently for them to figure things out.&quot;</td>
<td>This statement indicates that the teacher is using student-centered instruction and is attempting to increase the impact on the students through refining the innovation. Level of Use: IVB Refinement</td>
</tr>
<tr>
<td>5a</td>
<td>&quot;My plan is to...look for new ways to teach math. I am going to take more time to discuss and question my students...I am planning on spending more time focusing on what they do not understand and finding new and meaningful ways to teach the concepts.&quot;</td>
<td>This statement indicates that the teacher is beginning to combine student-centered instruction with other activities to increase the effect. Level of Use: V Integration</td>
</tr>
</tbody>
</table>

(table continues)
Response | Statement | Level of Use
--- | --- | ---
5b | “I have noticed that I do question more and try to understand how a student solved a problem before I correct them and say to do it another way. I have noticed that two of my students are more willing and able to explain how they solved a problem.” | Level of Use: V Integration

Note. Adapted from posts from the online version of “Making Sense of Numbers,” by UNI (2009).

Change was observed when the researcher read the online discussion board posts of teachers while taking the pilot course, *Making Sense of Numbers*. The changes clearly followed Hall and Hord’s Stages of Concern and Levels of Use measures of the concerns-based adoption model. As a result of the change observed by reading the online discussion board posts in the pilot study, the researcher began to ask questions:

- If teachers’ concerns about innovative pedagogical practices changed, how could those changes be measured and interpreted using CBAM diagnostic tools?
- If teachers’ Levels of Use of innovative pedagogical practices changed, how could those changes be measured and interpreted using CBAM diagnostic tools?
- The focus of *Making Sense of Numbers* included teachers’ mathematical content knowledge for teaching. How could changes be measured in teachers’ mathematical content knowledge for teaching?
During the same time the researcher began to ask questions, she was leading a team developing a second course, *Making Sense of Operations* (UNI, 2009) to follow the pilot course, *Making Sense of Numbers*. The teams developing and facilitating the course were interested in measuring teacher change, so it was decided to include three Concerns Based Adoption Model (CBAM) measures in *Making Sense of Operations*, the Stages of Concern Questionnaire, the Levels of Use protocol, and a third measure, the Innovation Configuration Map. In addition, a pretest and posttest to measure change in teachers' mathematical content knowledge for teaching was also developed. These four measures were included in the development of the second course, *Making Sense of Operations*.

The first of the CBAM tools included in the development of *Making Sense of Operations*, the Stages of Concern Questionnaire (SoCQ), was described earlier in the review of literature. The SoCQ measures teachers' concerns as they experience change. The SoCQ used in this study can be found in Appendix E.

The second of the CBAM tools included in the development of *Making Sense of Operations*, the Levels of Use (LoU), was also described earlier in the review of literature. The LoU measures behavior. This instrument measures the degree to which an innovative teaching practice is used. It does not measure the quality or fidelity of implementation. The branching list of questions used to determine LoU can be found in Appendix F.

Innovation Configuration Map

The third of the CBAM tools included in the development of *Making Sense of Operations*, the Innovation Configuration Map (ICM), was also described earlier in the
review of literature. An ICM describes the components of an innovation and what teachers do as they implement each component of the innovation with variations, from poor to ideal. The ICM developed for *Making Sense of Operations* focused on Meaningful Distributed Instruction: Preview for Symbolic Procedure, a strategy designed to provide distributed instruction of an algorithm (see Appendix D). The ICM paints a picture of what Meaningful Distributed Instruction: Preview for Symbolic Procedure looks like in the classroom. The Innovation Configuration Map is also an observational tool for the facilitators who observe the teachers in the classroom using the strategy to determine the quality of the implementation or the fidelity with which the teacher implements the innovation.

**Pretest and Posttest of Mathematical Content Knowledge**

A fourth measure developed, the pretest and posttest of the mathematical content knowledge for teaching focused on the content taught in the *Making Sense of Operations* course (see Appendix G). The content taught in the *Making Sense of Operations* course focused on the four operations for rational numbers: addition, subtraction, multiplication, and division. Rational numbers include any number that can be written as a fraction, including whole numbers.

These three Concerns Based Adoption Model measurement tools, Stages of Concern Questionnaire, Levels of Use, and Innovation Configuration Map, and mathematics content pre and posttest were administered during facilitation of *Making Sense of Operations* during the spring of 2010. A more detailed description of the research approach for this study follows.
Research Approach

Professional Development Course Selection

The *Making Sense of Operations* course was selected as the focus for this study because the pilot study course, *Making Sense of Numbers*, clearly revealed the need for additional study into the change process teachers experience as they deepen their understanding of mathematics content knowledge for teaching and as they implement innovative pedagogical practices. Additionally, methods of data collection were in place.

Participant Selection

Participants were selected based upon their participation in *Making Sense of Operations* spring 2010. The group of 20 teachers participating in *Making Sense of Operations* spring 2010 participated in *Making Sense of Numbers* fall 2009 as part of a grant funded through the American Recovery and Reinvention Act by Area Education Agency (AEA) 14. These 20 teachers all teach elementary students in districts within AEA 14.

Of the 20 teachers enrolled in *Making Sense of Operations*, six teachers were selected as the focus of this study based upon their mathematics content knowledge at the beginning of the course. The teacher with the highest pretest score and the teacher with the lowest pretest score in each of three grade bands: kindergarten through first grades; second through fourth grades; and fifth through eighth grades were selected for further study.

The teachers’ content knowledge was assessed using a 20-question pretest (see Appendix G) scored using the following system:
• Questions 1 through 3 worth a total of four points each (one point for a correct answer, up to three points for an accurate explanation and representation)

• Questions 4 through 6b worth a total of two points each (either zero or two)

• Questions 7 through 12 worth one point each

• Questions 13 through 19 worth a total of four points each (one point for a correct answer, up to three points for an accurate explanation and representation)

• Questions 20a and 20b worth a total of two points each (either zero or two)

Three mathematics education faculty members from the University of Northern Iowa independently scored the tests using the consistent scoring system described above. When scored independently, the three faculty members each identified the same teacher as the high and the same teacher as the low scorer in grade bands 5 through 8 and 2 through 4 and the same teacher as the high scorer in grade band kindergarten through first grade. There was slight discrepancy in identification of the low scorer in grade band kindergarten through first grade. To address this discrepancy, the low scorer in this grade band was selected based upon the average of the three scores and the lowest range among the scorers.

The Table 6 below includes each teacher selected for this study, identified by participant number and grade band, and each scorer’s total raw score of 58 possible points for each participant. The difference between the highest scorer and the lowest scorer is identified in the Range column and the average of the three scores is identified in the last column.
Table 6

Scoring of Pretest for Selected Participants

<table>
<thead>
<tr>
<th>Participant Number</th>
<th>Grade Band</th>
<th>Scorer #1</th>
<th>Scorer #2</th>
<th>Scorer #3</th>
<th>Range of scores of the 3 scorers</th>
<th>Average of the 3 scorers</th>
</tr>
</thead>
<tbody>
<tr>
<td>#15</td>
<td>K-1st</td>
<td>46</td>
<td>42</td>
<td>50</td>
<td>8</td>
<td>46</td>
</tr>
<tr>
<td>#17</td>
<td>K-1st</td>
<td>29</td>
<td>33</td>
<td>34</td>
<td>5</td>
<td>32</td>
</tr>
<tr>
<td>#13</td>
<td>2nd-4th</td>
<td>46</td>
<td>50</td>
<td>52</td>
<td>6</td>
<td>49</td>
</tr>
<tr>
<td>#16</td>
<td>2nd-4th</td>
<td>19</td>
<td>21</td>
<td>20</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>#4</td>
<td>5th-8th</td>
<td>38</td>
<td>32</td>
<td>46</td>
<td>14</td>
<td>39</td>
</tr>
<tr>
<td>#7</td>
<td>5th-8th</td>
<td>15</td>
<td>14</td>
<td>23</td>
<td>9</td>
<td>17</td>
</tr>
</tbody>
</table>

Content knowledge pretest results for each participant were relatively consistent among the three scorers, suggesting that the six participants selected met the criteria set: the high scorer and low scorer in each of three grade bands, kindergarten through first grade, second through fourth grade, and fifth through eighth grade. A pseudonym given for each of the participants in the study is identified in Table 7.

Data Collection

Making Sense of Operations has two main goals. First, the course aims to deepen teachers' content knowledge related to number operations. Second, the course aims to improve teachers' ability to implement research-based teaching practices, in this case Meaningful Distributed Instruction: Preview for Symbolic Procedure (MDI:PSP). Four
forms of data collection were embedded into the *Making Sense of Operations* course during course development. These four forms of data were collected from all teachers in the Making Sense of Operations course. These forms of data collection included the following collection methods:

1. Teachers took a pretest on the first day of the course and a posttest on the last day of the course focused upon number operations, the course's mathematics content knowledge for teaching (see Appendix G).

2. Teachers completed four Stages of Concern Questionnaires (SoCQ) during four of the five face-to-face instructional sessions, January 19, March 2, April 28, and June 10, to identify their stage of concern, their feelings, about implementing innovative pedagogical practice (see Appendix E).

3. Teachers responded to questions and to each other related to implementation of innovative pedagogical practices on the online discussion board (see Appendix C), which revealed behaviors that identified their Level of Use of innovative practices (see Appendix F).

### Table 7

**Pseudonyms for Selected Participants**

<table>
<thead>
<tr>
<th>Participant Number</th>
<th>Grade Band</th>
<th>Pseudonym</th>
</tr>
</thead>
<tbody>
<tr>
<td>#15</td>
<td>K-1st</td>
<td>Jane</td>
</tr>
<tr>
<td>#17</td>
<td>K-1st</td>
<td>Deb</td>
</tr>
<tr>
<td>#13</td>
<td>2nd-4th</td>
<td>Mary</td>
</tr>
<tr>
<td>#16</td>
<td>2nd-4th</td>
<td>Ann</td>
</tr>
<tr>
<td>#4</td>
<td>5th-8th</td>
<td>Sue</td>
</tr>
<tr>
<td>#7</td>
<td>5th-8th</td>
<td>Pat</td>
</tr>
</tbody>
</table>
4. Teachers were observed implementing innovative pedagogical practices. The Innovation Configuration Map was used as a tool to determine the fidelity with which teachers implemented MDI:PSP and the quality of the implementation (see Appendix D).

A more detailed description of the four methods of measuring teacher change, including the administration technique and analysis technique for each measurement tool follows.

Data Source #1: Pretest and Posttest of Mathematical Content Knowledge

A mathematical content knowledge test was administered to measure change in teachers’ content knowledge related to number operations at the beginning of the course and at the end of the course. A detailed description of how the test was developed, administered, and analyzed follows.

Development of the mathematical content knowledge test. In conjunction with development of Making Sense of Operations, a mathematical content knowledge was developed (Appendix G). This content knowledge test was based upon the content taught during the course facilitation. The 20-item test worth a total of 58 points included three types of questions:

- Questions that required an answer, representation and explanation (one point for a correct answer, up to three points for an accurate explanation and representation).
- Questions that required teachers to write word problems (either zero or two points).
- Questions requiring a multiple-choice response (one point).
Questions on the mathematical content knowledge focused on addition and subtraction (15 of the 58 points), multiplication and division (15 of the 58 points), and fractions (28 of the 58 points).

Data collection: Mathematical content knowledge test. The mathematical content knowledge pretest was administered on the first face-to-face day of class, January 19, 2010. Teachers were given approximately one hour to complete the pretest.

The mathematical content knowledge posttest was administered on the last face-to-face day of class, June 10, 2010. Teachers were again given approximately one hour to complete the posttest.

Data analysis: Mathematical content knowledge test. The content knowledge pretest and posttest were the same questions. The same method of analysis was also used. As stated earlier, three mathematics education faculty members scored the pretest to ensure reliability. The following three steps were taken to further ensure reliability of scoring:

1. A consistent scoring method was devised and communicated to the three scorers.

2. The scorers met to score one teachers’ pretest. The scorers each scored the test independently, then discussed discrepancies in scoring and agreed to common scoring methods.

3. The researcher analyzed the discrepancies in scoring after pretesting. (The results of the analysis of scoring were shared earlier p. 47)
Data Source #2 Stages of Concerns Questionnaire

The Stages of Concern Questionnaires (SoCQ) was administered to measure teachers' feelings about change through examination of their concerns (Appendix E). A goal of Making Sense of Operations was to change teachers' instructional strategies by using Meaningful Distributed Instruction: Preview for Symbolic Procedure as an innovative teaching strategy. The teachers were asked to change the way they taught mathematics, which elicited concern. A detailed description of how the SoCQ was administered and analyzed follows.

Data collection: Stages of Concern Questionnaires. Teachers completed four Stages of Concern Questionnaires (SoCQ). SoCQ were administered during four of the five face-to-face instructional sessions, January 19, March 2, April 28, and June 10, to identify their stage of concern, and their feelings, about implementing innovative pedagogical practice. During the face-to-face instructional sessions, as a group, teachers were simply asked to complete the questionnaire with the understanding that the innovation in this case was Meaningful Distributed Instruction: Preview for Symbolic Procedure.

While four SoCQ were administered, several teachers did not return the questionnaire completed on April 28. It was determined that only the questionnaires administered on January 19, March 2, and June 10 would be used in this study. The decision to use only three questionnaires was made based on analysis that revealed that insignificant change occurred during the shorter intervals. Significant change was evident only when the first and last questionnaire was considered.
Data analysis: Stages of Concern Questionnaires. The SoCQ were hand-scored using the Stages of Concern Quick Scoring Device (Appendix H). Each of the 35 items on the questionnaire relates to one of the seven stages of concern. Five questions on the survey relate to each of the seven stages. For example, as shown in Table 8, five concern statements taken from the questionnaire, items 4, 8, 16, 25, and 34, relate to the task phase, Stage 3 Management:

Table 8

Example of Concern Statements Stage 3 Management

<table>
<thead>
<tr>
<th>Stage 3 Management</th>
<th>Concern Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>I am concerned about not having enough time to organize myself each day.</td>
</tr>
<tr>
<td>8</td>
<td>I am concerned about conflict between my interests and my responsibilities.</td>
</tr>
<tr>
<td>16</td>
<td>I am concerned about my inability to manage all the innovation requires.</td>
</tr>
<tr>
<td>25</td>
<td>I am concerned about time spent working with nonacademic problems related to this innovation.</td>
</tr>
<tr>
<td>34</td>
<td>Coordination of tasks and people is taking too much of my time.</td>
</tr>
</tbody>
</table>

Note. Adapted from “Measuring Implementation in Schools: The Stages of Concern Questionnaire,” by A. A. George, G. E. Hall, and S. M. Stiegelbauer, p. 27. Copyright 2006 by SEDL

“Profile Interpretation” (p. 31) was selected for this study based upon review of the interpretation procedures recommended by George, Hall, and Stiegelbauer (2006) in Measuring Implementation in Schools: The Stages of Concern Questionnaire. Profile Interpretation examines the percentile scores for all seven stages and interprets the meaning of the highs and lows.

The SoCQ data was analyzed based upon the initial scores, the January questionnaire, and the final scores, the June questionnaire, for each participant. Raw
scores and percentile scores were used to analyze the data. According to George et al. (2006), “The percentile figures are not absolute; instead they are relative to the other stage scores for that individual” (p. 32). The percentile figures were reported in the findings section of this study.

Both individual and group data were analyzed. Teachers were compared individually over time; as a group by grade band; as a group by high-pretest/low-pretest; and as a whole group.

While interpreting Stages of Concern, guidelines included reference to paragraph definitions for each SoC (see Appendix I), establishment of a holistic perspective, examination of high and low stages of concern and individual item responses (George et al., 2006). An interpretation chart was used to examine relative high and low scores (see Appendix J).

Data Source #3: Levels of Use

Levels of Use (LoU) is a qualitative measure of change in teacher behavior. One goal of Making Sense of Operations was to change teachers’ instructional strategies by their use of Meaningful Distributed Instruction: Preview for Symbolic Procedure (MDI:PSP) as an innovative teaching strategy. Change in teacher behavior in the classroom was required to accomplish this goal.

According to Hall et al. (2006), “Whereas SoC addresses the affective aspects of change, such as people’s reactions, feelings, perceptions, and attitudes, LoU focuses on behaviors and shows how users are acting with respect to a specific change” (p. 1). The authors go on to state, “Levels of Use is a behavioral phenomenon. It does not deal with
attitudes, emotions, or feelings. It also does not deal with the quality of the innovation. Instead, LoU presents behavioral profiles of eight different approaches to using an innovation” (p. 5). These eight approaches include:

- Level 0, Nonuse.
- Level I, Orientation, the user is acquiring information about the innovation.
- Level II, Preparation, the user is preparing for first use of the innovation.
- Level III, Mechanical Use, the user focuses on day-to-day use, little reflection
- Level IVA, Routine, use of the innovation is stabilized.
- Level IVB, Refinement, user varies the use of the innovation to increase impact
- Level V, Integration, user combines own efforts with colleagues’ use
- Level VI, Renewal, user reevaluates the quality of use of the innovation and seeks modifications to increase impact (p. 5)

Data collection: Levels of Use. Typically Level of Use is measured through a focused interview. This focused interview uses a branching technique to establish LoU (see Appendix F). In this study, the branching technique was used to establish LoU through analysis of written teacher responses.

While participating in Making Sense of Operations, the Online Implementation Sessions required teachers to post eleven responses to prompts on the online discussion board. In addition, for each of the eleven original responses, teachers were required to respond to two other teachers’ posts. An example of the prompts and posts can be found in Appendix C.
To collect this data, eleven original responses to prompts and all teacher responses to other teachers were downloaded from the online discussion board and printed. For the purposes of this study, only the original posts were analyzed. The teacher responses to other teachers were not analyzed, however, occasionally original responses to prompts were contextualized through the ongoing discussion between teachers as they responded to one another. Therefore it was necessary to read the discussion in its entirety.

Data analysis: Levels of Use. The discussion board posts were read for statements that revealed Level of Use. Using the branching technique each posting received a rating to establish LoU at the time of the posting. To increase reliability, two mathematics education faculty members read each of the posts and assigned a LoU.

Data Source #4: Innovation Configuration Map

An ICM is a tool that qualitatively measures of the quality of implementation on an innovation. An Innovation Configuration Map (ICM) paints a picture of what an innovation looks like when in use. “It [ICM] describes the innovation in action” (Hord, Stiegelbauer, Hall, & George, 2006, p. 2). In conjunction with development of Making Sense of Operations, an ICM for Meaningful Distributed Instruction: Preview for Symbolic Procedure was developed (Appendix D).

