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The off-shell effects on the photodisintegration cross section of the deuteron are studied by means of a set of interactions which have identical on-shell, but different off-shell behaviour. All members of this set yield the same two-nucleon scattering phase shifts and deuteron binding energy, but wave functions which differ at short distances. The experimental photodisintegration results could therefore be used as a further check on the off-shell properties of phenomenologically determined interactions.

INDEX DESCRIPTORS: Off-shell effects, Photodisintegration, Deuteron.

1. *Introduction*

Fundamental to a theoretic description of nuclei is the nucleon-nucleon force. The knowledge of this force is based in part on the two-body scattering data and the two-body bound state. This information has been used to develop a number of phenomenological forces, all of which describe the two-body experimental data well, but which are nevertheless markedly distinct from one another. Apart from the fact that they have different functional form in coordinate space and that some are local and other nonlocal, these forces give often different predictions in systems involving more than two bodies. The nuclear force is not uniquely determined by two-body data.

In terms of the nucleon-nucleon T-matrix, the forces describing the same two-body data have identical on-energy-shell T-matrix elements, but may have different off-energy-shell T-matrix elements. Nuclear properties that are sensitive to the off-energy-shell behaviour will be affected by these differences.

Another uncertain element of the nuclear interaction is the possibility of three- and many-body forces; that is, the force between two nucleons may be altered by the presence of a third nucleon. It is usually assumed that the effects of many-body forces are negligible, as we have very little experimental evidence for their magnitude. Even if many-body forces are assumed to be negligible, it is difficult to study the off-shell effects of two-body forces in systems involving three or more nucleons, because of the complicatedness of the calculations. Often the off-shell effects in such systems are obscured by the approximations which are necessary to make the calculation possible.

It is our aim to study the magnitude of effects due to the off-shell part of the nuclear force in the photodisintegration of the deuteron. This reaction lends itself particularly to such a study because 1) there are no approximations due to a nuclear many-body calculation, 2) the many-body force, if it is not negligible, will not be operative in this system, 3) there is a third particle, namely the incident photon, to enable us to study the off-shell T-matrix elements, and 4) the electromagnetic interaction, unlike the nuclear, is well understood. Our approach will be to construct a set of interactions which are identical on-energy-shell, i.e. they all give the same two-body results, but which differ off-shell. We then study the variation in the photodisintegration cross section as a function of an off-shell parameter.

2. *Construction of Equivalent Interactions*

In constructing the set of on-shell equivalent interactions, we follow the method described by Coester et al. (1970). Suppose we have an interaction V which describes the two-nucleon system. The Hamiltonian is

$$H = H_o + V \tag{1}$$

where H_o is the free two-particle Hamiltonian. The corresponding Schrodinger equation is

$$H \mid \psi_k > = k^2 \mid \psi_k > \tag{2}$$

The observable properties of interest of a two-body system are the scattering phase shifts and the deuteron binding energy. These are related to the asymptotic form of the wave function in coordinate space. Thus if we make a unitary transformation on the wave function which leaves the asymptotic form unchanged, we will have constructed a new wave function with the same observable properties. Let Ω be the unitary transformation which transforms the wave function according to the relation,

$$\left|\tilde{\psi}_k\right> = \Omega \left|\psi_k\right> \tag{3}$$

The corresponding transformed Hamiltonian is

$$\tilde{H} = \Omega^{+} H \Omega \tag{4}$$

In order to preserve the asymptotic form of the wave function, Ω must satisfy the following restriction,

$$< \underset{\sim}{r} | \Omega | \underset{\sim}{r}^1 > \rightarrow \delta(\underset{\sim}{r} - \underset{\sim}{r}^1) \text{ as } r \rightarrow \infty \cdot \tag{5}$$

The interaction part of the transformed Hamiltonian is

$$\tilde{V} = \tilde{H} - H_o \cdot \tag{6}$$

The different potentials V and $\tilde{V}$ yield identical deuteron binding energy, and scattering phase shifts at all energies.

