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An analysis of equity-linked insurance pricing

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AN ANALYSIS OF
EQUITY-LINKED INSURANCE PRICING

A Thesis Submitted

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AN ANALYSIS OF EQUITY-LINKED INSURANCE PRICING

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Table of Contents

Introduction

Life insurance is seen as a necessity by most people, but how well do we actually understand the mechanics behind the products we purchase? Life insurance is a major and long-term investment for most, so a thorough understanding of the varying types of products that insurance companies offer is vital to making an informed purchasing decision. This has become increasingly difficult over recent years as companies are beginning to offer more and more new and innovative products. While exciting for those working in the insurance industry, such an abundance of product options may be overwhelming for potential life insurance customers.

The research conducted in this paper centers around equity-linked life insurance products and their different pricing methods. The most basic form of equity-linked insurance exists in the form of equity-linked certificates of deposit. Equity-linked products have been increasing in popularity over the past few decades as they provide opportunities for consumers to profit greatly depending on the status of the stock market. The profit of these products directly correlates to the performance of a designated index such as the S&P 500, Dow Jones Industrial Average, or the NASDAQ Composite Index. When the index performs well, the profit of the insurance product increases, and if the index does poorly, the profit will decrease. With continuous and rising competition among insurance providers, offering equity-linked products is one way they can differentiate themselves from their competitors. Equity-linked insurance is also an attractive option for carriers to offer as they have the potential to profit if the markets are in decline. Since these products are relatively new and the profit potential is so volatile, insurance companies must consider how to best price these products in order to best benefit both the consumer and the carrier.

Problem

Since equity-linked life insurance products depend on the performance of a specified stock market index which is difficult to predict, pricing them can be difficult for insurance companies. Different carriers may use different methods of pricing their equity-linked products and may be missing out on potential profits they could earn if they were to use another method. Additionally, consumers may not be purchasing the most cost-effective equity-linked products. Equity-linked life insurance products are relatively new compared to other more basic life insurance products. Therefore, not as much research exists on the effectiveness of the pricing models used for these products.

Purpose

This comprehensive study of equity-linked insurance options will explore the pricing of certificates of deposit and life insurance options using a present value method. With this study, I will be able to construct and price various equity-linked insurance products, with a focus on life insurance, that insurance companies could then sell to prospective customers. I will use concepts and formulas based in actuarial math, probability theory, and financial engineering in order to construct, price, and analyze new equity-linked insurance products. The fundamental methodology I will use involves applying pricing theory based on the expected value of the insurance payoff present value. To calculate this, I will need to make some assumptions with regard to factors such as interest rate, current stock price, and volatility. I will then draw conclusions based on the new products and prices I create.

There is a need for this type of study within the actuarial community as little research exists on the impacts of equity-linked insurance pricing. Equity-linked insurance products do not always offer the best returns, so I want to find and model a pricing method that still makes these products appealing to potential investors. While the methods and pricing theories I will be using are not new, the products I create using them will be innovative and original. My potential product could benefit both insurance carriers and consumers.

Research Question to Be Answered

What are reasonable prices for equity-linked life insurance products and how should they be priced?

Literature Review

Background

Equity-linked insurance products are relatively new compared to other products and are gaining popularity among investors. According to Edwards and Swidler (2005), equity-linked certificates of deposit were first offered in 1987, but only recently became more widespread. The first equity-indexed annuities emerged later in 1995 (Tiong, 2000). These types of products differ from typical insurance products because their interest rates depend on the market indices. They can be linked to a variety of indices including the S&P 500, the Dow Jones Industrial Average, or the NASDAQ composite index (Edwards & Swidler, 2005). As equity-linked products gain popularity and become more complex, insurance retailers must find new ways to competitively price them. My research will focus on equity-linked life insurance, but in order to fully understand equity-linked insurance as a whole, I will investigate a few of the types of products currently offered on the market.

Equity-Linked Certificates of Deposit

Certificates of deposit (CDs) are securities sold by banks that behave similarly to a savings account in that a deposit is made, and interest accrues at a fixed rate until the CD reaches maturity and is withdrawn. Equity-linked certificates of deposit (ELCDs) are similar except for the fact that their interest rates depend on the market indices. The payoff of ELCDs is equal to the principal deposit plus additional interest based on the market index at the time of maturity. If the market performs poorly, however, investors are guaranteed their principal return on ELCDs, meaning there is no money value risk to investors. According to Edleson and Cohn (1993), however, there is a trade-off between risk and return; by purchasing ELCDs, investors sacrifice the fixed interest they could earn on a regular CD but could potentially earn more during market upswings.