Data collection: Innovation Configuration Map. According to Hord, Stiegelbauer et al. (2006), “The most reliable method of data collection for research purposes is observation, or observation supplemented with interviews” (p. 31). In this study, field notes were compiled during observations or while watching video of the study
participants implementing Meaningful Distributed Instruction: Preview for Symbolic Procedure (MDI: PSP).

**Data analysis: Innovation Configuration Map.** In this study, the ICM was used to document the degree of fidelity to the original design of MDI: PSP. The ICM identifies six distinct components or appropriate teacher actions of MDI: PSP, listed in Table 9:

Notes resulting from observation or viewing video was compared to the ICM for alignment with the six components with acceptable variation. Also through the observation, the time required for teachers to implement MDI: PSP was compared with the five minutes ideally required to implement.

**Summary**

Table 10 summarizes the data source and the type of teacher change each data source documented in this study. Four types of data were collected to measure teacher change in four distinct ways. Each set of data provided valuable insight into the research questions posed by this study.

The chart found in Appendix K summarizes the format and focus of the *Making Sense of Operations* course, the content to be delivered, the strategy to be implemented as an innovative pedagogical practice, and the measurement tools to be used to collect data.
Table 9

*Components and Acceptable Variations MDI: PSP*

<table>
<thead>
<tr>
<th>Component</th>
<th>Teacher actions</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CUE</strong></td>
<td>1. Teacher cues the students that this is an MDI lesson and names the specific computational focus of the lesson</td>
<td>Teacher presents computational problem that previews a symbolic procedure to be taught. (Teacher makes the computation problem visible to the students in a horizontal format, i.e. 7+3)</td>
</tr>
<tr>
<td><strong>PRESENT</strong></td>
<td>2. Teacher presents a word problem that previews the specific computational procedure to be taught. (Teacher makes the computation problem visible to the students in a horizontal format, i.e. 7+3)</td>
<td></td>
</tr>
<tr>
<td><strong>MODEL</strong></td>
<td>3. Teacher sets up the problem. In whole class setting, teacher asks students how to use a model (selected by the teacher) to represent set up of the problem but not the answer (teacher sets up representation until students can assist). It is assumed that the students are familiar with the chosen model.</td>
<td>Teacher allows students to set up the problem at their desk if this can be done simply within the 5-minute time frame. It is assumed that the students are familiar with the chosen model.</td>
</tr>
<tr>
<td><strong>SOLVE</strong></td>
<td>4. Teacher solves the problem. Teacher asks a sequence of questions to “lead” students through the step-by-step actions with the model. Each action corresponds to the steps of the written symbolic procedure of the algorithm. The problem is solved only with the actions on the model not the written symbolic procedures. Teacher asks direct questions.</td>
<td></td>
</tr>
<tr>
<td><strong>ANSWER</strong></td>
<td>5. Answer the question. Teacher asks students, “what is the answer to the problem? How does this model show the answer to the problem?”</td>
<td></td>
</tr>
<tr>
<td><strong>SUMMARIZE</strong></td>
<td>6. Teacher briefly summarizes the step-by-step actions performed to solve the problem using the model.</td>
<td></td>
</tr>
</tbody>
</table>

*Note.* Adapted from “Making Sense of Operations.” Copyright 2009 by UNI

---

Table 10

*Data Source: Teacher Change*

<table>
<thead>
<tr>
<th>Data Source</th>
<th>Teacher change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. A mathematics content knowledge pre and posttest.</td>
<td>1. Change in teachers’ mathematical content knowledge</td>
</tr>
<tr>
<td>2. Stages of Concern Questionnaires.</td>
<td>2. Changes in teachers’ feelings, perceptions, or attitudes</td>
</tr>
<tr>
<td>3. Online Implementation Session discussion board posts.</td>
<td>3. Changes in teachers’ behaviors or actions in the classroom</td>
</tr>
<tr>
<td>4. Field notes from direct or video observations.</td>
<td>4. Teachers’ ability to implement innovative strategies</td>
</tr>
</tbody>
</table>
CHAPTER IV

RESULTS

The results chapter of this study is organized around the participants and the data sets collected. This section consists of the findings reported by individual teacher; by a comparison among groups, grade band groups and high or low pretest content knowledge groups; and by the whole group.

Findings

This section examines each of the teachers involved in this study through the lens of the data collected. Data were collected from 6 of the 20 participants involved in Making Sense of Operations. These six participants included two teachers from the kindergarten through first grade band, Jane and Deb; two from the second through fourth grade band, Mary and Ann; and two from the fifth through eighth grade band, Sue and Pat. The findings section includes the following information for each teacher:

- A brief description of each teacher and current teaching situation as stated in the first Online Implementation Session discussion board post.
- The results of their mathematics content pre and posttests.
- A description of the change in their concerns related to implementing new instructional strategies.
- A description of the change in their behavior changed related to implementing new instructional strategies.
• A comparison between Meaningful Distributed Instruction: Preview for Symbolic Procedure lesson taught by each teacher and the ideal lesson described in the Innovation Configuration Map.

This report of findings begins with a description of each individual teacher through the lens of the data sets and concludes with a comparison of these teachers by groups. The teachers will be grouped by grade band, and by either high pretest or low pretest scores. This report of findings ends with a summary.

Jane

Jane teaches first grade in a rural southwest Iowa town. As a student, Jane felt she was “somewhat” good at math. She recognizes that first graders demonstrate difficulty retaining concepts and ideas once they have completed a chapter. Jane feels that the first graders in her classroom are excited about math and do not like to miss math class.

Jane: Change in mathematics content knowledge. Jane scored the highest on the pretest in her grade band, Kindergarten through first grade. Jane’s raw score on the pretest was 46 of 58 or 79%. Jane’s raw score on the posttest was 51 of 58 or 88%. The difference between Jane’s raw pretest score and raw posttest score was 5 points from the first day of the Making Sense of Operations course to the last day of the course. The difference between Jane’s pretest percentage and posttest percentage was 9%, an 11% increase. Figure 4 graphically represents the change in Jane’s mathematics content knowledge.

Jane: Change in stages of concern. Based on the Stages of Concern Questionnaire (SoCQ) and using Profile Interpretation, Jane’s highest Stages of Concern in January
Figure 4. Change in Jane’s mathematics content knowledge January to June

were stage 0 Awareness and 1 Information. Jane’s high scores in these stages indicate that she wanted more information about Meaningful Distributed Instruction: Preview for Symbolic Procedure (MDI: PSP). Low scores in January were in the stages of 3 Management and 5 Collaboration in January reflect that Jane had little concern about managing the use of MDI: PSP and that she was not concerned about collaborating with others in implementation of MDI: PSP.

In June, Jane’s scores reflect that her initial concern about lack of information had been somewhat alleviated. Her concerns leveled and relative to other anxieties, Jane was more interested in consequences for her students and collaboration. Generally, Jane’s
degree of concern lessened between January and June. The change in Jane’s concerns is represented in Figure 5.

![Figure 5. Change in Jane’s concerns January to June](image)

**Jane: Change in level of use.** To determine change in Level or Use (LoU), online discussion board posts were examined. Jane’s first post to the online discussion board was made on January 27, and her last post was made May 12.

Jane remained at a level on nonuse until mid-February. Jane’s focus during these first weeks of the course was still on the strategy that she learned during the previous course. On February 15, Jane recognized the difference between the teaching strategy she learned during the previous course and the new strategy, Meaningful Distributed
Instruction: Preview for Symbolic Procedure (MDI: PSP). At this time, Jane also recognized the value of MDI: PSP. By mid-February, Jane’s level of use was LoU II Preparation.

By March 24, Jane was preparing to use MDI: PSP in her classroom. She was writing plans to do so, and was aligning her plans with the research she was reading and with the Iowa Core Curriculum. While planning, Jane viewed a demonstration video, and actually made attempts to refine the strategy to increase the impact on her students. This is evident in the following quote from the online discussion board:

It seems to me that in the videos, the teacher was saying that one idea should be presented for the 10 days. I think there should be a sequence of difficulty to follow and each should have its own preview. With 2-digit addition (no regrouping), I would start with numbers with zeros, like 20 + 30, and then move on to numbers that have zero and ones, i.e. 20 + 34. After spending 3 days on a group, I would then move on and end with 3 – 4 days of mixed zeros and no zeros.

At this time, Jane’s level of use was LoU II Preparation with evidence of LoU IVB Refinement. As Jane prepared to use MDI: PSP she was also refining the strategy. Her refinements were well within the original intent of MDI: PSP.

In April, Jane began to collaborate with a colleague in her building to choose problems for the MDI: PSP lessons. This indicates LoU V Integration, a level of use in which the user is combining her own efforts with the efforts of colleagues to achieve a collective effect on students.

In May, Jane posted about her inner voice and about her external voice at the time. In a third posting, she described the difference between Meaningful Distributed
Instruction: Preview for Number Sense, the strategy taught in the first course, and

Meaningful Distributed Instruction: Preview for Symbolic Procedure.

Inner voice
As this semester began with the introduction of previews for symbolic procedures, I experienced a sense of excitement about a strategy that might achieve the learning of meaningful math for my first grade students. It was well received by my students and they have seemed to master the algorithm with more understanding than in past years. I am concerned about another year when I will surely be using the previews earlier and more often for the understanding of many algorithms and skills. I am anxious about presenting them in a sequential order that will promote the most valuable and meaningful learning.

The conversations with my co-teacher and the other participants of the class were very helpful in monitoring my learning. The feedback of those conversations kept me going ahead when I began to doubt what I was doing. The online discussions helped me to know that I was not alone in what I was feeling. Visiting with other colleagues not involved in the course was another way of reflecting on my actions. Their questions about what I was doing and how I felt about the course served as good sounding boards to help me sort things out.

Initially I thought that I was too busy, anxious, set in my ways to try something so outside my comfort level. The way I always did it had worked for years. But on the other hand I knew that many of my students hadn't really understood the why of the math lessons and algorithms. I now feel that I need to document the skills and algorithms that I will present next time so that I will feel more prepared and less anxious.

Outer voice
I hope in the next school year to start sooner on previews for symbolic procedures. I want to use it for skills such as counting on and counting back, doubles, number lines, and other strategies that students need to be familiar with to understand the addition and subtraction algorithms.

(My colleague) and I will still be involved with the next sessions of this course. She has been my sounding board for these courses. I have gotten valuable feedback from her throughout. We will also be bringing another first grade teacher on board, so that will be something to look forward to.

The focus needs to remain, for me, on the addition and subtraction strategies for first grade students. I feel that I have overdone the addition and need to spend more time and energy on developing subtraction skills. I would like to work with
the other first grade teachers in my district to develop a sequence, or time line, for presenting the previews to help develop skills for adding and subtracting.

MDI
MDI Previews for Number Sense are short, daily repeated practices for problem solving. The teacher chooses a big idea for the grade level, such as addition and subtraction for first grade. Story problems are presented to students, who solve them using any strategy or method that they feel comfortable with. The teacher observes and questions students about their work. One student's strategy is then chosen to be highlighted for the rest of the students. The highlighted solution is presented to the class and another similar problem is given to students so that they may try the highlighted strategy for themselves. It presents students the opportunity to solve problems in a manner that is comfortable and "safe" for them and also gives them an opportunity to try another, different strategy. This procedure gives the teacher many opportunities to observe students using various strategies and allows for questioning to get to the core of the understanding behind the strategy.

MDI Previews for Symbolic Procedures are quick demonstrations given by the teacher, daily for 7-10 days, before the presentation of an algorithm, or a strategy. The demonstrations are teacher led and controlled with students observing and listening to the proper use of math language.

I have found both types of previews to be enlightening. I have discovered interesting ideas about how my students think and learn. The previews have helped to determine the next step in teaching new ideas, and also helped to guide reteaching and practice. I plan to use both more extensively next year and to start them earlier in the year.

By May, Jane was no longer confused about the difference between the strategy she had learned in the previous course and the new strategy, and could describe the differences in her last post. Jane clearly moved from a level of nonuse, LoU 0, to a level of integration, LoU V.

Jane: Lesson implementation. Jane implemented a Meaningful Distributed Instruction: Preview for Symbolic Procedure lesson on the double-digit addition algorithm. Of the six teacher actions during the lesson (See Appendix D), Jane enacted four with complete fidelity to the model. Jane completed both of the two planning
actions with complete fidelity to the model. Table 11 details an analysis of the lesson.

For each of the six teacher actions and each of the two planning actions, a score of zero to three was assigned. Jane’s total score for this lesson was 20 of a possible 24 as shown in Table 11.

Jane’s lesson was implemented with a high degree of fidelity to the model. Her use of mathematical language was concise and contributed to excellent classroom management during the lesson.

Table 11

<table>
<thead>
<tr>
<th>Teacher actions</th>
<th>Jane’s actions</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUE</td>
<td>Jane did not cue the students.</td>
<td>0</td>
</tr>
<tr>
<td>PRESENT PROBLEM</td>
<td>Jane presented the problem 31+42 in a vertical format, an acceptable variation because it is double-digit addition.</td>
<td>3</td>
</tr>
<tr>
<td>MODEL PROBLEM</td>
<td>Jane modeled the problem with minimal input from students.</td>
<td>3</td>
</tr>
<tr>
<td>SOLVE PROBLEM</td>
<td>Jane solved the problem using the step-by-step actions with the model.</td>
<td>3</td>
</tr>
<tr>
<td>ANSWER QUESTION</td>
<td>Jane asked students to answer the problem and showed the answer.</td>
<td>3</td>
</tr>
<tr>
<td>SUMMARIZE</td>
<td>Minimally.</td>
<td>2</td>
</tr>
<tr>
<td>Chosen representation</td>
<td>Base ten blocks were an excellent model.</td>
<td>3</td>
</tr>
<tr>
<td>Time frame (5-7 minutes)</td>
<td>1:52:07</td>
<td>3</td>
</tr>
</tbody>
</table>

Deb

Deb is a first grade teacher in rural southwest Iowa. This year her school started a new mathematics textbook series in which a new lesson was taught accompanied by “a LOT” of time reviewing old concepts. Deb thinks it is important for kids to share their
ideas because sharing might help another student figure out a strategy. Deb is excited about trying new things to help students learn.

Deb: Change in mathematics content knowledge. Deb scored the lowest on the pretest in her grade band, Kindergarten through first grade. Deb’s raw score on the pretest was 32 of 58 or 55%. Deb’s raw score on the posttest was 45 or 58 or 78%. The difference between Deb’s raw pretest score and raw posttest score was 13 points from the first day of the Making Sense of Operations course to the last day of the course. The difference between Deb’s pretest percentage and posttest percentage was 23%, a 42% increase.

Figure 6. Change in Deb’s mathematics content knowledge January to June
Deb: Change in stages of concern. Based on the Stages of Concern Questionnaire (SoCQ) and using Profile Interpretation, Deb’s highest Stages of Concern in January were stage 0 Awareness and stage 3 Management. In January, Deb’s high scores in these stages indicated that she needed more information about Meaningful Distributed Instruction: Preview for Symbolic Procedure (MDI: PSP) and that she was concerned about the logistics, such as time and classroom management, of implementing the new strategy. Deb’s concerns related to implementing MDI: PSP were all relatively high in January.

Deb’s concerns remained high in June, however, her concerns about consequences for her students related to MDI: PSP decreased. Figure 7 graphically represents the change in Deb’s concerns January to June.

Deb: Change in level of use. Deb’s first post to the online discussion board was made on January 27. Her last post was made May 12.

Deb remained at a level on nonuse until mid-April. The focus of Deb’s posts between January and April was on learning from others in the course. Deb posted information about her classroom and textbook series through February.

A post on February 16 revealed continued confusion about Meaningful Distributed Instruction: Preview for Symbolic Procedure (MDI: PSP), however, Deb was beginning to see value in using the strategy. Much confusion centered on the difference between the strategy taught in the previous course, Meaningful Distributed Instruction: Preview for Number Sense (MDI: PNS) and the new strategy, MDI: PSP.
A post on March 17 continued to reveal a Level of Use 0, nonuse, and continued confusion about the strategy. At this point in the course, planning the MDI: PSP lesson was to begin, and Deb still was unable to differentiate between MDI: PSP and MDI: PNS. April 14 Deb began planning her lesson and at this point some the confusion about the strategy seemed to be diminished. At this time, Deb's level of use was LoU II, Planning. The confusion about the strategy seemed to lessen, however, her comments indicate that she is still confused about who directs the lesson and would prefer to focus on student comments rather than her explanations.
Deb's reflection on May 12 indicates a LoU III, Mechanical Use. In the post, Deb comments about difficulty in transitioning from the MDI: PNS problems, which were more similar to strategies learned during previous professional development Cognitively Guided Instruction (CGI). Deb had extensive professional development in teaching number sense, teaching a preview for symbolic procedure, or an algorithm was a difficult change. Deb does express a desire to continue using MDI: PSP and also sees the value for the student. Following are two of Deb's posts on May 12 from the online discussion board:

After having worked with CGI type problems from last year, I was excited to try another year of learning new things with Math. I had a hard time in the beginning trying to transition from CGI type problems to MDI problems. I was used to spending lots of time with students explaining, and then demonstrating, that I wasn't sure if I could do the MDI's in just 5-10 minutes. Once I started it, I got used to the way that we could move quickly and efficiently through the problems. I like how the previews are just that - previews of what we will be learning in Math later on. I think that by doing the previews, the students have a better understanding once we learn the algorithm. I have used a lot of strategies to monitor my learning - the biggest would be from sharing with other teachers, my co-workers, and just observing how the students are learning and adjusting my teaching to try and match their learning styles. As for myself, I have gained lots of insights. I know that this is something that is important for students to learn. I know that it wasn't easy to adjust and move on from my old teaching style, but once I have, the students have benefited which is the most important thing. Overall, I am pleased with how the year went!

This year was our first year with our new Math series, so it was harder to know what to do with previews since I didn't know what concepts were coming up. Now that I've worked with this series for a whole year, I will be able to know when to start new previews because I'll have an idea of what is coming up for the students to learn. This year, I mainly worked with (a colleague) from Kindergarten, who was also taking this class. Next year, I'd still like to work with (a colleague), but I may use my other first grade teachers to "bounce ideas off of". This year, I mainly looked at addition, which is an important concept taught in first grade. Next year, I'd like to continue with addition, but also focus on subtraction, since that is another big concept that the students work with. I think it will be easier next year - the first year with something new is always harder, and I
had two new things - a new series and a new concept. Over the summer, I'm hoping to sit down and look at my current Math series and try to figure out when I can "plug-in" the previews, so that I'll kind of have an idea of when to start. Hopefully it will work!