We will perform a transformation on the deuteron wave function, which we label, ψ_B, and which for simplicity we take to be a pure S-wave. The unitary transformation that generates the equivalent wave functions has the form

$$< \underset{\sim}{r} | \Omega | \underset{\sim}{r}^1 > = \delta(\underset{\sim}{r} - \underset{\sim}{r}^1) - \frac{1}{4\pi} g_o(r) g_o(r^1) \tag{7}$$

There are several restrictions on the functioning g_o (r). The unitarity of Ω results in the condition

$$\int_o^\infty g_o^2(r) r^2 dr = 2. \tag{8}$$

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The asymptotic condition (5) yields

$$g_o(r) \rightarrow 0 \quad \text{as} \quad r \rightarrow \infty, \tag{9}$$

and finally in order to have a wave function that is finite at the origin

$$g_o(r) = \text{constant as } r \rightarrow 0. \tag{10}$$

There are infinitely many functions $g_o(r)$ that satisfy equations (8), (9), and (10), and therefore we can obtain an infinite number of equivalent interactions.

If the original interaction, V, is local, i.e.

$$< \underset{\sim}{r} \; V \; \underset{\sim}{r}^1 > \; = \; V(r)\, \delta(\underset{\sim}{r} - \underset{\sim}{r}^1) \tag{11}$$

then the equivalent potentials are nonlocal. In fact, the potential is the sum of the local potential and three additional separable terms. The extent or range of the nonlocality can be controlled by a proper choice of the function $g_o(r)$. Since the range of $g_o(r)$, is the range of the difference between ψ_B and $\bar{\psi}_B$, one can make this range sufficiently small so that the wave functions differ only at very short distances for which the actual shape of the function is not known.

In this analysis we have generated a class of equivalent potentials, which have identical scattering phase shifts and bound state energy. Van Dijk and Razavy (1973) have shown that additional properties can be held invariant by placing further restrictions on the function $g_o(r)$. For instance, it is possible to construct equivalent interactions in which the root-mean-square radius is also the same for all interactions.

3. *Photodisintegration Cross Section for Nonlocal Interactions*

The photodisintegration cross section is calculated using time dependent perturbation theory. The coupling of the proton-neutron system with the electromagnetic field takes place through the action of the electric and magnetic fields on the electric charge of the proton. The minimal coupling of the proton with the electromagnetic field is usually achieved by the replacement of the proton momentum operator,

$$\underset{\sim}{P}_p \rightarrow \underset{\sim}{P}_p - \frac{e}{c}\, \underset{\sim}{A}(\underset{\sim}{r}_p, t) \tag{12}$$

where A is the magnetic vector potential. This prescription is difficult to follow when nonlocal potentials are involved. For it is well known that a nonlocal potential can be expanded as a power series in the momentum.

The minimal interaction of the electromagnetic field can be introduced in an equivalent manner by means of the unitary transformation (see Yamaguchi (1954)),

$$e^{iF(\underset{\sim}{x})} \tag{13}$$

where

$$F(\underset{\sim}{x}) = \frac{e}{\hbar c} \int^{\underset{\sim}{x}} \underset{\sim}{A}(\underset{\sim}{x}^1) \cdot d\underset{\sim}{x}^1 \tag{14}$$

The transformed Hamiltonian is

$$\bar{H} = e^{iF} H e^{-iF} = H + i[F, H] + \ldots \tag{15}$$

$\bar{H}$ is the Hamiltonian which has the proton coupled to the electromagnetic field, whereas H is the Hamiltonian without the electromagnetic field. The neglected terms involve e/ħc of degree two or higher. The interaction part of the Hamiltonian is

$$H^1 = i[F, H] \tag{16}$$

The differential cross section of the photodisintegration of thr deuteron is given as

$$\frac{d\sigma}{d\Omega} = \frac{2\pi\omega}{S} \, |<f|H^1|i>|^2 \, \rho(E_f) \tag{17}$$

where $\hbar\Omega$ is the energy of the incident photons, S the incident energy flux; |i> and |f> are the initial and final two-nucleon states, and $\rho(E_f)$ is the density of final states. The differential cross section for unpolarized incident radiation, assuming no final state interaction, is