In their study, “Do Equity-Linked Certificates of Deposit Have Equity-Like Returns?,” Edwards and Swidler (2005) examined the return distribution of ELCDs. They found that ELCDs offer returns that are lower than the market returns and behave similarly to 5-year Treasury notes. Edwards and Swidler (2005) also examined the returns of synthetic ELCDs, made up from a portfolio of a zero-coupon bond and a call option. They found that the returns on synthetic ELCDs are higher than those of the regular ELCDs but still slightly lower than the market returns (Edwards & Swidler, 2005). In my study I will expand upon Edwards’ and Swidler’s work by investigating the initial pricing methods of ELCDs rather than their returns. I also plan on looking at both regular ELCDs and synthetic ELCDs.

Equity-Indexed Annuities

In order to better understand equity-linked insurance as a whole, I will also examine equity-indexed annuities (EIAs), which are deferred annuities with returns linked to the stock markets. Similar to certificates of deposit, EIAs exist with no money value risk to investors. According to Tiong (2000), EIAs have four main advantages to customers: minimum guaranteed interest rate provisions, locked-in interest rates to prevent the account value from decreasing due to market downturns, the opportunity to benefit from market growth, and, since EIAs are tax-deferred, investors do not pay taxes until they withdrawal. These characteristics make EIAs attractive to potential investors despite the volatility associated with the stock markets.

In her study, “Valuing Equity-Indexed Annuities,” Tiong (2000) analyzed the pricing methods of three different EIA products – point-to-point, cliquet, and lookback. Using Esscher transforms and other financial engineering methods, she was able to present a pricing formula for each of these products. This study will be valuable to my own analysis because I can reference some of her formulas and techniques and then use them to form my own pricing formulas for other equity-linked insurance products.

Equity-Linked Life Insurance

Lastly, I will analyze the pricing of equity-linked life insurance products. A large variety of equity-linked life insurance products exist and are becoming more popular among investors, so I will look at both term and universal products. True to their name, term life insurance policies provide coverage for a designated period of time, and universal life insurance policies provide coverage until the policyholder dies. Equity-linked life insurance products are a form of variable annuities, and similarly to other equity-linked products, their payoff depends on the market values at the time of expiration.

Gao, He, and Zhang (2011) expressed the need for research on equity-linked life insurance pricing, saying “many attempts have been made to provide consistent valuing approaches for equity-linked life insurance using financial and economical methods” (p. 147). They later went on to explain that some of these attempts are questionable in the assumptions and methods used (Gao et al., 2011). Equity-linked life insurance products are difficult to price since they depend on many variables such as future stock prices, mortality rates, surrender charges, and policy benefits.

In the article “Risk Management of Policyholder Behavior in Equity-Linked Life Insurance,” MacKay, Augustyniak, Bernard, and Hardy (2015) priced variable annuities based on predetermined assumptions. They considered various policy features and benefits, or riders, when pricing variable annuities. These include guaranteed benefit riders, surrender charges, mortality risk, and fees (MacKay et al., 2015). I plan on adopting some of the methods used in this article for my own analysis of equity-linked life insurance.

Stochastic Modeling

In "Quantile Hedging for Equity-Linked Life Insurance Contracts in a Stochastic Interest Rate Economy," the researchers took a different approach to equity-linked life insurance policies (Gao et al., 2011). They analyzed the pricing of equity-linked life insurance using both quantile hedging and stochastic interest rates (Goa et al., 2011). In developing the payoff amount of the products within my calculations, I will be using a different stochastic method. It is still beneficial to see how stochastic methods can be used in other ways and alongside other methods. I also may adapt the quantile hedging approach used in this article to fit my own study.

Stochastic modeling is widely used in the insurance industry to model complex variable interactions that other techniques would be unable to predict. Predictive models typically require two types of inputs: deterministic and stochastic (Wylde, 2014). Deterministic inputs are either constant values or can be determined using a fixed, non-random formula, while stochastic inputs are more complex and are assigned statistical distributions (Wylde, 2014). The stochastic approach I will be using in my calculations is a simple version of the Geometric Brownian Motion (GBM) method. This is a continuous time stochastic process often used to model stock prices (Dunbar, 2016). According to Steven Dunbar, a professor of mathematics at the University of Nebraska, GBM is preferred by many economists over other methods due to its simplicity and its characteristic of always being positive (2016). The GBM technique will help me to predict the payoffs of the equity-linked insurance products I create by allowing me to model the payoffs using different relations with the baseline index.