Deb's final post, on May 12, reveals better understanding of the difference between MDI: PSP and the previous strategy, MDI: PNS:

MDI Previews for Number Sense are when you give the students a problem, such as a story problem. The students solve the problem using their own strategy. The teacher then chooses a strategy to be shared to the whole class. Once the problem is shared, another problem is given and the students all try the strategy that the "chosen" student demonstrated. MDI Previews for Number Sense can be given all year long, and they are problems that are focused on a BIG Mathematical idea. In first grade, this might be addition or subtraction. Usually, the problems are given in context, such as a story problem.

MDI Previews for Symbolic Procedure are problems that are given two weeks prior to the students learning the algorithm. With these problems, the teacher mainly shows a problem on the white board or Elmo. The teacher will use manipulatives to model how to solve the problem. The teacher will then select a student to restate the steps that were taken to solve the problem. After this the teacher gives a quick summary. These MDI's usually last around 3-5 minutes. The problems usually do not have context.

With MDI's for Number Sense, it seems like the students are the ones who are mainly directing the problem whereas with the MDI's for Symbolic Procedures, the teacher is the one who is directing the learning.

Deb moved from a level of nonuse, LoU 0, to a level of mechanical use, LoU III. Her confusion about the difference between MDI: PNS which she learned in the previous course and MDI: PSP, the new strategy, decreased over time.

Deb: Lesson implementation. Deb implemented a Meaningful Distributed Instruction: Preview for Symbolic Procedure lesson on the double-digit addition algorithm. Of the six teacher actions during the lesson (See Appendix D), Deb enacted four with complete fidelity to the model. She implemented two with partial fidelity to the
model. Deb completed both of the two planning actions with complete fidelity to the model. Table 12 details an analysis of the lesson.

Table 12

Deb's Lesson Analysis

<table>
<thead>
<tr>
<th>Teacher actions</th>
<th>Deb's actions</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUE</td>
<td>Deb made reference to the computational focus</td>
<td>1</td>
</tr>
<tr>
<td>PRESENT PROBLEM</td>
<td>Deb presented the problem 43+24 in a vertical format, an acceptable variation because it is double-digit addition.</td>
<td>3</td>
</tr>
<tr>
<td>MODEL PROBLEM</td>
<td>Deb modeled the problem. Asking students how to use the model.</td>
<td>3</td>
</tr>
<tr>
<td>SOLVE PROBLEM</td>
<td>Deb solved the problem using the step-by-step actions with the model.</td>
<td>3</td>
</tr>
<tr>
<td>ANSWER QUESTION</td>
<td>Deb asked appropriate questions and answered the problem</td>
<td>3</td>
</tr>
<tr>
<td>SUMMARIZE</td>
<td>Minimally</td>
<td>2</td>
</tr>
<tr>
<td>Chosen representation</td>
<td>Base ten blocks were an excellent model</td>
<td>3</td>
</tr>
<tr>
<td>Time frame (5-7 minutes)</td>
<td>4:31</td>
<td>3</td>
</tr>
</tbody>
</table>

For each of the six teacher actions and each of the two planning actions, a score of zero to three was assigned. Deb's total score for this lesson was 21 of a possible 24.

Deb's lesson was implemented with a high degree of fidelity to the model. She struggled with delivery issues such as standing in front of the document camera and did not smoothly present the problem. Her use of mathematical language was less concise and contributed to a longer lesson and distracted students.

Mary

Mary, a second grade teacher in rural southwest Iowa, finds it hard to separate teaching mathematics from learning of mathematics. Mathematics has always been her favorite subject, both as a learner and a teacher. Through teaching mathematics, her learning has continued. She tries to help students make sense of mathematics, and
analyzes the steps needed to ensure understanding. Mary is teaching mathematics using the Saxon Mathematics Program and feels that her students' retention of mathematics is very good. She thinks Meaningful Distributed Instruction: Preview for Symbolic Procedure will fit with her current program nicely.

**Mary: Change in mathematics content knowledge.** Mary scored the highest on the pretest in her grade band, second through fourth grades. Mary also scored the highest on the pretest of all teachers enrolled in this section of *Making Sense of Operations*. Mary’s raw score on the pretest was 49 of 58 or 84%. Mary’s raw score on the posttest was 51 of 58 or 88%. The difference between Mary’s raw pretest score and raw posttest score was two points from the first day of the *Making Sense of Operations* course to the last day of the course. The difference between Mary’s pretest percentage and posttest percentage was 4%, a 5% increase. Figure 8 graphically represents the change.

**Mary: Change in stages of concern.** Based on the Stages of Concern Questionnaire (SoCQ) and using Profile Interpretation, Mary’s highest Stages of Concern in January were stage 0 Awareness, stage 1 Information, stage 2 Personal and stage 3 Management. High scores in these stages indicate the Mary needed more information about Meaningful Distributed Instruction: Preview for Symbolic Procedure (MDI: PSP), she had personal concerns about MDI: PSP, and that she was concerned about the logistics, such as time and classroom management, of implementing the new strategy. High concerns in these three areas in January indicated that Mary was seriously contemplating implementation of the new strategy and was considering all implications.
In June, Mary’s concerns shifted significantly toward the consequences of implementation for her students, collaborating with others, and refocusing on how she might improve implementation of the strategy. The significant increase in stage 6 has strong ideas about how to do things differently, and that her feelings about MDI: PSP may be negative.

**Mary: Change in level of use.** Mary’s first post to the online discussion board was made on January 25. Her last post was made May 11.

In January, Mary moved quickly from a level of nonuse, LoU 0, to a level of
orientation, LoU I, in her first posting. At LoU I, Mary was learning of the new strategy, Meaningful Distributed Instruction: Preview for Symbolic Procedure (MDI: PSP). Mary also expressed ideas about implementing MDI: PSP and was making plans for use in her classroom, LoU II Planning.

Through February and March, Mary continued to plan, LoU II, but her posts reflected strong connection to Cognitively Guided Instruction, the focus of a previous professional development experience, and her textbook series. She questions the teacher-directed nature of the MDI: PSP, “I think mathematical discourse is the best judge of a student’s depth of conceptual understanding.” In her view, students need to talk and
explain their thinking to ensure that the teacher clearly understands their level of comprehension.

In April, Mary begins to collaborate with colleagues, indicating LoU V Collaboration. However, she continues to question the value of MDI: PSP, based upon her prior experience with number sense.

The main challenge I face is that the students already know the algorithm for 2-digit numbers. Many of them want to use the algorithm instead of the physical model to explain. The other challenge is the age-old time factor. The plate is so full of things that I have to get done...that adding one more thing is difficult.

During a discussion of the value of MDI: PSP for formative assessment, Mary states:

A formative assessment is a tool for determining what students know so that an instructor can plan the next instructional step. MDI Previews for Symbolic Procedures can be used as formative assessment when a student explains the procedure that has been modeled by the teacher. If a student cannot explain the procedure, then the teacher has information about that particular student. When previewing for the whole class, it would take several days to gather information about all of your students. I'm not sure it's the best tool for formative assessment because the 10 days cycle of preview problems would not allow you to get information on all of your students.

In April, Mary seems somewhat conflicted about her views on developing number sense and listening to students and the more teacher-directed MDI: PSP, however, she does see value in MDI: PSP as stated below:

I think there is value in using MDI Previews for Symbolic Procedures in the classroom because it allows many opportunities for students to witness the steps that connect the concrete model to the symbolic procedure. It is inevitable that students will be exposed to the algorithm either at school or at home. Knowing how to do the algorithm doesn't guarantee an understanding of why the algorithm works. MDI Previews for Symbolic Procedures increase to likelihood that students will have a deeper understanding of the mathematics that happen within the procedure.
Ideally, students should be able to model the symbolic procedure themselves instead of just watch an instructor present the model.

Mary’s final posts, in May, reflect continued concern with use of the MDI: PSP:

As I reflect on my learning this semester, I think the main struggle I had was to find a blend of the two strategies that we have learned and the CGI that I learned last year. Allowing students to find their own solutions to problems seemed in conflict with demonstrating a commonly used algorithm. I understand the importance of building understanding of that algorithm prior to its introduction. Since students inevitably will be introduced to the algorithm, why not make sure they have the conceptual understanding to back it up? Most math programs introduce the algorithms, so this allows a blend of the new learning of these courses with the required curriculum for our district.

According to her posts Mary achieved LoU V. She plans to implement MDI: PSP next year, “Next year, I plan to incorporate Making Sense of Operations into my classroom by focusing on addition and subtraction throughout the year. I will preview the algorithm prior to its introduction to increase my students’ conceptual understanding.”

However, Mary continues to question the value of this strategy.

Mary: Lesson implementation. Mary implemented a Meaningful Distributed Instruction: Preview for Symbolic Procedure lesson on three-digit subtraction with renaming. Of the six teacher actions during the lesson (See Appendix D), Mary enacted four with complete fidelity to the model. She implemented two with partial fidelity to the model. Mary completed both of the two planning actions with complete fidelity to the model. Table 13 details an analysis of the lesson.

For each of the six teacher actions and each of the two planning actions, a score of zero to three was assigned. Mary’s total score for this lesson was 19 of a possible 24.
Table 13

Mary’s Lesson Analysis

<table>
<thead>
<tr>
<th>Teacher actions</th>
<th>Mary’s actions</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUE</td>
<td>Mary cued that this was a preview lesson. No reference to the computational focus</td>
<td>1</td>
</tr>
<tr>
<td>PRESENT PROBLEM</td>
<td>Mary presented the problem 403-211 in a vertical format.</td>
<td>3</td>
</tr>
<tr>
<td>MODEL PROBLEM</td>
<td>Mary modeled the problem with base-ten blocks. Asking students how to use the model.</td>
<td>3</td>
</tr>
<tr>
<td>SOLVE PROBLEM</td>
<td>Mary solved the problem using the step-by-step actions with the model.</td>
<td>3</td>
</tr>
<tr>
<td>ANSWER QUESTION</td>
<td>Mary asked students for the answer and where the answer was within the model.</td>
<td>3</td>
</tr>
<tr>
<td>SUMMARIZE</td>
<td>No summary.</td>
<td>0</td>
</tr>
<tr>
<td>Chosen representation</td>
<td>Base ten blocks were an excellent model.</td>
<td>3</td>
</tr>
<tr>
<td>Time frame (5-7 minutes)</td>
<td>4:30</td>
<td>3</td>
</tr>
</tbody>
</table>

Mary’s lesson was implemented with a high degree of fidelity to the model. Her use of mathematical language was concise and contributed to a clear lesson and few management problems.

Ann

When Ann, a second grade teacher in rural southwest Iowa, thinks back to when she was in school, she remembers sitting in the classroom listening to the lesson, sometimes going to the board for practice, and then getting the assignment. She doesn’t remember doing much hands-on work nor going over problems she missed. She remembers that there were many times she didn’t understand fully what she was doing or how math made sense. She realized, after taking Making Sense of Numbers in the fall, that she did not have a deep understanding of mathematics at all. She hopes that through taking Making Sense of Operations, she will learn to give her students the opportunities they need to deepen their understanding with conceptual and procedural knowledge by
modeling strategies and giving them distributed practice. She recognizes that if she wants her students to be and feel successful she will have to make sure they get the chance to practice strategies over a period of time rather than in just one lesson.

**Ann: Change in mathematics content knowledge.** Ann scored the lowest on the pretest in her grade band, second through fourth grades. Ann’s raw score on the pretest was 20 of 58 or 40%. Ann’s raw score on the posttest was 36 of 58 or 72%. The difference between Ann’s raw pretest score and raw posttest score was 16 points from the first day of the *Making Sense of Operations* course to the last day of the course. The difference between Ann’s pretest percentage and posttest percentage was 32%, an 80% increase. Figure 10 graphically represents this change.

**Ann: Change in stages of concern.** Based on the Stages of Concern Questionnaire (SoCQ) and using Profile Interpretation, Ann showed high levels of concern in January in every area except stage 4 Consequence and stage 6 Refocusing. Ann’s high stage 1 Information and high stage 5 Collaboration suggests that she was interested in learning from what others know and are doing, rather than a concern for leading the collaboration. High concerns in these areas in January indicated that Ann was seriously contemplating implementation of the new strategy and was considering all implications.

Like Mary, in June, Ann’s concerns shifted significantly. The decrease in concerns about management and personal concerns suggests that she felt more comfortable in implementation of MDI: PSP. The increase in 6 Refocusing indicates that she is ready to adjust the innovation to better meet her students’ needs.
Ann: Change in level of use. Ann’s first post to the online discussion board was made on January 24. Her last post was made May 11.

Ann’s initial posts indicated nonuse in January, but quickly moved to LoU I Orientation and LoU II Planning. Ann was anxious to learn more about Meaningful Distributed Instruction: Preview for Symbolic Procedure (MDI: PSP), and was planning to implement MDI: PSP in her classroom.

Ann taught her MDI: PSP lesson in May, a LoU III Mechanical Use. Ann’s post in April expresses a concern that was apparent when observing her in the classroom, “I have also struggled some with saying too much as making it more of a lesson than just a...
preview and then I am getting into my lesson time for the day.” Ann again mentioned the need for understanding vocabulary in May:

Through this second MDI I found that many students needed to know why they are doing the steps, not just to do them as I modeled them. It gave them a deeper understanding to the standard algorithm and their math vocabulary.

In May, Ann identified a colleague she plans to work with next year to implement MDI: PSP and the content she plans to focus upon, addition and subtraction, which indicated LoU V Integration. In Ann’s last post, she wrote of her growth:

I have learned right along with my students on both of these previews. Not only have I been teaching my students, but they have been teaching me too. I discovered new ways of solving problems that I would never have thought of and I have learned both of these MDI’s can help to deepen the understanding of number sense and operation to students. Both of these previews are valuable tools
for a teacher to have in their "little basket". I only wish that the rest of the teachers in my district could have had this opportunity this year to incorporate into their math class.

**Ann: Lesson implementation.** Ann was observed implementing a Meaningful Distributed Instruction: Preview for Symbolic Procedure (MDI: PSP) lessons, on three-digit subtraction with renaming. Ann’s MDI: PSP lesson lasted just over four minutes.

Ann did not start the lesson with a cue. The problem was written on the whiteboard: "There are 200 dandelions in the yard. First graders picked 122. How many dandelions are left?" Ann stated the problem and then promptly presented the problem 200 - 122 in a vertical format. Ann then modeled the problem with little input from students. When Ann asked questions, they were direct and students could answer succinctly. Ann then solved the problem and answered the question. This portion of the lesson lasted two minutes and thirty seconds. Ann asked a student to summarize by retelling the lesson. This was an inefficient summarization technique as the student’s retelling was difficult to understand. Table 14 contains a summary of Ann’s lesson.

For each of the six teacher actions and each of the two planning actions, a score of zero to three was assigned. Ann’s total score for the lesson she taught was 18 of a possible 24. Ann’s lesson was implemented with some degree of fidelity to the model.

**Sue**

Sue teaches sixth grade and is the fourth through sixth grade mathematics teacher in an elementary building in rural southwest Iowa. Sue is amazed to think about all of the changes that have been made in teaching from the time when she was a student to a first-
### Table 14

**Ann's Second Lesson Analysis**

<table>
<thead>
<tr>
<th>Teacher actions</th>
<th>Ann's actions</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUE</td>
<td>No cue</td>
<td>0</td>
</tr>
<tr>
<td>PRESENT PROBLEM</td>
<td>Ann presented the problem 200 – 122 in a vertical format.</td>
<td>3</td>
</tr>
<tr>
<td>MODEL PROBLEM</td>
<td>Ann set up the problem by modeling with base-ten blocks.</td>
<td>3</td>
</tr>
<tr>
<td>SOLVE PROBLEM</td>
<td>Ann solved the problem asking minimal, direct questions of the students.</td>
<td>3</td>
</tr>
<tr>
<td>ANSWER QUESTION</td>
<td>Teacher answers. Lesson length to this point 2:30.</td>
<td>3</td>
</tr>
<tr>
<td>SUMMARIZE</td>
<td>Ann asked a student to summarize. This was time-consuming and ineffective.</td>
<td>0</td>
</tr>
<tr>
<td>Chosen representation</td>
<td>Base ten blocks were an excellent model</td>
<td>3</td>
</tr>
<tr>
<td>Time frame (5-7 minutes)</td>
<td>4:19</td>
<td>3</td>
</tr>
</tbody>
</table>

year teacher eight years ago. When she first arrived as a new teacher with new ideas, some teachers looked down on things like working in groups and not sitting in rows. As a student, Sue remembers sitting in rows and never talking unless she was called on. She sees that things are different now. Her students rarely raise their hands when discussing and reviewing concepts and she feels they have great discussions without raising hands. She thinks education has made great strides.

**Sue: Change in mathematics content knowledge.** Sue scored the highest on the pretest in her grade band, fifth through eighth grades. Sue’s raw score on the pretest was 39 of 58 or 67%. Sue’s raw score on the posttest was 38 of 58 or 66%. The difference between Sue’s raw pretest score and raw posttest score was a decrease of one point from the first day of the Making Sense of Operations course to the last day of the course. The difference between Sue’s pretest percentage and posttest percentage was a decrease of 1%, a 1% decrease.
The only participant to complete the course with a lower mathematical content knowledge score was Sue, but the decrease was minimal. Figure 12 graphically represents Sue’s change in mathematics content knowledge.

![Figure 12: Change in Sue’s mathematics content knowledge January to June](image)

**Figure 12.** Change in Sue’s mathematics content knowledge January to June

**Sue: Change in stages of concern.** Sue’s concerns about implementing MDI: PSP were high in January and remained unchanged. High stage 1 Information combined with high stage 5 Collaboration suggested interest in learning from others, rather than concern for leading collaboration.

While Sue’s level of concern remained high in June, her need for information about the innovation decreased. High stage 2 Personal indicated intense personal
concerns about MDI: PSP and consequences. High management concerns revealed Sue’s struggled with logistics, time and management, of MDI: PSP. The combination of high stage 6 and high stage 3 expose frustration with unresolved Management concerns and strong ideas about changing MDI: PSP. Increase in stage 6 Refocusing stage 0 Awareness most likely indicated Sue’s feelings about MDI: PSP became negative.