$$\frac{d\sigma}{d\Omega} = \frac{\pi^2}{4} \frac{e^2}{c\hbar} \, k\, (\propto^2 + k^2 + \tfrac{1}{4} K^2) \sin^2\theta \left| \int_o^1 ds \left(\frac{d\psi_B}{da} \right) \right|^2 \tag{18}$$

where k^2 is the c.m. energy in the final state, a = $(k^2 + skK\cos\theta + \tfrac{1}{4}S^2K^2)^{1/2}$ $\propto^2$ is the deuteron binding energy, and θ is the angle between the incident photon momentum and the outgoing proton momentum in the c.m. system. When we limit ourselves to electric dipole transitions only, the expression for the total cross section becomes remarkably simple,

$$\sigma = \frac{2\pi^3}{3} \frac{e^2 k}{\hbar c} (k^2 + \propto^2) \left| \frac{d\psi_B}{dk} \right|^2 \tag{19}$$

4. *Results*

For a model wave function of the deuteron we choose the function of Hulthén (1957),

$$\psi_B = N \frac{e^{-.23r} - e^{-.93r}}{r} \tag{20}$$

To generate the set of equivalent interactions we take

$$g_o(r) = (2\beta)^{3/2} e^{-\beta r} \tag{21}$$

This function determines the deformation of the Hulthén wave function, and also the range of the nonlocality of the equivalent potential. In fact, β is the parameter that indicates the range of the transformation and nonlocality. The total cross section, when we assume no final state interaction and electric dipole transitions only, equation (19), shows substantial variation as a function of the parameter β. In Fig. 1, when the incident photon energy is about 10 MeV, the cross section can take on values from zero to the value obtained using the Hulthén interaction. At larger energies the variation becomes even more pronounced, Fig. 2. At about 40 MeV photon energy the cross section varies between zero and four times the Hulthén cross section. In both cases the range of the nonlocality of the equivalent interactions is less than the range of the Hulthén potential.

We also calculated the angular dependence of the differential cross section, again assuming no final state interaction, equation (18). In this equation all electric multipole transitions are included. Figures 3 and 4 illustrate that differential cross section changes by the transformation; at higher energies the peak of the curve can be shifted either to the larger or smaller angle region by choosing β appropriately.

5. *Conclusion*

The variation in the cross section manifested by the set of equivalent potentials is much larger than the experimental errors in the measure-

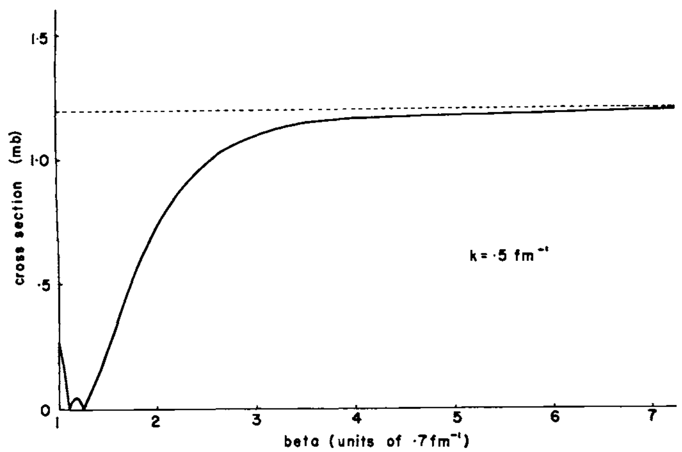

Fig. 1 Variation in the total photodisintegration cross section as a function of β . The dashed line corresponds to the cross section of the Hulthén wave function (equation 20).