Methodology

Preliminary Data

In conducting my research of equity-linked insurance, I used multiple techniques rooted in actuarial math and probability theory. I first gathered mortality data from the Social Security Administration (SSA). This data, gathered in 2015 and shown in Table 1, breaks down the life expectancies of American males and females by current age from 0 to 119. It also includes probability of death within a year and number of survivors out of a base number of 100,000. The variable primarily used in my calculations was the probability of death. Using Microsoft Excel, I inputted the data from the SSA website. I then modified the mortality table by calculating the probability of dying at each specific age. For example, the probability of a 65-year-old male dying at age 78 is approximately 3.38% and the probability of a 25-year-old female dying at age 56 is approximately 0.8%. I calculated these probabilities by multiplying the probability of death for a certain age by the probability that the person does not die for each age prior to that. For instance, if I am calculating the probability that a 30-year-old dies at age 45, I would multiply the probability of death for a 45-year-old from the mortality table by the probability of surviving the previous 15 years, or one minus the probability of death for each of those years. Rather than calculating each of these values individually, I utilized the "VLOOKUP" function in Excel to cross reference a table of the survival probabilities I created and the mortality table with the current age and age of death. I then copied this formula across all 119 years for both male and females to obtain tables that show the probability of a person dying at a specific age. This is necessary in computing the price of equity-linked life insurance.

Black-Scholes Model

After creating these tables, I then used the Black-Scholes formula to predict the price of a call option. The Black-Scholes formula consists of three main steps. The first of these is calculating the d_1 value for each age using the following formula, where S_0 is the current stock price, X is the strike price, r is the risk-free interest rate, σ is the stock volatility, and T is the time to expiration:

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + (r + \sigma^2)T}{\sigma\sqrt{T}}$$

I next calculated d_2 for each age with the following formula:

$$d_2 = d_1 - \sigma\sqrt{T}$$

Finally, I used these values to compute the call option price for each age by using the following formula, where N means finding the normal approximation of the value in parentheses:

$$C = S_0 N(d_1) - Xe^{-rT}N(d_2)$$

Assumptions

For the sake of these calculations, I had to make assumptions for some of the variables used in these formulas. For the value of S_0 I assumed a value of 2740.37, which was the value of the S&P 500 on November 1, 2018. I chose this assumption as it was consistent with the stock prices from the last few months and it was from the first day of the month, making it a reasonable estimate for the current stock price in my calculations. For the risk-free rate assumption, I looked at recent Treasury Bill rates and chose a comparable rate of 2.5%. In my initial calculations, I assumed a value of 1520 for the strike price, X, but this is a variable that can change depending on the specific call option purchased by the investor. In my later analysis, I changed the strike price assumption to determine the effects it would have on the overall price of the policy.

Lastly, to determine the volatility assumption, I initially analyzed recent points on the CBOE Volatility Index or VIX, which measures the stock market's expectation of implied volatility of the S&P 500. Volatility according to the VIX over the past month has been around 0.18, so I used this as my initial volatility assumption. Later on in my research, however, I added a more accurate parameter estimation into my file. I imported historic S&P 500 prices from Yahoo Finance and added them to an Excel spreadsheet. For my research I used one year, or 252 trading days, of data. I inputted the stock prices into the following formula:

$$\ln(\frac{S_t}{S_{t-1}})$$

This formula simply takes the natural log of each price divided by the price from the trading day before. I then found the standard deviation of these values to be approximately 0.009 by using the "STDEV.S" function in Excel. This function estimates the standard deviation of a specific sample. Finally, I had to annualize this value since I used a year of trading data by multiplying by the square root of 252 days. This calculation resulted in a final volatility parameter estimation of approximately 0.142, which I used in the remainder of my calculations as needed.

Price Calculations

After inputting the assumptions for current stock price, strike price, rate, and volatility into the formulas, I calculated the d_1 and d_2 values for each time of expiration, T. To find the expiration values, I took the age of death minus the current age. I then used the d_1 and d_2 values to compute the call price for each value of expiration, or T, using the Black-Scholes call option pricing formula above, giving me a new triangle shaped table of values. For all of these calculations, I used various Excel functions.

In order to calculate the life insurance price using for each age, I then had to multiply the calculated call price by each of the death probabilities. I did this for both the male and female probabilities and put them into new tables. The last step in calculating the prices was to sum up all of the values over each age of death. This gave me the total call-based equity-linked life insurance prices for both males and females from ages 0 to 118.