![Figure 13. Change in Sue’s concerns January to June](image)

*Sue: Change in level of use.* Sue’s first post to the online discussion board was made on January 29. Her last post was made May 12. Generally Sue’s posts were brief throughout the course.
Sue's posts indicate that, while she was interested in learning about Meaningful Distributed Instruction: Preview for Symbolic Procedure (MDI: PSP), she did not move beyond superficial use of the innovation.

Sue's posts also indicate that she was felt a lack of confidence in her own ability to understand mathematics:

I really liked this article and it made me really see how my thinking has changed. As a student and as a beginning teacher I never broke these problems up. I would do the work and work it all out completely rather than finding user-friendly numbers and working with them. Maybe this shows me I did not have a good understanding of the number system and how they work. I still question this as we do problems in class.

As was apparent during observation of Sue teaching MDI: PSP, Sue also lacked clear understanding of appropriate methods of representing problems. The following posts alludes to her struggle:

My preview sounds very similar to the ones that people have already talked about. I plan on starting with 1/5+2/5=3/5. I plan to use the graph paper at first and then use the other resources that I have. I have the "pie" pieces, but want to stay away from those because that seems to be the only way students can look at fractions and I want to stay away from that. I chose the denominator of 5 because I have 20 students in my room so can easily use the class as my groups of 5 without any "remainders". We will spend the first few times with this concept and then I can go to mixed numbers etc. The biggest problems I have seen before are trying to find like denominators, so I am dreading that!! Does anybody have any good lessons for this?

Sue's suggestion to "stay away" from "pie" pieces indicates that she did not realize that the area model would have been a much stronger model for this problem. Her request for lessons also indicates that she is struggling to understand.
Sue continued to express concerns when posting in May, and writes of frustration with her own learning experience. She writes of growth she experienced and anticipation of continued growth in her ability to teach mathematics:

As I look back on my thinking and learning in this class, it has been a bumpy road. It has taken so much mentally to change your thinking as a learner and teacher. I often wish I could go back and go through some of my high school classes again because I have gained so much as a learner that I would like to try again. At this point in our learner I have become a much stronger teacher and I look at all processes in a different light. I am excited to have a new start next year in my organizing of the year and the concepts.

In describing the difference between Meaningful Distributed Instruction: Preview for Number Sense and Meaningful Distributed Instruction: Preview for Symbolic Procedure, Sue returns to her concerns and lack of confidence in her ability to teach mathematics:

It is not that one technique is easier in my planning or implementing I just feel like I am struggling to get a flow for what I am doing to best use my math block. We have learned some great techniques but just such intense learning that it seems overwhelming in how my math block should look. Does anyone else ever feel like this?

Based on Levels of Use analysis of Sue’s online discussion board prompts, during the Making Sense of Operations course, Sue became a superficial user of MDI: PSP, LoU III Mechanical.

Sue: Lesson implementation. Sue implemented a Meaningful Distributed Instruction: Preview for Symbolic Procedure lesson on adding fractions with common denominators. Of the six teacher actions during the lesson (See Appendix D), Sue enacted one with fidelity to the model. She implemented four with partial fidelity to the
model, and one with none. Sue completed the lesson in the appropriate time frame, but used an inappropriate model. Table 15 details an analysis of the lesson.

Table 15

*Sue’s Lesson Analysis*

<table>
<thead>
<tr>
<th>Teacher actions</th>
<th>Sue’s actions</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUE</td>
<td>Sue cued that this was a lesson on adding fractions.</td>
<td>2</td>
</tr>
<tr>
<td>PRESENT PROBLEM</td>
<td>Sue presented the problem 1/5 + 2/5 in a vertical format.</td>
<td>3</td>
</tr>
<tr>
<td>MODEL PROBLEM</td>
<td>Sue modeled the problem using a set model, blocks with fractions written on top. A more effective model would have been an area or region model.</td>
<td>1</td>
</tr>
<tr>
<td>SOLVE PROBLEM</td>
<td>Sue solved the problem using the step-by-step actions with the model but the solution was not clearly a fraction. It appeared to be the whole number 3.</td>
<td>2</td>
</tr>
<tr>
<td>ANSWER QUESTION</td>
<td>Sue answered the question.</td>
<td>2</td>
</tr>
<tr>
<td>SUMMARIZE</td>
<td>To summarize, Sue asked a student to solve another problem, 2/8 + 3/8 using the same model. Another student said mental math could be used, that the denominator stays the same, and then 2+3 could be added mentally.</td>
<td>2</td>
</tr>
<tr>
<td>Chosen representation</td>
<td>Ineffective model</td>
<td>0</td>
</tr>
<tr>
<td>Time frame (5-7 minutes)</td>
<td>2:17</td>
<td>3</td>
</tr>
</tbody>
</table>

For each of the six teacher actions and each of the two planning actions, a score of zero to three was assigned. Sue’s total score for this lesson was 15 of a possible 24.

Sue implemented her lesson with a moderate degree of fidelity to the model. The representation or model used for conceptual understanding of the symbolic procedure was a set model; a more effective model would have been an area or region model. Sue asked a student to solve another problem to summarize, which in some lessons is an acceptable variation. Sue’s use of mathematical language was concise and contributed to a clear lesson and few management problems.
Pat teaches fifth grade in a rural classroom in southwest Iowa. When Pat thinks back on her experiences in mathematics classes, most of the emphasis was on competition: speed and how fast she could get the work done. Pat sees that this is not the best method for struggling students who need more practice and very explicit instruction. She sees the need to build a strong foundation on math skills for her students. Pat loves when students make connections between what they are learning at home, school, and just on their own.

**Pat: Change in mathematics content knowledge.** Pat scored the lowest on the pretest in her grade band, fifth through eighth grades. Pat’s raw score on the pretest was 17 of 58 or 29%. Pat’s raw score on the posttest was 32 of 58 or 55%. The difference between Pat’s raw pretest score and raw posttest score was fifteen points from the first day of the *Making Sense of Operations* course to the last day of the course. The difference between Pat’s pretest percentage and posttest percentage was 26%, a 90% increase.

**Pat: Change in stages of concern.** Based on the Stages of Concern Questionnaire (SoCQ) and using Profile Interpretation, Pat’s concerns were highest in the first four stages: 0 Awareness, 1 Information, 2 Personal, and 3 Management. High levels of concern in these areas are common when beginning to implement a new strategy, and
indicate the need for information and support. In January, Pat had a low level of concern about consequences for students.

Pat’s levels of concern increased in most stages in June. Pat’s concerns about consequences of implementation for her students increased. Pat’s high concern about management and her high concern with refocusing could indicate that she has become frustrated with not having management concerns resolved and has developed ideas about how the situation should be changed. As with Sue, the significant increase in stage 6 Refocusing and continued high concern in stage 0 Awareness most likely indicate that Pat may have strong ideas about how to do things differently, and that her feelings about MDI: PSP may have become negative.
Figure 15. Change in Pat's concerns January to June

Pat: Change in level of use. Pat's first post to the online discussion board was made on January 29. Her last post was made May 22.

Pat's initial posts indicated an interest in building a strong foundation of math skills for her students and an interest in learning about Meaningful Distributed Instruction: Preview for Symbolic Procedure (MDI: PSP), but never became a user, LoU 0.

By the end of March, Pat's posts indicated an interest in the strategy. She planned and conducted a MDI: PSP lesson using a set model teach addition of fractions. Pat describes the lesson in a post in March and continued discussion of the lesson in April.
The lesson Pat describes is concept of fraction, but not symbolic procedure, and she later indicates, "if a good discussion is going, or an example presents itself, I can’t just stop mid-stream...I keep going." The intention of a MDI: PSP lesson is to be a short, five minute or less. Pat does not recognize the discrepancy between the intent of MDI: PSP and the lesson she taught.

Pat’s last post shows little understanding of the contrast between Meaningful Distributed Instruction: Preview for Number Sense and MDI: PSP:

MDI’s cover 7-10, 5 minute lessons, where the students discover a way to figure out an answer to a problem that works for them. They may be given the opportunity to demonstrate their "way" to the rest of the class.

A preview is exactly what it says: preview a concept before the algorithm is taught. It is a way to demonstrate the "why" of an algorithm before it is actually taught. The teacher demonstrates this for the students.

I think both ways are useful. I will use both ideas next year. I will start using the MDI’s right away and then work into the previews for future skills taught. The beginning of the year is primarily review anyway, so this is an excellent time to use the MDI’s. I feel confident I will cover more ground next year; especially with the continuing on of these courses. I feel my awareness of math concepts broadening. I WISH I would have had this type of instruction back in the day!!!!

As written in the last three sentences of the post above, Pat felt confident that she was making progress and learning.

Pat: Lesson implementation. Pat implemented a lesson on adding fractions with common denominators, however it did not fit the model for Meaningful Distributed Instruction: Preview for Symbolic Procedure. Pat’s lesson was nine minutes and fifty-nine seconds in length. Of the six teacher actions during the lesson (See Appendix D), Pat did not enact any with fidelity to the model.
Pat started by placing Skittles on the document camera. She asked how she
should show ¼. She then presented one Skittle on one side of the screen and three on the
other. Students responded that this represented ¼ + ¾. This portion of the lesson was
approximately one minute and thirty seconds in length.

Students then began to ask questions and the discussion moved from the problem
to common denominators and using them to solve problems. Pat then modeled 2/4 + 2/4,
followed by ¼ + 2/4. The model did not match the second problem, which the students
pointed out. Pat pointed out that there were four equal parts.

Pat then started to give out the candy and a discussion started about the possibility
of getting their favorite color. The discussion went on until Pat elicited the word ratio
from the students.

The lesson concluded after nearly ten minutes by Pat stating that the lesson was
about adding with like what? Students replied denominators after some prompting.

It was very clear that Pat cared about the students, wanted them to learn, and
wanted to answer their questions. She did not implement the lesson with fidelity to any
part of the model. Pat’s score was 4 of 24 points. Table 16 summarizes rational for Pat’s
score.

Change by Groups: Grade Band Groups and High/Low Pretest Scorer Groups

This next section compares the data among groups. Comparisons were made
between grade band groups and high or low pretest content knowledge groups. Finally,
comparisons were made for the group as a whole.
Table 16

*Pat’s Lesson Analysis*

<table>
<thead>
<tr>
<th>Teacher actions</th>
<th>Pat’s actions</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUE</td>
<td>N0 cue</td>
<td>0</td>
</tr>
<tr>
<td>PRESENT PROBLEM</td>
<td>Pat eventually presented the problem $\frac{1}{4} + \frac{3}{4}$ in a horizontal format, later presented a second problem, followed by a third problem involving probability, and included a brief lesson on common denominators</td>
<td>1</td>
</tr>
<tr>
<td>MODEL PROBLEM</td>
<td>Pat modeled some problems using a set model, but the model did not match the actions at times.</td>
<td>1</td>
</tr>
<tr>
<td>SOLVE PROBLEM</td>
<td>Pat solved problems, but it was difficult to tell which problem she was solving.</td>
<td>1</td>
</tr>
<tr>
<td>ANSWER QUESTION</td>
<td>Pat answered the questions, but it was difficult to tell which questions she was answering.</td>
<td>1</td>
</tr>
<tr>
<td>SUMMARIZE</td>
<td>No summary</td>
<td>0</td>
</tr>
<tr>
<td>Chosen representation</td>
<td>Ineffective model</td>
<td>0</td>
</tr>
<tr>
<td>Time frame (5-7 minutes)</td>
<td>9:59</td>
<td></td>
</tr>
</tbody>
</table>

**Grade band group change: Mathematics content knowledge.** In comparing change in mathematics content knowledge by grade band, in this group of teachers, the kindergarten through first grade (K-1st) teachers started the course with the highest mathematics content knowledge. The pretest score average of the K-1st grade teachers was 67%. The lowest grade band, the fifth through eighth grade teachers started the course with a pretest score average of 48%. The percent increase from pretest to posttest was highest in the second through fourth grade band, a 29% increase. Table 17 contains pretest, posttest, and percent increase data.
Figure 16 graphically represents the grade band data from the previous table.

Figure 16. Change in each grade band’s content knowledge January to June

Grade band group change: Stages of concern. Comparing change in concerns by grade band in January shows that concerns were similar in all grade bands, but in June,
the grade bands held very different concerns from one another. The kindergarten through first grade (K-1st) and second through fourth grade bands’ concerns changed in the following ways:

- Concerns generally decreased from January to June.
- Stage 0 to 2 concerns generally decreased and stage 4 to 6 generally increased from January to June.

The fifth through eighth grade band’s concerns changed in the following ways:

- Concerns generally increased from January to June.
- Concerns in stages 0 through 3 did not decrease, and concerns in stages 4 through 6 increased.

As reflected in Figure 17 and 18 on the next page, overall the pattern for two grade bands, K - 1st and 2nd - 4th was similar. The pattern for grade band 5th - 8th was different from the other two.

**Grade band change: Level of use.** Based upon their online discussion board posts, a Level of Use (LoU) was established for teachers. The LoU established for each teacher did not consider the teachers’ actual implementation of the Meaningful Distributed Instruction: Preview for Symbolic Procedure lesson. Table 18 represents the Level of Use achieved by each participant based upon online discussion board posts. Grade band second through fourth achieved the highest level of use, followed by the kindergarten through first grade band.
Figure 17. Each grade band’s concerns in January

Figure 18. Each grade band’s concerns in June
Table 18

*Highest Level of Use by Grade Band*

<table>
<thead>
<tr>
<th>Grade Band</th>
<th>Participant</th>
<th>Highest Level of Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>K - 1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>Jane</td>
<td>LoU V Integration</td>
</tr>
<tr>
<td></td>
<td>Deb</td>
<td>LoU III Mechanical Use</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; - 4&lt;sup&gt;th&lt;/sup&gt;</td>
<td>Mary</td>
<td>LoU V Integration</td>
</tr>
<tr>
<td></td>
<td>Ann</td>
<td>LoU V Integration</td>
</tr>
<tr>
<td>5&lt;sup&gt;th&lt;/sup&gt; - 8&lt;sup&gt;th&lt;/sup&gt;</td>
<td>Sue</td>
<td>LoU III Mechanical Use</td>
</tr>
<tr>
<td></td>
<td>Pat</td>
<td>LoU 0 Nonuse</td>
</tr>
</tbody>
</table>

Grade band change: Implementation with fidelity to the model. A score was recorded based upon each participant’s ability to implement a lesson with fidelity to the model presented in the Innovation Configuration Map (ICM) for Meaningful Distributed Instruction: Preview for Symbolic Procedure MDI: PSP. Using field notes resulting from observation and the ICM points were assigned to each of the six steps in the MDI: PSP. Points were assigned for effective use of a model to represent the problem and for completing the lesson within five to seven minutes. Table 19 lists the grade band, teacher, and her score. Greatest fidelity to the model was achieved by grade band kindergarten through first grade.
Table 19

*Fidelity to the Model by Grade Band*

<table>
<thead>
<tr>
<th>Grade Band</th>
<th>Participant</th>
<th>Score on Fidelity to the ICM</th>
</tr>
</thead>
<tbody>
<tr>
<td>K - 1st</td>
<td>Jane</td>
<td>20 of 24</td>
</tr>
<tr>
<td></td>
<td>Deb</td>
<td>21 of 24</td>
</tr>
<tr>
<td>2nd - 4th</td>
<td>Mary</td>
<td>19 of 24</td>
</tr>
<tr>
<td></td>
<td>Ann</td>
<td>18 of 24</td>
</tr>
<tr>
<td>5th - 8th</td>
<td>Sue</td>
<td>15 of 24</td>
</tr>
<tr>
<td></td>
<td>Pat</td>
<td>4 of 24</td>
</tr>
</tbody>
</table>

High/low pretest scorer change: *Mathematics content knowledge*. In comparing change in mathematics content knowledge by high or low pretest content knowledge, in this group of teachers, the group of teachers with low pretest content knowledge increased their content knowledge through this course by 65%. The group of teachers with the high pretest scores increased their content knowledge by 5%.

The initial gap between the high scorers and low scorers on the pretest was 35%. At the end of the course, gap between the high scorers and the low scorers had been reduced by 65%, to a 12% gap.

Table 20 contains data comparing the high pretest scorers and low pretest scorers by their pretest scores, posttest scores, and percent of change. This table also contains information about the gap between the groups.
Table 20

*High Pretest Scorers, Low Pretest Scorers Content Knowledge*

<table>
<thead>
<tr>
<th>High/Low</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Percent Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>41%</td>
<td>68%</td>
<td>65%</td>
</tr>
<tr>
<td>High</td>
<td>77%</td>
<td>81%</td>
<td>5%</td>
</tr>
<tr>
<td>Gap</td>
<td>35%</td>
<td>12%</td>
<td>-65%</td>
</tr>
</tbody>
</table>

Figure 19 on the next page graphically represents the high pretest scorers and low pretest scorers’ data from the previous table.

**High/low pretest scorer change: Stages of concern.** Comparing teachers with high pretest scores and teachers with low pretest scores shows that the high scorers’ concerns leveled out from January to June. Initially, high scorers’ concerns were stage 0 to 3. By June, high scorers’ lower level concerns were decreasing and the stage 4 to 6 concerns were increasing.
Figure 19. Change in high/low scorers' content knowledge January to June

The concerns of teachers that scored lower on the pretest were relatively the same from January to June, with the exception of Stage 6 Refocusing concerns. High-level stage 0 and stage 1 concerns combined with high Stage 6 concerns indicate that the teachers may have been experiencing frustration with the changes that were required of them. Figure 20 represents the change in teachers that scored high on the content pretest stage of concern from January to June. Figure 21 represents the change in teachers that scored low on the content pretest stage of concern from January to June.
Figure 20. Change in the high group’s concerns January to June

Figure 21. Change in the low group’s concerns January to June
High/low pretest scorer change: Level of use. Table 21 represents the level of use achieved by each participant. The high pretest scorers achieved a higher level of use as a group.