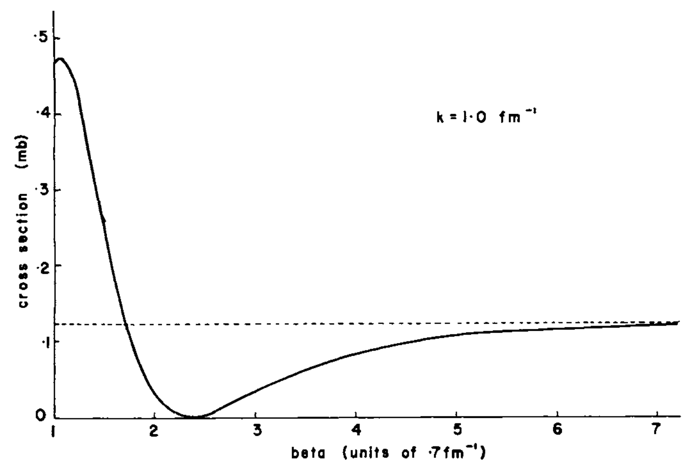

Fig. 2 See caption of Fig. 1.

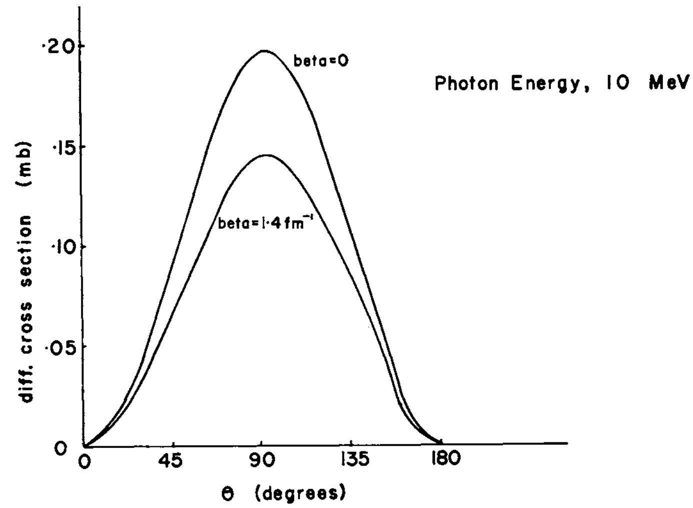

Fig. 3 The differential cross section for two values of β . β = 0 corresponds to the Hulthén wave function case.

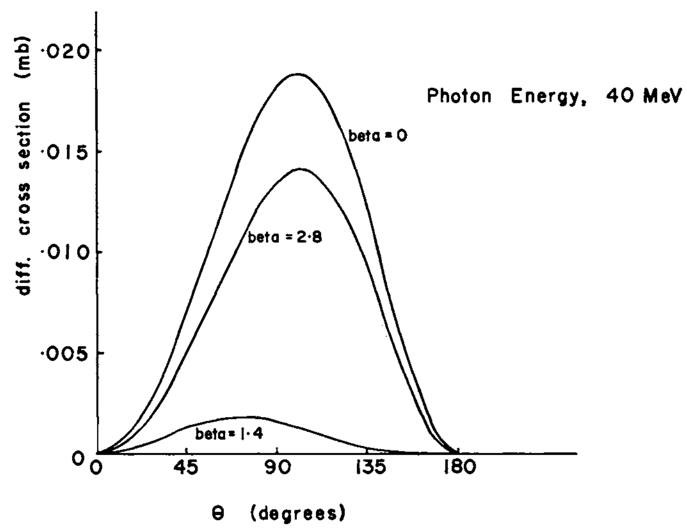

Fig. 4 The differential cross section for three values of β in units of fm^{-1}.

ments made on them, as shown by Skopik at al. (1974). Although this calculation does not purport to use realistic forces, it is evident that differing off-shell behavior can have unrealistic effects on the results. The experimental photodisintegration results therefore can be used as a check on proposed realistic potentials as to the correctness of the off-shell behavior.

Similar sets of equivalent potentials have been used to show the sensitivity of the off-shell behavior of the nuclear force on the nuclear matter properties by Coester et al. 1970), on the proton-proton effective range parameters by Sauer (1974), and on nucleon-nucleon bremsstrahlung cross sections by Nyman (1974). All these phenomena, as well as the photodisintegration of the deuteron provide us with means of delimiting the off-shell properties of the realistic nucleon-nucleon force.

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