After computing the call-based prices, I then determined what the price of the policy would be if it were based on a put option. The fundamental formula I used to calculate the put price was the put-call parity formula, where P is the put price, C is the call price, Xe^{-rT} is the present value of the strike price, and S_0 is the current stock price:

$$P - C = Xe^{-rT} - S_0$$

Since I already calculated the call prices for each expiration, T, and I already had my assumptions for rate, strike price, and current stock price, I was then able to rearrange the put-call parity formula to solve for the put premium:

$$P = C + Xe^{-rT} - S_0$$

I used this formula to calculate the put price for each expiration. I calculated each value of expiration, T, by subtracting the current age from the age of death. This, again, gave me a new triangle shaped table of values.

I next used the same technique as with the call prices and multiplied each value by the probability of death for each age and made separate tables for male and female values. I again summed up these prices across each age of death in order to obtain the total put-based equity-linked life insurance prices for both males and females from ages 0 to 118. I added these values to the tables with the call prices. A combination of these tables is shown in Table 2.

In order to compare my calculated life insurance prices with a baseline price, I lastly computed the price of a one-dollar death benefit. To do this, I used the same assumption for risk-free rate as I did in the above calculations and again computed the time of expiration by subtracting current age from age of death. I then inputted these values into the following formula, where P is the price of a one-dollar death benefit:

$$P = e^{-rT}$$

This gave me a triangle table of values which I summed up across each age of death to determine the total prices of a one-dollar death benefit for each age from 0 to 119. This more basic pricing method serves as a control with which I can compare my computed equity-linked prices.

Results

From my computations explained in the previous section, I was able to compile my calculated equity-linked life insurance prices into two tables, separated by male and female policyholders. Table 2 is a combination of these tables and shows the equity-linked life insurance prices of both males and females, based on both call and put options. These prices reflect the assumptions for current stock price, strike price, risk-free interest rate, and volatility that I explained in the methodology section. However, for this analysis, I will vary each of these assumptions individually in order to determine the effect each one has on the overall prices of both call and put-based products.

Using my initial assumptions for stock price, strike price, risk-free rate, and volatility, the equity- linked life insurance prices based on calls were much larger than those using puts, as shown in Figure 1. The call-based prices follow a fairly uniform downward slope as age increases. While difficult to see in this figure since the values are so low, the put-based prices follow a trend in which they increase until around the age of 50 when they begin decreasing. These patterns apply for both males and females; however, the female prices are greater for both calls and puts, which is to be expected since women have greater life expectancies.