Table 21

Highest Level of Use by High/Low Pretest Scorers

<table>
<thead>
<tr>
<th>Participant</th>
<th>Highest Level of Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jane</td>
<td>LoU V Integration</td>
</tr>
<tr>
<td>High Pretest Scorers</td>
<td></td>
</tr>
<tr>
<td>Mary</td>
<td>LoU V Integration</td>
</tr>
<tr>
<td>Sue</td>
<td>LoU III Mechanical Use</td>
</tr>
<tr>
<td>Deb</td>
<td>LoU III Mechanical Use</td>
</tr>
<tr>
<td>Low Pretest Scorers</td>
<td></td>
</tr>
<tr>
<td>Ann</td>
<td>LoU V Integration</td>
</tr>
<tr>
<td>Pat</td>
<td>LoU 0 Nonuse</td>
</tr>
</tbody>
</table>

High/low pretest scorer change: Implementation with fidelity to the model. A score was recorded based upon each participants ability to implement a lesson with fidelity to the model presented in the Innovation Configuration Map (ICM) for Meaningful Distributed Instruction: Preview for Symbolic Procedure MDI: PSP. Using field notes resulting from observation and the ICM points were assigned to each of the six steps in the MDI: PSP. Points were also assigned for effective use of a model to represent the problem and for completing the lesson within five to seven minutes. Table
22 groups the high and low scorers and scores for teachers in each group. Greatest fidelity to the model was achieved by grade band kindergarten through first grade. The average score for the high pretest scorers was 18 of 24 or 75% fidelity, the average score for low pretest scorers 17 of 24 or 60% fidelity.

Table 22

*Fidelity to the Model by High/Low Pretest Scorers*

<table>
<thead>
<tr>
<th>Participant</th>
<th>Score on Fidelity to the ICM</th>
</tr>
</thead>
<tbody>
<tr>
<td>High pretest scorers</td>
<td></td>
</tr>
<tr>
<td>Jane</td>
<td>20 of 24</td>
</tr>
<tr>
<td>Mary</td>
<td>19 of 24</td>
</tr>
<tr>
<td>Sue</td>
<td>15 of 24</td>
</tr>
<tr>
<td>Deb</td>
<td>21 of 24</td>
</tr>
<tr>
<td>Low pretest scorers</td>
<td></td>
</tr>
<tr>
<td>Ann</td>
<td>18 of 24</td>
</tr>
<tr>
<td>Pat</td>
<td>4 of 24</td>
</tr>
</tbody>
</table>

Change by the Whole Group

The whole group was examined by the same four lenses as individuals, grade bands, and high and low pretest scorers. The group consisted of six individuals, a subset of twenty teachers participating in *Making Sense of Operations*. The purpose of reporting these results is not to assess the *Making Sense of Operations*. Rather, these results
provide a benchmark for comparison of the group of six individuals. The results do not provide a comprehensive evaluation of *Making Sense of Operations*.

**Whole group change: Mathematics content knowledge.** As a group of six teachers, their average pretest score was 59%, their average posttest score was 75%, a difference of 16%. This difference represents a 26% increase from pretest to posttest.

![Bar chart showing change in whole group's content knowledge from January to June](chart.png)

*Figure 22. Change in the whole group’s content knowledge January to June*

**Whole group change: Stages of concern.** As a group of six, these teachers' concerns about gaining information about Meaningful Distributed Instruction: Preview for Symbolic Procedure, and their personal and management scores decreased from January to June. These teachers’ concerns for refocusing, or adjusting the teaching
strategy to their situation, increased. Their concerns about use of the strategy and the consequences for their students remained low throughout the time period.

Figure 23. Change in the whole group's concerns January to June

Whole group change: Level of use. As a group of six teachers, three obtained a Level of Use V Integration of Meaningful Distributed Instruction: Preview for Symbolic Procedure (MDI: PSP). Two of these LoU V users were in the high pretest score group, and one was in the low. At this level of use, the teachers had accomplished the following:

- Acquired sufficient information about implementation of MDI: PSP
- Prepared to use MDI: PSP
- Mastered the tasks required to use the MDI: PSP
- Varied the use of the MDI: PSP to increase the impact on their students

- Combined their own efforts to use MDI: PSP with the efforts of colleagues to achieve a collective effect on students

Two of the teachers obtained a level of Use III Mechanical Use, as described in the list above. One teacher did not become a user of MDI: PSP during this time frame.

Table 23

*Level of Use by Whole Group*

<table>
<thead>
<tr>
<th>Participant</th>
<th>Highest Level of Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jane</td>
<td>LoU V Integration</td>
</tr>
<tr>
<td>Mary</td>
<td>LoU V Integration</td>
</tr>
<tr>
<td>Ann</td>
<td>LoU V Integration</td>
</tr>
<tr>
<td>Sue</td>
<td>LoU III Mechanical Use</td>
</tr>
<tr>
<td>Deb</td>
<td>LoU III Mechanical Use</td>
</tr>
<tr>
<td>Pat</td>
<td>LoU 0 Nonuse</td>
</tr>
</tbody>
</table>

Whole group change: Implementation with fidelity to the model. Of the group of six teachers in this study, all but two implemented Meaningful Distributed Instruction: Preview for Symbolic Procedure 75% fidelity using the scale developed to align with the Innovation Configuration Map model.
Table 24

*Fidelity to the Model by Whole Group*

<table>
<thead>
<tr>
<th>Participant</th>
<th>Score on Fidelity to the ICM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deb</td>
<td>88%</td>
</tr>
<tr>
<td>Jane</td>
<td>83%</td>
</tr>
<tr>
<td>Mary</td>
<td>79%</td>
</tr>
<tr>
<td>Ann</td>
<td>75%</td>
</tr>
<tr>
<td>Sue</td>
<td>63%</td>
</tr>
<tr>
<td>Pat</td>
<td>17%</td>
</tr>
</tbody>
</table>

This group of six teachers increased their content knowledge by 26% from January to June. Their concerns about implementing Meaningful Distributed Instruction: Preview for Symbolic Procedure MDI: PSP decreased related to personal concerns and management concerns. The focus of their concerns shifted to collaboration with other teachers and to adjusting the strategy to better meet the needs of their students. Reflections on an online discussion board revealed that five of the six teachers mastered the tasks required to use MDI: PSP. All but two of the teachers implemented MDI: PSP with acceptable fidelity to the model.

**Summary**

This chapter examined the results of this study and was organized around the participants and the data sets collected. Findings were reported by individual teacher; by
a comparison among groups, grade band groups and high or low pretest content knowledge groups; and by the whole group.

This report of findings included a description of each individual teacher through the lens of the data sets and concludes and compared these teachers by grade band groups and high or low pretest scorers. The findings concluded with a whole group report. The next chapter uses these findings to address the research questions posed in this study.
CHAPTER V
DISCUSSIONS

This chapter addresses the research questions by examining the relationship between the findings and each question. This study sought to understand the process of change elementary teachers experienced as they participated in mathematics professional development. Questions posed by this study include:

1. What is the impact of mathematics professional development on six elementary teachers’ mathematical content knowledge for teaching?

2. What is the impact of mathematics professional development on six teachers’ implementation of innovative pedagogical practices?

3. What impact does teachers’ content knowledge for teaching have on six teachers’ ability to implement innovative pedagogical practices?

The first question, related to content knowledge, is addressed by the findings of the mathematics content knowledge pre and posttest. The second question, related to implementation of innovative practice, is addressed by the findings of the Stages of Concern Questionnaire (SoC), the Levels of Use (LoU) identified by teachers’ online postings, and comparison of the Innovation Configuration Map (ICM) with notes resulting from observation. The third question, related to the impact of content knowledge on ability to implement, is addressed by findings of all four data sets.

Research Questions

The next section addresses each research question by examining the relationship between the findings of this study and each question. This study sought to understand the
process of change elementary teachers experienced as they participated in mathematics professional development

Research Question 1

The first question posted by this study, what is the impact of mathematics professional development on six elementary teachers’ mathematical content knowledge for teaching, was answered by the findings of the mathematics content knowledge for teaching pretest and posttest.

Of the 20 teachers that participated in Making Sense of Operations, six were selected to participate in this study. This group of six teachers achieved an average increase of 26% from the pretest in January to the posttest in June. Grouped by grade band, the highest percent of increase in this group of six was achieved by the second through fourth grade teachers, an increase of 29%. The lowest percent of increase was achieved by the kindergarten through first grade teachers, an increase of 24%. The difference in percent increase between the low grade band and the high grade band was 5%. Grouped by grade band, there was little difference in percent increase.

Grouped by high or low mathematics content knowledge pretest score, the highest percent of increase on the content knowledge test was achieved by the low pretest scorers, an increase of 65%. The high pretest scorers increased their scores by 5%. The gap at the time of the pretest between content knowledge of the low pretest scorers and the high pretest scorers was a difference of 35%. The gap at the time of the posttest between the low pretest scorers and the high pretest scorers was a difference of 12%. The
change in the gap between the pretest and the posttest was a 65% decrease. The low pretest scorers increased their scores significantly more than the high pretest scorers.

Individually, Pat increased her pretest score to posttest score by 90%, Ann by 80%, Deb by 42%, Jane by 11%, and Mary by 5%. Sue’s pretest to posttest score decreased 1%.

The first question posted by this study was answered. The impact of mathematics professional development on six elementary teachers’ mathematical content knowledge for teaching was increased as a group by 26%. The low pretest scorers achieved a 65% increase in mathematical content knowledge for teaching, the high pretest scorers a 5% increase. Individually, five of the six teachers increased their mathematics content knowledge for teaching through this mathematics professional development.

**Research Question 2**

The second question posted by this study, what is the impact of mathematics professional development on six teachers’ implementation of innovative pedagogical practices, was answered by the findings of the Stages of Concern Questionnaire (SoC), the Levels of Use (LoU) identified by teachers’ online postings, and comparison of the Innovation Configuration Map (ICM) with notes resulting from observation.

Of the 20 teachers that participated in Making Sense of Operations, six were selected to participate in this study. Teachers’ ability to implement innovative teaching practices was determined by their feelings about the innovation, their behavior related to the innovation, and observation of implementation of the innovative teaching practice.
As a group of six teachers, concerns about understanding the innovation, personal concerns and management concerns about implementing the innovation decreased. Concerns about adjusting the innovation to their teaching situation increased. All but one participant achieved a basic level of use of the innovation and all but two implemented the innovation with an acceptable level of fidelity.

Grouped by grade band, concerns about the innovation were similar at the beginning of the course and by the end of the course concerns decreased for two grade bands, Kindergarten through first and second through fourth. Concerns increased for one grade band, fifth through eighth. The level of use of the innovation and ability to implement the innovation with an acceptable level of fidelity was highest for the Kindergarten through first and second through fourth grade bands and lowest for the fifth through eighth grade band.

Grouped by high or low mathematics content knowledge pretest score, concerns about the innovation leveled and decreased for the high pretest scorers and remained high for the low pretest scorers. The level of use of the innovation was at a mechanical use or higher for the high pretest scorers and ranged from nonuse to a mechanical use for the low pretest scorers. The ability to implement the innovation with an acceptable level of fidelity was higher for the high pretest scorers, an average of 75%, and lower for the low pretest scorers, an average of 54%.

Individually, three of the teachers' concerns about the innovation decreased and three increased. Three achieved a level of use of the innovation that included refinement and collaboration with others, two remained at mechanical use of the
innovation, and one remained a nonuser. Four of the teachers implemented the innovation with an acceptable level of fidelity to the model.

The second question posted by this study was answered. The mathematics professional development did impact six elementary teachers' implementation of innovative pedagogical practices. The mathematics professional development improved teachers' feelings about the innovative teaching practice. All but one of the teachers achieved basic level of use of the innovation. All but two of the teachers implemented the innovation with an acceptable level of fidelity to the model.

Research Question 3

The third question posted by this study, what impact does teachers' content knowledge for teaching have on six teachers' ability to implement innovative pedagogical practices, was addressed by findings of all four data sets.

Content knowledge for teaching was most improved in the low pretest scorer group. This improvement impacted the answer to this question. The gap in the data that existed in January between the low pretest scorer group and the high pretest scorer group diminished significantly by June, from 35% to 12%. The original assignment to a low pretest scorer group or a high pretest scorer group was accurate in January, but from January to June the accuracy of the group assignment blurred. As Table 25 shows, the low pretest scorer in the kindergarten to first grade group and the low pretest scorer in the second to fourth grade group scored higher on the posttest than the high pretest scorer in the fifth to eighth grade group.
Table 25

*Change in Content Knowledge*

<table>
<thead>
<tr>
<th>Participant</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Difference</th>
<th>Percent Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jane, High K - 1st</td>
<td>79%</td>
<td>88%</td>
<td>9%</td>
<td>11%</td>
</tr>
<tr>
<td>Deb, Low K - 1st</td>
<td>55%</td>
<td>78%</td>
<td>23%</td>
<td>42%</td>
</tr>
<tr>
<td>Mary, High 2nd – 4th</td>
<td>84%</td>
<td>88%</td>
<td>4%</td>
<td>5%</td>
</tr>
<tr>
<td>Ann, Low 2nd – 4th</td>
<td>40%</td>
<td>72%</td>
<td>32%</td>
<td>80%</td>
</tr>
<tr>
<td>Sue, High 5th – 8th</td>
<td>67%</td>
<td>66%</td>
<td>-1%</td>
<td>-1%</td>
</tr>
<tr>
<td>Pat, High 5th – 8th</td>
<td>29%</td>
<td>55%</td>
<td>26%</td>
<td>90%</td>
</tr>
</tbody>
</table>

Though the change in content knowledge blurred the high pretest content and low pretest content group assignment, four patterns emerged from this collection of data:

1. Based on a questionnaire, the concerns of teachers with high pretest content knowledge decreased and shifted from concerns for self to concerns for students and collaboration between January and June. The concerns of teachers with low pretest content knowledge remained high or increased and did not shift from self-concerns.

2. Based on what teachers said in online posts, levels of use of the innovation were higher among the high pretest content knowledge scorers and lower for low pretest scorers.

3. Based on observations of teachers implementing the innovation, two grade bands achieved a higher degree of fidelity, kindergarten through first and
second through fourth. The degree of fidelity to the model was lower for grade band fifth through eighth.

4. When teachers were observed implementing the innovation, less time was required for teachers with high pretest content knowledge to implement the innovation, an average of approximately two minutes forty-five seconds, than the low pretest content knowledge teachers, an average of approximately six minutes fifteen seconds.

The third question posted by this study, what impact does teachers' content knowledge for teaching have on six teachers’ ability to implement innovative pedagogical practices, was answered with less clarity. Teachers with higher content knowledge tended to adapt to change more easily, as was evidenced by a survey of their concerns. Teachers with higher content knowledge tended to be better able to write about and reflect on their use of the innovation, as was evidenced by their online posts. Higher content knowledge did not tend to impact teachers’ ability to implement innovative teaching practices with fidelity to the model. Rather in this study the grade band impacted ability to implement innovative teaching practices with a lower level of fidelity found at the upper grade levels.

Higher content knowledge impacted the time required to implement the innovation. Efficiency of mathematical language tended to be evident in teachers with higher content knowledge resulting implementation that was direct and succinct. Efficient mathematical language tended to reduce content wandering during the implementation and tended to reduce classroom management problems.
Summary

This study sought to understand the process of change elementary teachers experienced as they participated in mathematics professional development. Questions posed by this study include:

1. What is the impact of mathematics professional development on six elementary teachers’ mathematical content knowledge for teaching?
2. What is the impact of mathematics professional development on six teachers’ implementation of innovative pedagogical practices?
3. What impact does teachers’ content knowledge for teaching have on six teachers’ ability to implement innovative pedagogical practices?

Mathematical content knowledge for teaching was impacted by this professional development by an increase of twenty-six percent from January to June. The greatest increase occurred in the group that started with the lowest pretest scores.

This study identified three ways in which the mathematics professional development impacted implementation of innovative pedagogical practice:

- The mathematics professional development improved teachers’ feelings about the innovative teaching practice.
- Five of the six teachers achieved a basic level of use of the innovation.
- Four of the six teachers implemented the innovation with an acceptable level of fidelity to the model.

This study identified four patterns regarding the impact of teacher content knowledge on implementation of innovative pedagogical practices:
• Teachers with higher content knowledge tended to adapt to change more easily, as was evidenced by a survey of their concerns.

• Teachers with higher content knowledge tended to write about and reflect behaviors that indicate a higher level of use of the innovation, as was evidenced by their online posts.

• Higher content knowledge did not tend to impact teachers’ ability to implement innovative teaching practices with fidelity to the model.

• Higher content knowledge was correlated with the time required to implement the innovation. Efficiency of mathematical language tended to be evident in teachers with higher content knowledge resulting in implementation that was direct and succinct. Efficient mathematical language tended to reduce content wandering during the implementation and tended to reduce classroom management problems.

**Discussions**

Guskey’s model of the process of teacher change (1986) provided a suitable framework for this study. Guskey suggests professional development leads first to change in teachers’ classroom practices, then leads to change in student learning outcomes, and finally to change in teachers’ beliefs and attitudes as shown in Figure 1 on the next page.

Guskey’s model provided a roadmap to observe teacher change. This study sought to further research on teacher change, the components of change, and the relationship
between the components. This study focused on the first two steps in Guskey’s model, staff development followed by change in teacher' classroom practices.

Hall and Hord (2006) provided a lens through which teacher change could be defined and examined. Concerns Based Adoption Model served as a tool and technique for examining and measuring teacher change. These two theoretical frameworks each uniquely grounded this study.

In *Taking Charge of Change*, Hord, Rutherford et al. (2006) identified six key concepts of teacher change. Four of these concepts were especially apparent when examining the results of this study.

1. The concept that change is a process, not an event; and that change occurs over time, usually several years was apparent in this study. The findings confirm that change is a process rather than an event, and therefore point out a limitation of the study. The findings of this study would be different had the time frame been three years, rather than six months.

*Figure 1.* Guskey’s model of the process of teacher change was adapted from a figure in an article entitled “Staff Development and the Process of Teacher Change,” by T.R. Guskey, 1986, *Educational Researcher, 15*(5), 7.
2. The concept that change is a highly personal experience and that each person responds to change differently was also evident in the findings of this study. The findings from the six individual teachers that participated in the study were each unique.

3. A third concept, that change involves developmental growth and that this growth is expressed in terms of feelings and skills was highly apparent. The feelings teachers express in the process of change and the skills and behaviors they exhibit must be considered during the change process.

4. Finally, the concept that the focus of facilitation should be on individuals, innovations, and the context and that the real meaning of any changes lies within people was apparent. Change agents must focus on the individuals, understand the innovation, and consider the context as teachers move through the change process.

**Implications for Practice**

This study provided valuable insight into implications for further course development and further course facilitation. Five implications were identified and will be discussed:

1. The importance of including assessment into any professional development experience.

2. The importance of differentiating content for adult learners.

3. The importance of understanding and addressing feelings related to change that teachers experience as they attempt to improve their craft.
4. The importance of direct observation and coaching as part of any professional development process.