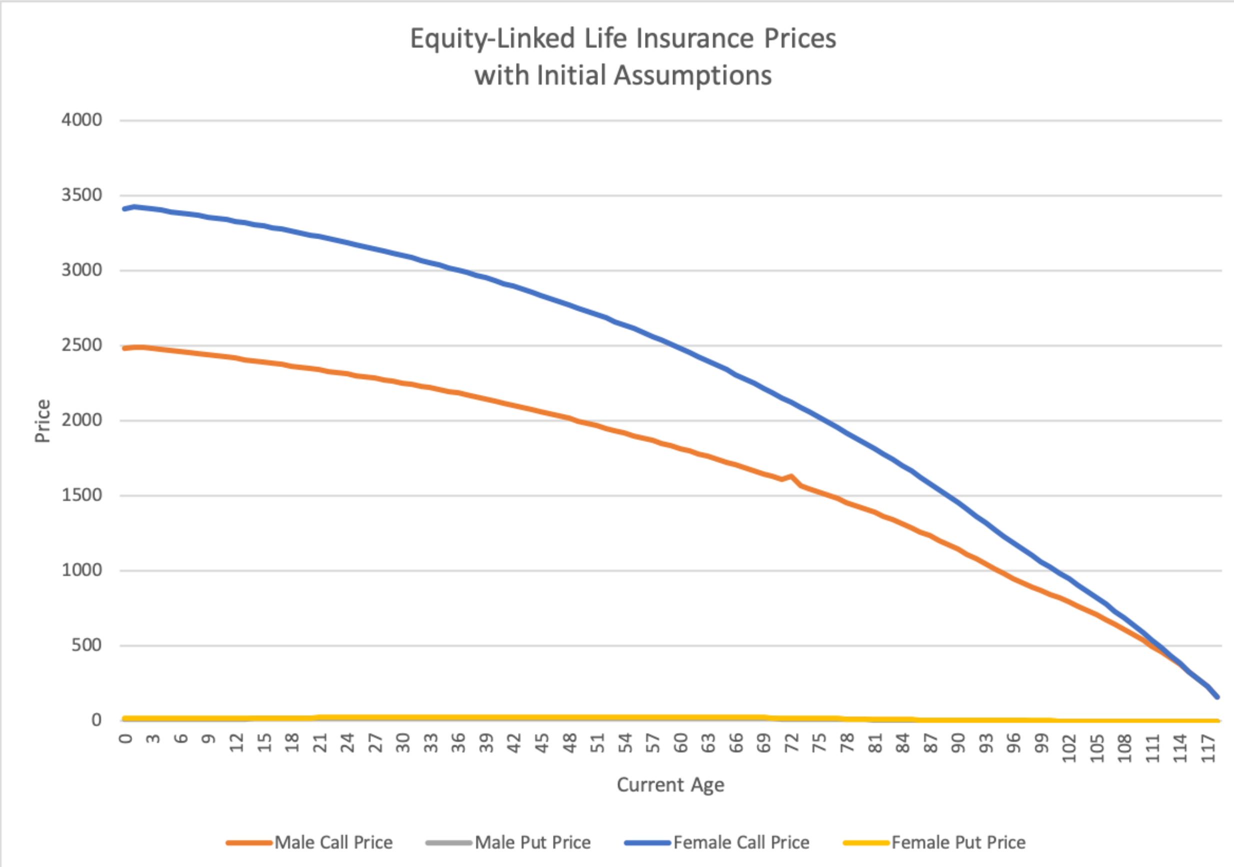

Figure 1

The put prices in Figure 1 are so low compared to the call prices because my initial strike price assumption is much lower than my assumed current stock price. This means that the call option is in-the-money and worth exercising. The purchaser would have the right to buy the stock at the strike price of $1520 and then sell the stock at the market price of $2740.37. On the other hand, the put option would be out-of-the-money in this example. The purchaser of the put would have the right to sell the stock at the strike price of $1520, which they would not exercise since the market price is much higher. Therefore, the call option price is much more expensive than the put option price in this case, because the call option has a much better potential for profit. If the strike price is increased to equal the current stock price assumption of

2740.37, as shown in Figure 2, both options become at-the-money. The call-based prices decrease for both males and females when compared to the initial assumptions, and the put-based prices increase. This implies that the call and put-based equity-linked products have inverse relationships with each other in terms of strike price.

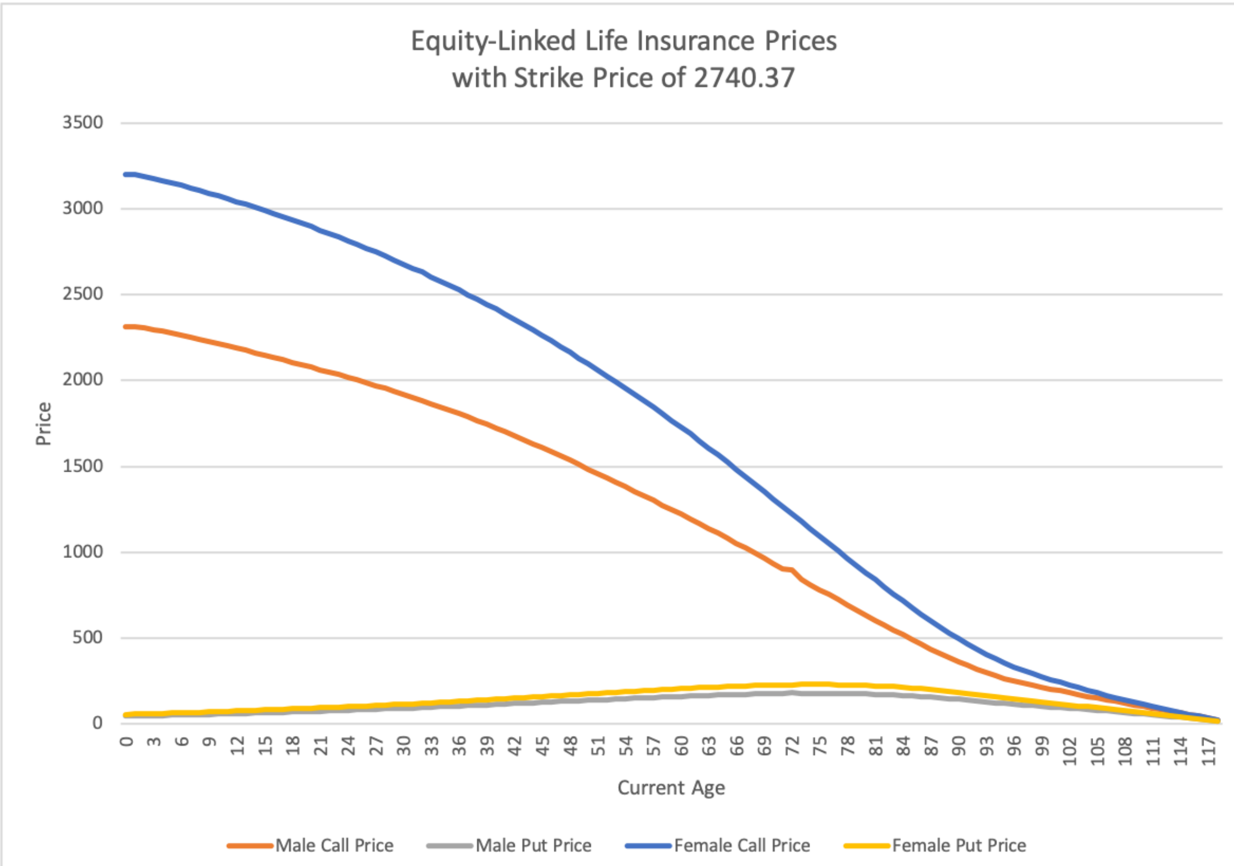

Figure 2

This is further proven in Figure 3 where the strike price is increased even more to 3500. In this case, the put option is considered in-the-money, while the call option is out-of-the money. This causes the put price to increase even more from the initial assumption and the call price to continue decreasing. Furthermore, as the current age increases, the put option price becomes greater than the call option price. This happens at age 77 for males and age 79 for females. From these graphs, it can be determined that as strike price increases, the put-based prices increase, and the call-based prices decrease. For the rest of my analysis, I will assume a strike price of 2740.37, equal to the current stock price, in order to maintain consistency.