5. The importance of sustained involvement over time.

Including assessment into all professional development experiences is critical. Often professional development experiences end with teachers completing a short evaluation form that asks simple, superficial questions related to satisfaction with the experience. Multi-dimensional assessment of professional development is critical. Dimensions that must be pre assessed, formatively assessed, and post assessed include:

- Participant’s content knowledge
- Participant’s feelings about the changes they are implementing
- Participant’s behavior related to changes they are implementing
- Participant’s ability to implement changes with fidelity to a model

Assessment of students is required. Assessment of teachers is often avoided. Assessment of change resulting from professional development is critical and should become commonplace.

Differentiating the content taught through professional development experiences is critical. The results of this study pointed out the need to differentiate the content provided in professional development experiences. The high pretest group of teachers made minimal gain pretest to posttest because their pretest scores were near eighty percent, creating a ceiling effect. A pretest in an elementary classroom is often an opportunity for teachers to identify students for whom a differentiated curriculum is
appropriate. Professional development experiences should include the option of differentiated content.

A second result of this study that pointed out the need to differentiate professional development content was teachers' inability to use efficient language in mathematics instruction. Clear differences were observed between teachers with and without the ability to efficiently communicate mathematics. Shulman (1986) pointed out the need for subject matter knowledge, pedagogical content knowledge, and curricular knowledge. Hill and Ball (2009) extended Shulman's work that described pedagogical content knowledge to more clearly articulate the type of content knowledge that is needed to effectively teach mathematics. This study clearly pointed out the need for mathematical knowledge for teaching, and particularly the need to address efficient mathematical language.

Understanding and addressing feelings that teachers experience as they make change in their practice is critical. Analyzing the Stages of Concern Questionnaires and reading the online discussion board prompts provided a window into the feelings teachers experience as they attempt to change. If feelings are not understood or addressed, change is undermined and will stagnate. Professional development experience must include multiple measures of teachers' feelings to accurately identify feelings and concerns so they can be addressed.

Direct observation of teachers as they attempt to change teaching practices, combined with coaching is a needed component of professional development experiences. Reading online discussion board posts provided a viewpoint of self-reported
teacher change. Direct observation of teacher practice provides a needed viewpoint and
provides the opportunity for coaching.

The results of this study confirmed the need for sustained, long-term professional
development if lasting change is to occur. The minimal shift in teachers' feelings about
implementation of Meaningful Distributed Instruction: Preview for Symbolic Procedure
(MDI: PSP) and the continued concern that teachers did not have adequate information
about MDI: PSP, even after five months of professional development pointed out the need
for long-term professional development experiences.

Recommendations for Further Research

The assessment tools used in this study included a mathematics content
knowledge test and Concerns Based Adoption Model instruments. Further research
should address methods of assessing professional development experiences that are
efficient and effective. A second recommend area for further research is the need to
better understand how to develop the type of content knowledge teachers need to
efficiently and effectively use language in the mathematics classroom.
REFERENCES


APPENDIX A

INSTRUCTIONAL SESSION SUMMARY

Making Sense of Operations
Ten Half-Day Instructional Sessions

Session 1: Making Sense of Operations
Making Sense of Operations uses the text, Making Meaning for Operations: Casebook (Schifter et.al. 1999b). Additional course materials include the Iowa Core Curriculum, Iowa Professional Development Model, and pertinent articles from practitioner journals. This course continues the exploration of children’s thinking that was begun in Making Sense of Numbers by focusing on the operation involved in the problems that children solve and the way that understanding is constructed. The course will challenge your definition of operation to expand beyond computation. In addition, the course makes explicit connections to the Iowa Core Curriculum and best practice for professional development as defined by the Iowa Professional Development Model. This session will introduce you to the course, the technology you will need for the assignments, and to your classmates.

Session 2: Counting Up, Counting Back, and Counting By
In this session you will read Cases and view videos to help you consider the ways students use counting to solve problems. You will have opportunities to see how children often follow the structure of the problem in order to solve it. Through the mathematics activity you will reflect on your own counting strategies. In addition, you will continue to explore Distributed Practice that is Purposeful and Meaningful.

Session 3: Addition and Subtraction as Models
The Cases for Session 3 highlight children demonstrating more sophisticated understandings of operations than what was demonstrated by the children in Sessions 1 and 2. The children solve problems using methods their teachers did not expect, which allow the participants to examine the children’s reasoning.

Session 4: What is Multiplication? What is Division?
The Cases for Session 4 focus on how students approach multiplication and division problems. You will explore what multiplication and division are, and how all four operations are related to each other. Through the Cases presented in Chapter 3 of the Casebook, you will examine the understanding students have of the connections between multiplication and division and how they build on their understanding of addition and subtraction.

Session 5: When Dividing Gives an Answer Less than One
In this session participants read two cases in which the same class addresses a problem twice: first as fourth graders and then, a few months later, as fifth graders. The students struggle with thinking about 5 divided by 39. Their idea of number is stretched as they try to make sense of the problem. Participants will also engage in a mathematics activity to stretch their ideas about fractions.

**Session 6: Combining Shares, or Adding Fractions**
In this session you will read Cases in which students try to make sense of adding fractions with unlike denominators. You will continue to think about children’s as well as your own understanding of fractions as you make sense of the representations that children show in the Cases and work through a mathematics activity.

**Session 7: Taking Portions of Portions or Multiplying Fractions**
This session looks at multiplication of fractions with factors less than one whole. The Cases highlight children making sense of problem situations that push them to think deeply about their understanding of fractions. You will use two models, an area model and a number line model for multiplying fractions that extend beyond the students’ work in the case studies.

**Session 8: Multiplying Mixed Numbers**
This session is a continuation of multiplying fractions emphasizing mixed fractions. The case highlights students making sense of an area model. After reading and discussing the students’ work, you will solve multiplication of mixed fractions problems using the area model.

**Session 9: Expanding Ideas about Division in the Context of Fractions**
In this session participants read two Cases in which students and their teachers try to make sense of dividing fractions. Participants will continue to think about children’s as well as their own understanding of fractions and the impact of operations on them as they make sense of the representations that children show in the Cases and complete a mathematics activity on solving a division problem involving fractions.

**Session 10: Highlights of Related Research**
This session highlights the research that supports instructional strategies throughout the course and the Iowa Core Curriculum. Participants will look at the mathematics and pedagogy through the eyes of a researcher and will make connections to current and desired classroom practice.
APPENDIX B

IMPLEMENTATION SESSION SUMMARY

Making Sense of Operations
Six Two-Week Implementation Sessions

Implementation 1: What is Distributed Practice that is Meaningful and Purposeful?
In the first course, Making Sense of Numbers, implementation of Distributed Practice that is Meaningful and Purposeful focused on previews that developed number sense. You will be asked to refine your learning by implementing MDI Previews for Number Sense throughout this course. In addition, you will broaden and deepen your understanding of Meaningful Distributed Instruction with new learning. The implementation focus that is new for this course connects actions and representations of an operation to algorithms for that operation in MDI Previews for Symbolic Procedures. In implementation 1 you will review definitions from the Iowa Core, preview an article by Dr. Rathmell that you will read in Session 2, and highlighted related research. In addition, this implementation asks you to reflect upon your experience as a learner and as a teacher regarding distributed instruction.

Implementation 2: Representing
This implementation focuses on Distributed Practice that is Meaningful and Purposeful in the form of a preview. More specifically, these previews use representations of algorithms prior to learning algorithms. Effectively using representations is an Essential Skill and making connections is an Essential Characteristic in the Iowa Core Curriculum. You will analyze a standard algorithm or procedure in mathematics and explain why the algorithm or procedure works using connected representations.

Implementation 3: Connecting Deep Procedural and Conceptual Knowledge
Throughout the course, both the teachers and students in the vignettes have been trying to deeply understand the mathematical content. This understanding includes both deep procedural knowledge and deep conceptual knowledge. This implementation explores what it means to build both procedural and conceptual knowledge. You will also create the first of ten MDI Previews for Symbolic Procedures for a mathematical concept and receive feedback. The focus of this type of MDI will be to build conceptual knowledge and then connect this knowledge to the procedures students are asked to learn.

Implementation 4: Understanding
One of the Characteristics for Effective Instruction is Teaching for Understanding. This course has been exploring how students' build understanding and meaning for different mathematical operations. In this implementation, you will review the ICC definition of Teaching for Understanding and discuss what implications it has for their classrooms. In this implementation you will create the remaining nine previews for symbolic procedures.
Implementation 5: Reflecting on Your Practice
You have had the opportunity to learn from each other and to support each other throughout the Course and through Implementation Class Discussion. In this session, you will explore the power of your own internal voice as well as the external voices of collaborative colleagues as you refine your journey of becoming a reflective practitioner. You will read an article to help you consider the habit of reflection and its potential impact on your own efficacy and develop a plan for continued implementation of the mathematical ideas explored in this course in collaboration with a colleague. You will also begin to develop a reflective response describing your learning that occurred from implementing Meaningful Distributed Instruction Previews for Symbolic Procedures into your classroom routines.

Implementation 6: Course Evaluation
We started this course with four overarching goals:

- You will think through the major ideas of K-6 mathematics and examine how children develop these ideas.
- You will explore children’s thinking to discover what they understand (and misunderstand) about operations.
- You will explore children’s thinking to discover how making meaning of operations affects their work with computation.
- You will implement Meaningful Distributed Instruction in your classrooms, specifically Previews for Symbolic Procedures.

You will have the opportunity to reflect upon and evaluate your learning in this course.
Online Implementation Session 4
The following was taken from the online portion of *Making Sense of Operations*:

**Connecting Deep Conceptual and Procedural Knowledge**

**Summary**
Throughout the course, both the teachers and students in the vignettes have been trying to deeply understand the mathematical content. This understanding includes both deep procedural knowledge and deep conceptual knowledge. This implementation explores what it means to build both procedural and conceptual knowledge. You will also create the first of ten MDI Previews for Symbolic Procedures for a mathematical concept and receive feedback. The focus of this type of MDI will be to build conceptual knowledge and then connect this knowledge to the procedures students are asked to learn.

**Objectives**
In this implementation, you will
- make connections between course content and the *Iowa Core Curriculum*;
- analyze and discuss connecting deep conceptual and procedural knowledge;
- create a Preview for Symbolic Procedures using the five-minute lesson design for MDI; and
- share your Preview for Symbolic Procedures and receive feedback.

**Reading**
"The ways in which mathematical ideas are represented is fundamental to how people can understand and use those ideas. When students gain access to mathematical representations and the ideas they represent, they have a set of tools that significantly expand their capacity to think mathematically." (NCTM, 2000, p. 67)

1. Review, again, the explanation of deep conceptual and deep procedural knowledge from the *Iowa Core Curriculum Essential Characteristics for K-12 Mathematics*

**Activity**
In Implementation 4 you will be asked to complete Part 1 of the assignment *Creating Meaningful Distributed Instruction: Previews for Symbolic Procedures*. You will want to read through this assignment before completing this activity.

*Creating Meaningful Distributed Instruction: Previews for Symbolic Procedures*
Select a key algorithm for your grade level and create previews that students can solve and discuss in 3-5 minutes. The preview should be structured so students use representations to solve the problems. In addition to the student task, design a set
of questions that require students to connect the actions done to the representation to the meaning of the operation. Be sure to follow the guidelines for previews. The tasks should be student centered and encourage the use of models or representations.

You will be asked to share your preview in the discussions for participants and the facilitator to give feedback. This activity and the feedback you receive will help you as you create a set of ten previews for the assignment in Implementation 4. This may be used as the first preview of the set of ten previews for this assignment. It may be helpful to view the sample sets of videos on Previews for Symbolic Procedures found in the Videos icon on the course Home Page.

Class Discussion

Part A
1. Go to the Implementation 3 Part A topic in the Discussions tool and start a new post. As you consider the following questions, you may want to think about new learning from the course, the Essential Characteristics, Concepts and Skills from the ICC, and readings. Address the following in your post:
   - Share the Preview for Symbolic Procedures you created for the activity in this implementation.
   - Provide feedback to others about the strengths of the Preview for Symbolic Procedures they shared as well as ideas for improving their preview.
   - How do you believe the use of Previews for Symbolic Procedures can help the connection of deep conceptual and procedural knowledge?
   - How do you (will you) promote the connection of deep conceptual and procedural knowledge in your classroom?

2. Respond to at least two participants’ posts by sharing questions, ideas, and resources. You may review the Rubric for Evaluating Class Discussion to better understand how you will be assessed.

Part B
1. Go to the Implementation 3 Part B topic in the Discussions tool. Start a new thread and answer the following questions:

   The ICC Essential Characteristics for K-12 mathematics characterizes deep-level knowledge as “comprehension, abstraction, flexibility, critical judgment, and evaluation.”
   - What information would you use to consider addressing a new mathematical concept in your MDI Previews for Number Sense?
   - Must students develop all five attributes of “deep-level knowledge” prior to moving on? Why, or why not?

2. Respond to at least two participants’ posts by sharing questions, ideas, and resources. You may review the Rubric for Evaluating Class Discussion to better understand how you will be assessed.

Creating Meaningful Distributed Instruction: Preview for Symbolic Procedures

Complete Part 1 of the assignment and submit it to your course facilitator as an attachment to the Assignments tool. You will submit Part 2 and Part 3 of the assignments in Implementation 5 or 6. Part 2 requires you to implement your
sequence of tasks. You will want to begin now so you have enough time by the end of the course to complete all the tasks and write a reflection over the experience.

**Part 1:** Select a key topic for your grade level and plan a series of 7-10 tasks that students can solve and discuss in 3-5 minutes. Follow the guidelines for Previews for Symbolic Procedures identified in the Rathmell article. The tasks should be student centered and encourage the use of models or representations.

**Submit your tasks to your facilitator by the end of Implementation 4.**

**Part 2:** Present a task in consecutive days. Collect students' work so you can monitor their progress. If students' written work is not appropriate, take photos of the students completing the tasks or collect written observations. Typically the lessons are structured as shown below:

- A computational problem is posed. (10 seconds)
- Students are asked to solve that computational problem using the model. (30-40 seconds)
- The teacher guides students to use thinking that is consistent with the algorithm with questions. (1 minute 30 seconds)
- A student is asked to explain the step-by-step actions with the model and confirm their answer. (30 seconds)
- The teacher summarizes the step-by-step actions. (10 seconds)

**Part 3:** Write a paper that includes

- the tasks you used;
- samples of three different students' work (paper, audio, video or teacher's note of a discussion, etc.) that show how their understanding of the given concept progressed over the days, and
- a reflection of your fidelity to the MDI: Preview for Symbolic Procedures strategy using the *Innovation Configuration Map* Observation Tool. Notice the left most column describes the strategy as individual components as well as acceptable variations of those components. Towards the right, some unacceptable variations of those components are listed.
  - In what ways are you confident that you are delivering the strategy with fidelity?
  - In what ways do you need to adjust?
  - Would this change effect student learning?

Part 2 and Part 3 of this assignment need to be completed by the end of **Implementation 6.** Submit your assignment to your course facilitator as an attachment to the *Assignments* tool.
The following was taken from the online portion of *Making Sense of Operations*. Teachers’ names have been replaced with identification numbers:

Compiled Messages

**Topic:** Grades 2-4  
**Date:** Monday, April 19, 2010  
**Subject:** More thoughts on MDI Previews for Symbolic Procedures  
**Author:** #19

How did you go about designing or choosing your MDI Previews for Symbolic Procedures? I chose double-digit multiplication because it is a challenging concept for fourth grade students, and it is part of my curriculum. What have been the challenges? One of the biggest challenges is that the students are not strong on the basic math facts. That makes it difficult to keep track of what you are doing in a double digit by double digit problem because you have to go back and figure out the fact before you can move on to the next step of the problem. The students also have had some difficulty in realizing we are working with tens and ones and hundreds instead of single digits throughout the problem. How is MDI Previews for Symbolic Procedures a strategy for formative assessment? I think MDI Previews for Symbolic Procedures can be used as a strategy for formative assessment. If demonstrated correctly by the teacher, students should have a clear model to follow when demonstrating and explaining work of their own. What is the value of using MDI Previews for Symbolic Procedures with students in your class? One of the biggest values I have noticed is giving students a better understanding of concepts like why you use a zero to hold a place when multiplying the tens in a double digit times a double-digit multiplication problem. Demonstrating and explaining at the same time can help reinforce concepts like place value and how to use it to solve problems. What tasks or lessons would you plan to help further develop the students' understanding of the topic? Make sure to include a rationale for the tasks you select. I am going to continue to use the graph paper to show how to multiply. I am also going to have the students cut out the different parts of the graph paper to show them the different parts of the multiplication problem. Before I do this, I am going to revisit arrays and the commutative property so they understand you can change the order of two numbers and still get the same product. I think this is important because sometimes they read the problem from the bottom up and another time the top down. It also will help when looking at the sections we cut out from the graph paper.

**Topic:** Grades 2-4  
**Date:** Tuesday, April 20, 2010  
**Subject:** Re: More thoughts on MDI Previews for Symbolic Procedures  
**Author:** #2

#19, I have found that using the graph paper does help them visualize what they are doing a little better. I think that breaking apart the numbers into smaller chunks helps them too. I think presenting
multiplication (or for that matter any math concept) in multiple ways will help kids in their understanding.

Topic: Grades 2-4  
Date: Friday, April 23, 2010  
Subject: Re:More thoughts on MDI Previews for Symbolic Procedures  
Author: #19
I agree with the multiple ways for sure. Even though the graph paper has gotten easier for some of my students. Breaking the numbers apart is easier for others and then there are those that just use the algorithm or still add the sets. I think patience is pretty important too!

Topic: Grades 2-4  
Date: Wednesday, April 14, 2010  
Subject: #11 4A  
Author: #11
I'm doing 2-digit multiplication and using graph paper for my Previews for Symbolic Procedure. I like that it "shows" the breakdown of the place values as you are performing the steps of multiplication. My students are still struggling some with the order of the steps, but they do see the results of ones times ones, ones times tens, tens times ones, and tens times tens. I have cut the graph paper sections apart to show them as I perform the sequential steps. I hope they will visualize these in their minds as they begin to work the algorithm. I believe these MDI's have helped them understand the placement of 0 to hold the ones when multiplying with the tens spot. I am just beginning to use these as an assessment tool with the students drawing using the graph paper and explaining what each section represents. I want them to begin using the algorithm and showing each graph section with each step used in the algorithm. Slow process to begin with having all students perform it, I think I'll switch to small groups for awhile, then go to individuals.