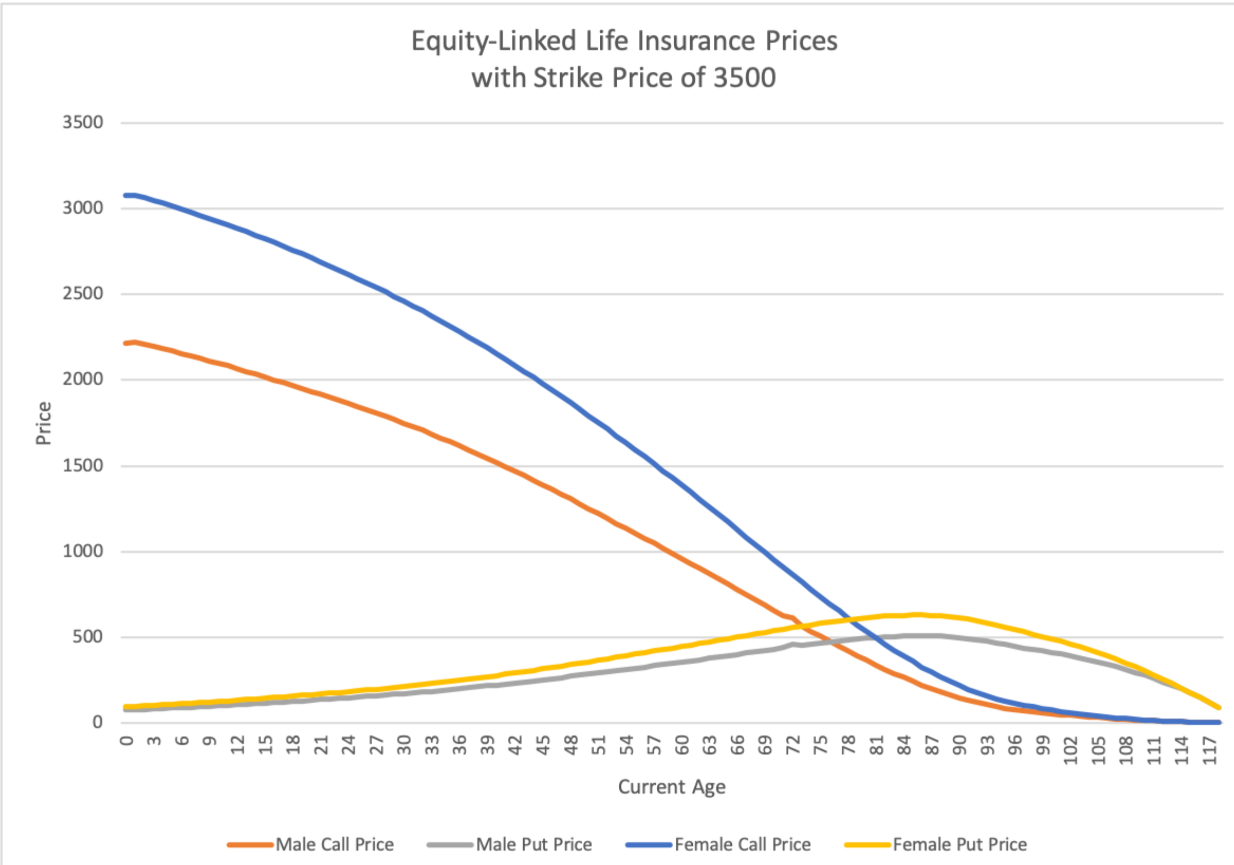

Figure 3

Next, I will increase the risk-free rate to determine the effect its changes have on the overall prices of the call and put-based products. Since I am now using 2740.37 as my new strike price assumption, Figure 2 will serve as a reference with which to compare the rest of my graphs. Figure 4 shows what the equity-linked life insurance prices would be if the risk-free rate were to decrease to 1% from the initial 2.5%. In this case, decreasing the rate causes the call prices to also decrease and the put prices to increase, implying the different options have an inverse relationship in regard to rate shifts. Figure 5 further proves this by showing that when the interest rate is increased to 5%, the call-based product prices increase, and the put-based product prices decrease.