Topic: Grades 2-4  
Date: Monday, April 19, 2010  
Subject: Re:#11 4A  
Author: #19
I agree that this is a great way to help them understand the use of a zero to hold the ones place when multiplying with the tens.

Topic: Grades 2-4  
Date: Tuesday, April 20, 2010  
Subject: Re:#11 4A  
Author: #16
I like the idea of using graph paper for multiplication. Hopefully over time your students will overcome their struggling with the steps, but yet see how valuable each step is when getting the end result. I don’t teach 2 digit multiplication, but I have noticed through past tutoring experiences and observation that students do struggle with the placement of the 0. It just doesn’t make sense to them. The preview with the graph paper sounds like it would make it more real for them and hopefully help them understand the value of the 0. I think your have the right idea of starting with groups and them move on into individual work.
I'm impressed with your use of the model for assessment. If your students can draw a representation of their 2-digit multiplication problem and explain their meaning of each section, they would demonstrate a deep understanding of the concept. It would be very time-intensive, but would give you valuable insight into the level of understanding each student has. I had never thought of representing the model with graph paper for 2-digit multiplication. I had used arrays for multiplication facts, but that is a much simpler model. How did your students respond to your introduction of the model? I know I found it fascinating, but complicated. Were they confused or intrigued?

At first a little confused, but then they started catching on. They really liked it when we started cutting out the areas and using them to explain the 1s X 1's, 1 X 10's, 10's x 1's, and the 10's x 10's. Then the 10 39's X 1 39's and 10 39's X 10 39's. Just had to make sure they saw the difference between 1 X 10's and 10's X 1's.

That sounds really good. I know by using the graph paper to work on using arrays to work with the basic facts have really helped my students. I think this use of arrays will be a good jumping off point for next year as they begin working with multiplying larger numbers. My kids even wanted to use the graph paper and arrays when we multiplied 3 factors. It was really cool to see my kids making the connection with the strategy we have been working on and connecting that to other problems.

I chose to work on basic multiplication facts and using arrays to split basic facts into smaller more friendly numbers to multiply with. I decided to focus on 3's, 4's, 6's, 7's, and 8's. I decided to focus on those facts because my kids have had problems with those in the past. Some of the challenges I started with at first was getting the kids to understand how to draw the array and then which number they were splitting into smaller parts. They seemed to have trouble understanding if they were splitting the number of rows or the number in each row and which factor from the multiplication sentence that was represented by. I have seen tremendous value in this. They have come up with some very interesting ways to split the arrays into smaller parts to help them
solving the problem. It has been a very valuable strategy that they have learned to use. I even started using it to multiply 3 factors. In fact one of the kids asked if we could use arrays to help us figure it out. They even suggested we split the array when one of the factors was not a friendly number to multiply with. This has really given me great insight to see what the kids are doing and their thinking as they explain what they have done. It's been great.

Topic: Grades 2-4
Date: Wednesday, April 14, 2010
Subject: Re: Implementation 4A
Author: #19
I am so glad you are working on the multiplication facts. It will be very helpful next year in fourth grade. As much as I have emphasized learning the facts and practicing them just 5 minutes a night, some of the kids still don't know them very well. I believe they understand the idea, and they can figure out the product, it just takes them more time than it should. Very frustrating for them and me!

Topic: Grades 2-4
Date: Wednesday, April 14, 2010
Subject: Re: Implementation 4A
Author: #11
Ditto to #19. One of my hugest hangups this year is my students’ lack of multiplication facts knowledge. They continue to break apart numbers and solve problems, but STILL struggle with basic facts. Anything that gives them strategies for applying these facts will help teachers trying to teach larger multiplication problems will help immensely. You’re seeing the same difficulties that I see when it comes to the 6-8 facts too. Keep plugging away, next years teacher will love you for it.

Topic: Grades 2-4
Date: Wednesday, April 14, 2010
Subject: #2 Implementation 4A
Author: #2
My students have been learning more effective strategies for figuring out their basic multiplication facts. I found that while our math book was quickly moving on to other concepts my students did not know effective ways to find the answer to their multiplication facts. They were continually using their multiplication tables in the book to do their facts. We began by skip counting with hundreds charts. We did this for 2s, 3s, 6s, 7s, and 8s. I chose to do these numbers because I had other strategies in mind for the other numbers. Many of my kids do not understand what a double is. This really surprised me, so we have been working on multiplying by 2 and dividing by 2. We have also used the graph paper to show dividing numbers into two different equations. After my kids started understanding doubling we used that strategy to do a “double double” equation for 4s. I think MDI is a great assessment tool because I can walk around and glance at my kids’ papers and immediately tell if they are understanding the concept or not. I think the procedure tells way more about their real understanding of numbers than simply doing an assignment.

Topic: Grades 2-4
Date: Wednesday, April 14, 2010
That sounds great #2. I too feel that the math book moves too quickly. I feel that some kids really don’t understand a concept or skill in the manner I think they should and the book is moving onto the next topic. I have been moving a lot slower but still using the book more as a resource for my teaching and for some problem examples. In fact we got out our books the other day and the kids commented that we haven’t used them in awhile. I kind of laughed to myself because I have been using them as a resource for my teaching. The MDI does give a good picture into the kid’s thinking.

I agree with you and #18 about the book moving on too quickly. I’ve been told all year that the book “spirals” and to keep moving forward, the concept will be presented again. It’s hard for me to move on when I don’t feel my students have grasped the concept being presented. I’ve been struggling all year to balance the district’s desire for us to cover as much as possible with my belief that we slow down for understanding.

It’s interesting that your kids have having a hard time with the doubling concept. I have been pleasantly surprised at the level of understanding my second graders are showing with their multiplication facts. They really seem to get the doubling concept for multiplying by 2. The 4’s are introduced tomorrow and I will be interested to see if anyone makes the connection with your double double strategy. Of course, we are just at the initial phase of these multiplication. We started with 5’s, then went to 2’s, and then to 3’s. I wonder if you will see benefits next year, since we are spending much more time on this concept this year. We have been drawing and building equal groups to solve multiplication problems. Our lesson tomorrow is building arrays with tiles and then transferring arrays to grip paper. I am excited to see how they do with that.
such as taking away 40 instead of 4, and that my students already have an idea of how the standard algorithm works. I have also struggled some with saying too much as making it more of a lesson than just a preview and then I am getting into my lesson time for the day. As far as formative assessment, I feel that the MDI previews for Symbolic Procedure is a very good tool to have in my basket. It allows me to see if the students can actually model and say what I do correctly. It also tells me what I need to work on with that student to make sure he or she is understanding the procedure. Making sure that I call on all of my students throughout a period of previews has been a challenge because they all want a chance to retell and model. The value of using the MDI's for Symbolic Procedure is that the students can actually see why they have to regroup in the model on a daily basis and they start making the connection with the standard algorithm when you do it with pencil and paper. I do have a few students that are still struggling with the pencil and paper, but yet they can do it with the model. Any suggestions on how I can help them make the connection clearer to them? I have been developing lessons for those students who are struggling with then tens and hundreds place. I have been having them use the base ten blocks, draw pictures, and next week we will use money. When I design my previews I have also been using the students names or teachers' names they know in the building to keep them interested as well as relating the problem to them on an individual basis such as baseball, tv shows, and flowers. I am also having my higher-ended students work with these students so I can observe more where the students are struggling and ask questions. I have noticed that even my higher ended students have been asking questions to the students of how did you get that answer and why are we regrouping.

Topic: Grades 2-4
Date: Friday, April 16, 2010
Subject: Re:#16's Implementation 4A
Author: #10
#16, Sounds like your previews were well thought out. I like how you started with regrouping ones, then moved on to tens, then zeros in both positions. I agree with you that there are challenges involved to make sure we use the correct terminology during the previews and making sure they don't last too long...even though was just modeling I sometimes found myself talking too much! That's great that you are developing lessons for those who struggle with tens and ones. You'll really be able to take this to the next level!

Topic: Grades 2-4
Date: Monday, April 12, 2010
Subject: #13 Implementation 4A
Author: #13
At our last face-to-face meeting, I had an opportunity to collaborate with other second grade teachers. Through our discussion, we determined that previewing 3-digit subtraction with trading would be a good symbolic procedure to preview. I decided to focus on numbers with zero, as those types of problems give many students difficulty. The problems I chose to preview have either a zero in the ones place, a zero in the tens place, or both. The main challenge I face is that the students already know the algorithm for 2-digit numbers. Many of them want to use the algorithm instead of the physical model to explain.
The other challenge is the age-old time factor. The plate is so full of things that I have to get done that adding one more thing is difficult. A formative assessment is a tool for determining what students know so that an instructor can plan the next instructional step. MDI Previews for Symbolic Procedures can be used as formative assessment when a student explains the procedure that has been modeled by the teacher. If a student cannot explain the procedure, then the teacher has information about that particular student. When previewing for the whole class, it would take several days to gather information about all of your students. I'm not sure it's the best tool for formative assessment because the 10 days cycle of preview problems would not allow you to get information on all of your students. I think there is value in using MDI Previews for Symbolic Procedures in the classroom because it allows many opportunities for students to witness the steps that connect the concrete model to the symbolic procedure. It is inevitable that students will be exposed to the algorithm either at school or at home. Knowing how to do the algorithm doesn't guarantee an understanding of why the algorithm works. MDI Previews for Symbolic Procedures increase the likelihood that students will have a deeper understanding of the mathematics that happen within the procedure. Ideally, students should be able to model the symbolic procedure themselves instead of just watch an instructor present the model. Getting the place value blocks in their hands and having them demonstrate the procedure that matches the algorithm would be the next step. I would first provide group practice with manipulatives. Hopefully, not all students would require extensive experiences with the manipulatives. Through observation and questioning, I could identify those students who would benefit from an intervention that provides additional hands-on practice to allow that deep conceptual understanding to develop. It might also be helpful to have students solve problems using the algorithm in conjunction with the physical model to help better make those connections.

Topic: Grades 2-4
Date: Friday, April 16, 2010
Subject: Re:#13’s Implementation 4A
Author: #10

#13, it is hard to get students away from thinking solely about the standard algorithm and to get them away from that and use the physical models to explain. As a tool for formative assessment, I think it's just one tool of many we can use. Within a 10-day time frame we might not get to hear from every student, but at least we have some knowledge of what some students know and understand. The previews should help them have a deeper understanding of the procedure. After my 10 days of procedures I had students use the base ten blocks along with the standard algorithm. For some, having the base ten blocks there was a great help. For others, they just wanted to work it through on paper. It was beneficial for my students to get that extra hands-on practice, though.

Topic: Grades 2-4
Date: Tuesday, April 20, 2010
Subject: Re:#13 Implementation 4A
Author: #2
I agree that the challenging part of the symbolic procedure method is that most of the kids have all learned the algorithm by now and want to use it - whether they understand it or not. Presenting it in a different way may still help them see the why of what they are doing. Subtracting 2 and 3 digit numbers is still a big problem for a lot of kids in 3rd grade. It's hard to get them to go back and discover the why when they think they know the process. I think there are many teachers out there who believe they don't really need to know why, but they need to know how.

Topic: Grades 2-4
Date: Saturday, April 10, 2010
Subject: #10 Implementation 4A
Author: #10

I designed/chose my MDI Previews for Symbolic Procedures with the other 2nd grade teachers when we met at our last face-to-face session. After we decided that our previews were going to focus on three-digit subtraction with regrouping, I later wrote the ten tasks for the students to solve. I made the tasks into word problems that would be relevant to their everyday life. I wrote these tasks to introduce the standard algorithm for three-digit subtraction with regrouping so students would have an understanding of this procedure before going to third grade. The challenges with the Previews for Symbolic Procedures have been making sure I involve as many students as I can from day-to-day...trying not to call on the same ones with each different task. Also, making sure I use the correct terminology (not just saying, "I need to take away"; but rather, "I need to take away 2 tens or twenty") so the students do not get confused. MDI Previews for Symbolic Procedures is a good strategy for formative assessment because it can give me (within a short amount of time) who is understanding the concept or procedure being shown. If the student can state what to do or retell it to the class, I have a quick way of gathering information on what he/she understands. The MDI Previews for Symbolic Procedures are ongoing for a two-week period and serve as practice for the students. They are done consecutively in a short amount of time. This allows me to gain data on what each student knows and their level of understanding, provided that I call on different students each time to gain this information. Then, I can make changes or adjust my tasks based on what they understand to increase student learning. The value of using MDI Previews for Symbolic Procedures is that they really help students see and understand they "why" behind what we do. Students will know more than just "how" to do the procedure. They will be able to make connections to ideas they already know and have seen and make the connection between the model and the standard algorithm. It will help them make sense of why we do a task a particular way. I've already developed lessons that progressed from no regrouping needed to regrouping a ten for ten ones to regrouping a hundred for 10 tens, regrouping both tens and hundreds, and regrouping involving 1 or 2 zeros (such as 400-137). I feel these lessons have helped to further develop the students' understanding of the topic because they have progressed from "easier" to more difficult with each task. They have added to what they already have learned so students can make connections to do the more difficult problems. Other things I could do to further develop understanding of three-digit subtraction with
regrouping would be to let students help guide the lesson and share in asking the questions instead of just me doing it. This would help students gain confidence in what they are doing and would help me know who really understands the procedures behind the task. I would also let one student use the magnetic board with the hundreds, tens, and ones manipulatives (in front of the class) while the task is being completed. Students need to become more independent with the procedures and this would help increase their independence with the given tasks. I would also bring real-life situations into this, for example, by using money (dollars, dimes, and pennies) instead of the base ten blocks. This could make it more meaningful for students because it would be something they could relate to. Another task I would plan to help further develop the students' understanding of the topic would be to use math centers (that cover three-digit subtraction with regrouping) because centers are flexible and can focus on different levels of readiness to meet the needs of my students' many interests. They would help reinforce the skill/task. I could use the centers as assigned learning, choice-based learning, used in their free time, on their own, in a group, or with a partner. The centers could help students who don't quite have the skill, or challenge other by taking them above and beyond the task/concept we're covering. I could also further develop the topic of place value with hundreds, tens, and ones by having students explore three-digit addition with regrouping.

Topic: Grades 2-4
Date: Thursday, April 8, 2010
Subject: Thoughts about the new MDI
Author: #19
I like using this MDI. My students have been pretty good about paying attention. They are having some difficulty repeating the steps. I am sure there are probably numerous reasons for that, but they are starting to comment that it is getting a little easier to follow along. (I am using the graph paper to show how to do double digit by double-digit multiplication). I have done 5 demonstrations to far, tomorrow will be number six. They have tried a couple of “practice” problems and a few 7-8 of them have done them correctly. It is fun to see their faces and hear their “Yes!” when they find out the answer is correct. Some are finding numbers easier to break apart and multiply. It has also helped a couple with their division some because they are looking at the dividend as hundreds, tens, ones. I’ll have more to post in a few days.

Topic: Grades 2-4
Date: Wednesday, April 14, 2010
Subject: Re: Thoughts about the new MDI
Author: #11
I’m having the same problem with students repeating the steps #19. I’m going to try having the students cut apart the graph paper as they draw the sections, then have them work the algorithm and show the section that goes with each step. It will either enhance understanding or completely confuse them! But like I said in my post, this has helped them understand why they have to use a 0 to hold when multiplying by the tens place.

Topic: Grades 2-4
Date: Monday, April 19, 2010
Subject: Re: Thoughts about the new MDI
Author: #19
#11, that is a great idea. I will try that tomorrow. Several of my kids say it is getting easier to understand, but they still have trouble explaining and do the work. Maybe cutting will help! It is worth a try.

Topic: Grades 2-4
Date: Tuesday, April 20, 2010
Subject: Re: Thoughts about the new MDI
Author: #16
I like using this MDI also. I noticed #11 had mentioned in his post that his students are struggling repeating the steps also. Hopefully over time it will become easier for them especially the more previews that you do. I had never thought of using graph paper to show multiplication and division until it was demonstrated in our last class. I am amazed at how much sense it makes and how you can break it apart and multiply. Have you always used graph paper or is this something that you started after our last class. I was just thinking that this would be a good idea for the third grade teacher here when she is working on multiplication, but was wondering what your thought on this was?
## APPENDIX D

### MDI: PSP INNOVATION CONFIGURATION MAP

<table>
<thead>
<tr>
<th>Component</th>
<th>Variation</th>
<th>Variation</th>
<th>Variation</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Teacher actions during the preview</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>CUE</strong></td>
<td>Teacher starts before students understand that this is a brief preview (no more than 2-5 specific computational focus minutes)</td>
<td>Teacher starts before naming the lesson</td>
<td>Teacher announces the lesson</td>
<td>Teacher starts the lesson after the students understand the lesson</td>
</tr>
<tr>
<td><strong>PRESENT PROBLEM</strong></td>
<td>Teacher presents computational procedure that previews the specific computational procedure to be taught. (Teacher makes the computation problem visible to the students in a horizontal format, i.e. 7+3)</td>
<td>Teacher presents a word problem that previews the specific computational problem to be taught. (Teacher makes the computation problem visible to the students in a horizontal format, i.e. 7+3)</td>
<td>Teacher makes the problem too difficult for the students to solve</td>
<td>Teacher continues to present the problem even though the students are confused</td>
</tr>
<tr>
<td><strong>MODEL PROBLEM</strong></td>
<td>Teacher allows students to use a model (selected by the teacher) to represent (set up) the problem but not the answer (teacher sets up representation until students can assist). It is assumed that the students are familiar with the chosen model.</td>
<td>Teacher presents a word problem using the model.</td>
<td>Teacher continues to present the problem even though the students are confused</td>
<td>Teacher continues to present the problem even though the students are confused</td>
</tr>
<tr>
<td><strong>SOLVE PROBLEM</strong></td>
<td>Teacher models the procedural without asking the students questions to &quot;lead&quot; them when students can explain or assist in explaining.</td>
<td>Teacher asks students to solve the problem on their own or in pairs, not in a whole class setting</td>
<td>If students are asked to do this on their own or in pairs, not in a whole class setting</td>
<td>Teacher asks students to solve the problem on their own or in pairs, not in a whole class setting</td>
</tr>
<tr>
<td><strong>ANSWER QUESTION</strong></td>
<td>Teacher asks students, &quot;what is the answer to the problem? How does this model show the answer to the problem?&quot;</td>
<td>Teacher reports the answer</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>SUMMARIZE</strong></td>
<td>A second summary is lengthy and student input was elicited during the summary that confused the students.</td>
<td>Summary is lengthy and student input was given during the summary that confused the students.</td>
<td>No summary of the lesson was given.</td>
<td>No summary of the lesson was given.</td>
</tr>
</tbody>
</table>

### Planning Actions

1. Chosen model represents the algorithm
2. Plan for a five to seven minute preview
3. Prior to the preview the teacher practices the "leading questions" after watching the video
4. Prior to the preview the teacher practices the summary of the step-by-step actions

**Note.** Variations to the left of the line are acceptable. Variations to the right are unacceptable. Adapted from “Making Sense of Operations” by UNI, 2009.
APPENDIX E

STAGES OF CONCERN QUESTIONNAIRE

Name (optional): ________________________________________________________________

The purpose of this questionnaire is to determine what people who are using or thinking
about using various programs are concerned about at various times during the adoption
process.

The items were developed from typical responses of school and college teachers who
ranged from no knowledge at all about various programs to many years' experience using
them. Therefore, many of the items on this questionnaire may appear to be of little
relevance or irrelevant to you at this time. For the completely irrelevant items, please
circle “0” on the scale. Other items will represent those concerns you do have, in varying
degrees of intensity, and should be marked higher on the scale.

For example:

This statement is very true of me at this time. 0 1 2 3 4 5 6 7

This statement is somewhat true of me now. 0 1 2 3 4 5 6 7

This statement is not at all true of me at this time. 0 1 2 3 4 5 6 7

This statement seems irrelevant to me. 0 1 2 3 4 5 6 7

Please respond to the items in terms of your present concerns, or how you feel about
your involvement with this innovation. We do not hold to any one definition of the
innovation so please think of it in terms of your own perception of what it involves.
Phrases such as “this approach” and “the new system” all refer to the same innovation.
Remember to respond to each item in terms of your present concerns about your
involvement or potential involvement with the innovation.

Thank you for taking time to complete this task.
1. I am concerned about students' attitudes toward the innovation.  
2. I now know of some other approaches that might work better.  
3. I am more concerned about another innovation.  
4. I am concerned about not having enough time to organize myself each day.  
5. I would like to help other faculty in their use of the innovation.  
6. I have a very limited knowledge of the innovation.  
7. I would like to know the effect of reorganization on my professional status.  
8. I am concerned about conflict between my interests and my responsibilities.  
9. I am concerned about revising my use of the innovation.  
10. I would like to develop working relationships with both our faculty and outside faculty using this innovation.  
11. I am concerned about how the innovation affects students.  
12. I am not concerned about the innovation at this time.  
13. I would like to know who will make the decisions in the new system.  
14. I would like to discuss the possibility of using the innovation.  
15. I would like to know what resources are available if we decide to adopt the innovation.  
16. I am concerned about my inability to manage all that the innovation requires.  
17. I would like to know how my teaching or administration is supposed to change.  
18. I would like to familiarize other departments or persons with the progress of this new approach.  
19. I am concerned about evaluating my impact on students.  
20. I would like to revise the innovation's approach.  
21. I am preoccupied with things other than the innovation.  
22. I would like to modify our use of the innovation based on the experiences of our students.  
23. I spend little time thinking about the innovation.
<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Irrelevant</td>
<td>Not true of me now</td>
<td>Somewhat true of me now</td>
<td>Very true of me now</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

24. I would like to excite my students about their part in this approach. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
25. I am concerned about time spent working with nonacademic problems related to the innovation. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
26. I would like to know what the use of the innovation will require in the immediate future. |
27. I would like to coordinate my efforts with others to maximize the innovation’s effects. |
28. I would like to have more information on time and energy commitments required by the innovation. |
29. I would like to know what other faculty are doing in this area. |
30. Currently, other priorities prevent me from focusing my attention on the innovation. |
31. I would like to determine how to supplement, enhance, or replace the innovation. |
32. I would like to use feedback from students to change the program. |
33. I would like to know how my role will change when I am using the innovation. |
34. Coordination of tasks and people is taking too much of my time. |
35. I would like to know how the innovation is better than what we have now. |

1. How long have you been involved with the innovation, not counting this year?
   Never ______ 1 year ______ 2 years ______ 3 years ______ 4 years ______ 5 or more years ______

2. In your use of the innovation, do you consider yourself to be a:
   Non-user ______ Novice ______ Intermediate ______ Old hand ______ Past user ______

3. Have you received formal training regarding the innovation (workshops, courses)?
   Yes ______  No ______

4. Are you currently in the first or second year of use of some major innovation or program other than this one?
   Yes ______  No ______

If yes, please describe briefly:

Thank you for your help!
APPENDIX F

LEVEL OF USE BRANCHING CHART

Question 1:
Is the teacher using the Meaningful Distributed Instruction: Preview for Symbolic Procedure (MDI:PSP)?

If no, this response represents LoU 0, I, or II (continue to Question 2)

Question 2:
Has the teacher decided to use MDI:PSP and set a date to begin use?
If yes, LoU II
If no, LoU 0 or II (continue to Question 3)

Question 3:
Is the teacher currently looking for information about MDI:PSP?
If yes, LoU 1
If no, LoU 0

If yes, this response represents Level of Use (LoU) III, IVA, IVB, V, or VI (continue to Question 2)

Question 2:
What kinds of changes is the teacher making in their use of MDI:PSP?
If User-Oriented, LoU III
If nothing unusual, LoU IVA
If Impact-Oriented, LoU IVB, V, or VI (continue to Question 3)

Question 3:
Is the teacher coordinating use of MDI:PSP with other teachers, including another not in the original group of teachers?
If yes, LoU V or VI (continue to Question 4)
If no, LoU VB or VI (continue to Question 4)

Question 4:
Is the teacher planning to make major modification or replace MDI:PSP?
If no and question 3 was no, IVB
If yes and question 3 was yes or no, VI
If yes and question 3 was no, V

Levels of Use of the Innovation

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Nonuse: State in which the user has little or no knowledge of the innovation, has no involvement with the innovation, and is doing nothing toward becoming involved.</td>
</tr>
<tr>
<td>1</td>
<td>Orientation: State in which the user has acquired or is acquiring information about the innovation and/or has explored or is exploring its value orientation and its demands upon the user and the user system.</td>
</tr>
<tr>
<td>II</td>
<td>Preparation: State in which the user is preparing for first use of the innovation.</td>
</tr>
<tr>
<td>III</td>
<td>Mechanical Use: State in which the user focuses most effort on the short-term, day-to-day use of the innovation with little time for reflection. Changes in use are made more to meet user needs than client needs. The user is primarily engaged in a stepwise attempt to master the tasks required to use the innovation, often resulting in disjointed and superficial use.</td>
</tr>
<tr>
<td>IVA</td>
<td>Routine: Use of the innovation is stabilized. Few if any changes are being made in ongoing use. Little preparation or thought is being given to improving innovation use or its consequences.</td>
</tr>
<tr>
<td>IVB</td>
<td>Refinement: State in which the user varies the use of the innovation to increase the impact on clients within immediate sphere of influence. Variations are based on knowledge of both short- and long-term consequences.</td>
</tr>
<tr>
<td>V</td>
<td>Integration: State in which the user is combining own efforts to use the innovation with the related activities of colleagues to achieve a collective effect on clients within their common sphere of influence.</td>
</tr>
<tr>
<td>VI</td>
<td>Renewal: State in which the user reevaluates the quality of use of the innovation, seeks major modifications or alternatives to the present innovation to achieve increased impact on clients, examines new developments in the field, and explores new goals for self and the system.</td>
</tr>
</tbody>
</table>
1. Is the following problem an addition problem or a subtraction problem? Explain your solution.

   Jack has $6. He wants to buy a CD that costs $9. How many more dollars does he need?

   Circle your answer: addition subtraction

   Explanation:

2. A student counts back to solve the following problem. If the student gives the answer 7 pencils, what error do you think the student made? Explain your solution.

   Jill has 9 pencils. She gives 3 of them to Jack. How many pencils does she still have?

   Explanation:

3. Is the following problem a multiplication problem or a division problem? Explain your solution.

   Seventy-five students and teachers are going to take a field trip. Each bus can hold 25 people. How many buses do they need?

   Circle your answer: multiplication division

   Explanation:

4. Create a subtraction word problem where two sets of objects are joined and the total is 9.
5. Create an addition word problem where two sets are being compared, but you add to find the answer.

6. Create two division word problems.
   a. In the first, you need to find the number of groups.
   b. In the second, you need to find the number in each group.

7. Which number sentence cannot be used to represent the following problem? Circle the letter for the correct response.

   Larry picked 15 apples from one tree. Then he picked some more at another tree. Altogether he picked 42 apples. How many did he pick at the second tree?

   a. 42 - 15 = ______
   b. 15 + _____ = 42
   c. 42 - _____ = 15
   d. 15 + 42 = ______

8. Which of the following number sentences cannot easily be solved by using the knowledge 37 + 45 = 82? Circle the letter for the correct response.

   a. 39 + 45 = ?
   b. 82 - 36 = ?
   c. 37 + 44 = ?
   d. 46 + 82 = ?

9. Which of the following solution strategies does not efficiently makes use of doubles to solve the problem 9 + 7? Circle the letter for the correct response.

   a. Seven and 7 is 14, then 2 more is 16.
   b. Take 1 from the 9 and give it to the 7. Then double 8 to get 16.
   c. Ten and 7 is 17, so 9 and 7 is 1 less or 16.
   d. Nine and 9 is 18, so 9 and 7 is 2 less or 16.

10. Which number sentence cannot be used to represent the following problem? Circle the letter for the correct response.
There were 78 books. Curly put the same number of books on each of 4 shelves. How many were on each shelf?

a. $78 \div ____ = 4$
b. $78 - 4 = 74 - 4 = 70 - 4 = 66 - 4 = 62 - 4 = 58 - 4 = 54 - 4 = \ldots 4 - 4 = 0$
c. $4 \times ____ = 78$
d. $78 \div 4 = ____$

11. Which of the following number sentences cannot easily be solved by using the knowledge $5 \times 17 = 85$? Circle the letter for the correct response.

a. $18 \times 5 = ____$
b. $85 \div 6 = ____$
c. $90 \div 5 = ____$
d. $5 \times 16 = ____$

12. Which of the following solution strategies does not efficiently make use of the distributive property to solve the problem $64 \div 4$? Circle the letter for the correct response.

a. Sixty divided by 4 is 15 and 4 divided by 4 is 1, so the answer is $15 + 1$.
b. Thirty-two divided by 4 is 8 and another 32 divided by 4 is 8, so the answer is $8 + 8$.
c. Sixty-four is $4 \times 4 \times 4$. $4 \times (4 \times 4) = 4 \times 16$, so the answer is 16.
d. Forty divided by 4 is 10 and 24 divided by 4 is 6, so the answer is $10 + 6$.

13. Draw a diagram to show how 5 people might share 3 cookies. What fraction of a cookie does each person get? Explain your solution.

14. Draw a diagram to show how 3 people might share 7 cookies. How many cookies does each person get? Explain your solution.

15. Moe and his friends ate $\frac{2}{3}$ of a pizza. Later Moe ate another $\frac{1}{4}$ of the pizza. What fraction of the pizza was eaten? Draw a diagram to show this and explain your solution.
16. There was 3/4 of a pan of brownies left. Jo and her family ate 2/3 of what was left. What fraction of the pan did they eat? Draw a diagram to show 2/3 of 3/4 and explain your solution.

17. Jim and Bob each got cakes that were the same size. Jim cut his cake into 12 equal-sized pieces and ate 3 of them. Bob cut his cake into 4 equal-sized pieces and ate 1 of them. Who ate more cake? Draw a diagram to compare 3/12 and 1/4. Explain your solution.

18. It takes 1 3/4 cups of flour to make a recipe. How much flour will it take to make 2 1/2 batches of the recipe? Draw a diagram and explain your solution.

19. Mrs. Jones has 12 cups of juice. She is pouring 3/4 cup of juice in each small paper cup. How many cups can she fill? Draw a diagram to show 12 ÷ 3/4 and explain your solution.

Challenge problem

20. Create two division word problems where in each you start with 11 objects, divide it by 4.
   a. In the first, you end up with 4 groups of 2 with 3 extras.

   b. In the second, you end up with 2 groups of 4 with 3 extras.
APPENDIX H
SOCQ QUICK SCORING DEVICE

Stages of Concern Quick Scoring Device

<table>
<thead>
<tr>
<th>Stage</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Taken from “Measuring implementation in schools: The stages of concern questionnaire” by A. A. George, G. E. Hall and S. M. Stiegelbauer, 2006, p. 86. Copyright 2006 by SEDL.
APPENDIX I

SOC DESCRIPTION OF THE STAGES

The Stages of Concern About an Innovation

<table>
<thead>
<tr>
<th>Impact</th>
<th>Task</th>
<th>Self</th>
<th>Stage</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td></td>
<td></td>
<td>Refocusing</td>
<td>The individual focuses on exploring ways to reap more universal benefits from the innovation, including the possibility of making major changes to it or replacing it with a more powerful alternative.</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td>Collaboration</td>
<td>The individual focuses on coordinating and cooperating with others regarding use of the innovation.</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>Consequence</td>
<td>The individual focuses on the innovation's impact on students in his or her immediate sphere of influence. Considerations include the relevance of the innovation for students; the evaluation of student outcomes, including performance and competencies; and the changes needed to improve student outcomes.</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>Management</td>
<td>The individual focuses on the processes and tasks of using the innovation and the best use of information and resources. Issues related to efficiency, organizing, managing, and scheduling dominate.</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>Personal</td>
<td>The individual is uncertain about the demands of the innovation, his or her adequacy to meet those demands, and/or his or her role with the innovation. The individual is analyzing his or her relationship to the reward structure of the organization, determining his or her part in decision making, and considering potential conflicts with existing structures or personal commitment. Concerns also might involve the financial or status implications of the program for the individual and his or her colleagues.</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>Informational</td>
<td>The individual indicates a general awareness of the innovation and interest in learning more details about it. The individual does not seem to be worried about himself or herself in relation to the innovation. Any interest is in impersonal, substantive aspects of the innovation, such as its general characteristics, effects, and requirements for use.</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td>Unconcerned</td>
<td>The individual indicates little concern about or involvement with the innovation.</td>
</tr>
</tbody>
</table>

Note. Adapted from "Measuring implementation in schools: The stages of concern questionnaire" by A. A. George, G. E. Hall and S. M. Stiegelbauer, 2006, p. 8. Copyright 2006 by SEDL.
# APPENDIX J

## SOCQ INTERPRETATION CHART

### Stage 0: High and Low Scores

| High Stage 0 | Indicates a person who is not concerned about the innovation |
| Low Stage 0 – High Other Stages | Suggests intense involvement with the innovation |
| Low Stages 0-3 | Indicates an experienced user who is still actively concerned about the innovation |

Caution: If the Stage 0 percentile is particularly high relative to the other scores, the other stage scores may have little significance.

### Stages 1 and 2: High and Low Scores

| High Stage 1 | Indicates a person who wants more information about the innovation |
| Low Stage 1 | Indicates respondents who feel they already know enough about the innovation |
| High Stage 2 | Suggests that respondents have intense personal concerns about the innovation and its consequences for them. Although these concerns reflect uneasiness regarding the innovation, they do not necessarily indicate resistance |
| Low Stage 2 | Indicates that the person feels no personal threat in relation to the innovation |

High Stage 1 – Low Stage 2 Suggests that the person needs more information about the innovation. These respondents generally are open to and interested in the innovation.

Low Stage 1 – High Stage 2 Indicates a person who has self concerns. These individuals may be more negative toward an innovation and generally are not open to information about it.

Note: Stage 1 and Stage 2 scores usually are similar. If they are not, check them closely.

### Stages 3 and 4: High and Low Scores

| High Stage 3 | Indicates concerns about logistics, time, and management |
| Low Stage 3 | Suggests that the person has minimal to no concerns about managing use of the innovation |
| High Stage 4 | Indicates concerns about the consequences of the use of the innovation for students |
| Low Stage 4 | Suggests that the person has minimal concerns about the effects of the innovation on students |

### Stage 5: High Scores

A high 5 score is complex.

| High Stage 5 | Indicates concerns about working with others in relation to use of the innovation. A person scoring high on Stage 5 and low on all other stages is likely to be an administrator, coordinator, or team leader. Coordinating others is the priority |
| High Stage 5 with Some Combination of Stages 3, 4, and 6 also High | Suggests concerns about a collaborative effort in relation to the other stages with high scores |
| High Stage 5 – High Stage 1 | Suggests a desire to learn from what others know and are doing, rather than a concern for leading the collaboration |

### Stage 6: High Scores

| High Stage 6 – Low Stage 1 | Indicates a person who is not interested in learning more about the innovation. The person is likely to feel that he or she already knows all about the innovation and has plenty of ideas for improving the situation. |
| High Stage 6 – High Stage 3 – Low Stages 0-2 | Indicates a person who has become frustrated with not having Management concerns resolved and has developed strongly held ideas about how the situation should be changed. The high Stage 6 score indicates that the person has ideas about how to change the innovation or situation from his or her point of view |
| Stage 6 Tailoring-up for Nonusers | Suggests the person has strong ideas about how to do things differently. These ideas may be positive, but are more likely to be negative toward the innovation |

Note. Adapted from “Measuring implementation in schools: The stages of concern questionnaire” by A. A. George, G. E. Hall and S. M. Stiegelbauer, 2006, p. 53. Copyright 2006 by SEDL.
APPENDIX K

SUMMARY OF TOOLS

Professional Development Course
1. Format: 10 half-day instructional sessions and six online implementation sessions
2. Focus areas:
   • Mathematics content knowledge for teaching
   • Implementation of innovative pedagogical practice

Mathematics Content Knowledge for Teaching
Content:
- Mathematics operations (addition, subtraction, multiplication, and division) for rational numbers

Measurement tools:
- Pretest of Mathematics Content Knowledge for Teaching
- Posttest of Mathematics Content Knowledge for Teaching

Innovative Pedagogical Practice
Strategy:
- Use of Meaningful Distributed Instruction: Preview for Symbolic Procedures

Measurement tools:
- Stages of Concern Questionnaire to identify teachers' concerns and feelings about the innovation
- Analysis of teacher responses to implementation questions on the online discussion board to identify Level of Use of the innovation
- Classroom observations using the Innovation Configuration Map to identify the fidelity and quality of implementation