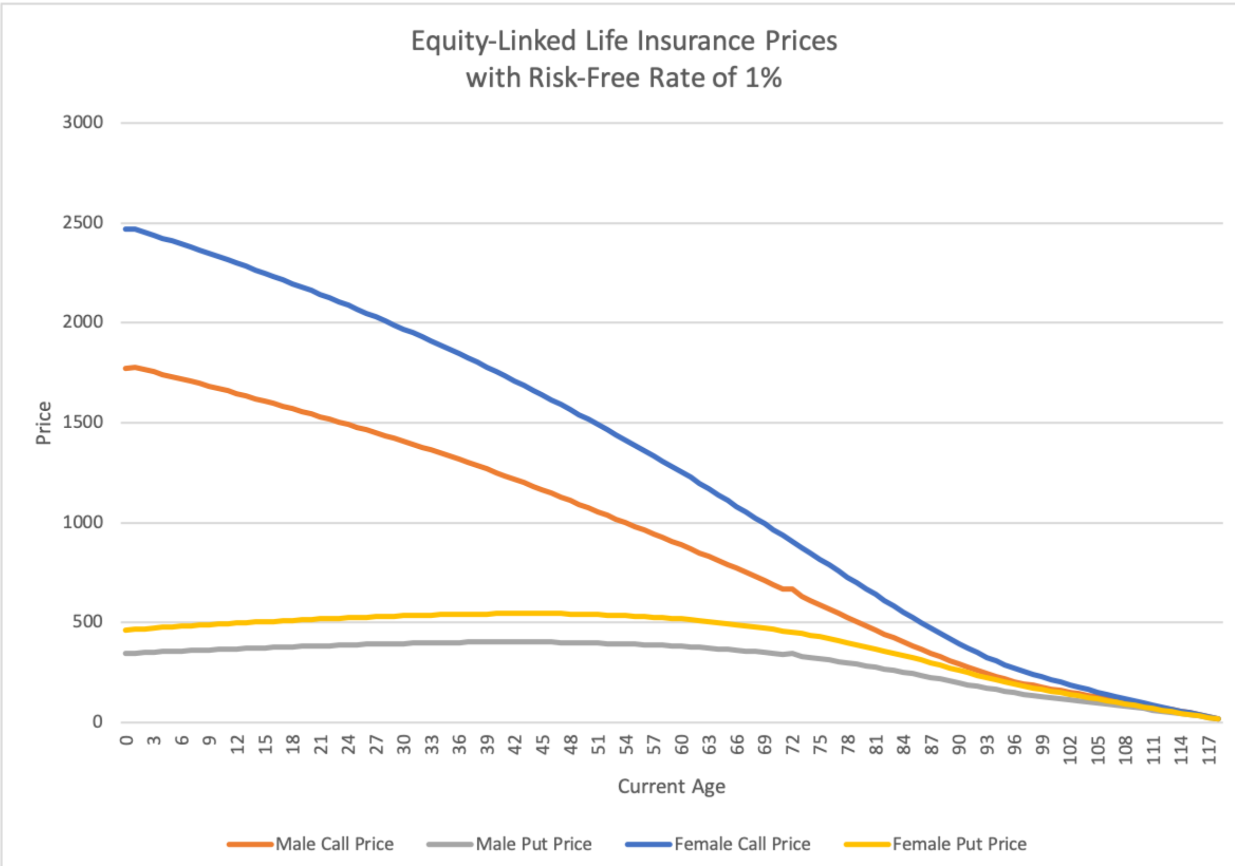

Figure 4

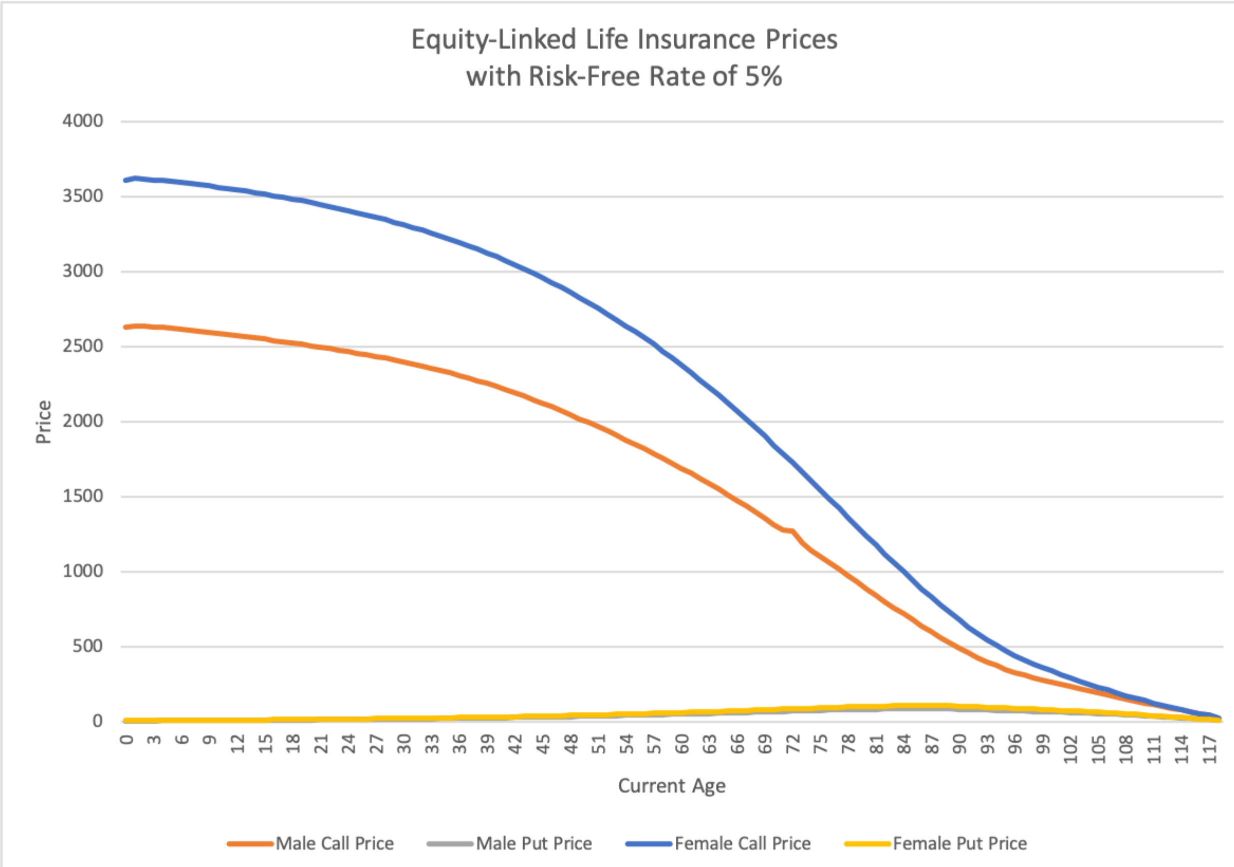

Figure 5

I also varied the current stock price to determine its effects on the overall call and put-based products. Again, Figure 2 can be a reference for these changes. Figure 6 shows the effect that decreasing the current strike price from 2740.37 to 2000 has on the overall prices for males and females. Decreasing the current stock price causes the call prices to decrease and the put prices to increase. Additionally, this shows that the put price actually becomes greater than the call price at age 72 for males and age 74 for females. On the contrary, increasing the stock price to 3500, as shown in Figure 7, causes the call-based prices to increase and the put-based prices to decrease. This again demonstrates an inverse relationship between the call and put-based equity-linked life insurance products in terms of stock price.

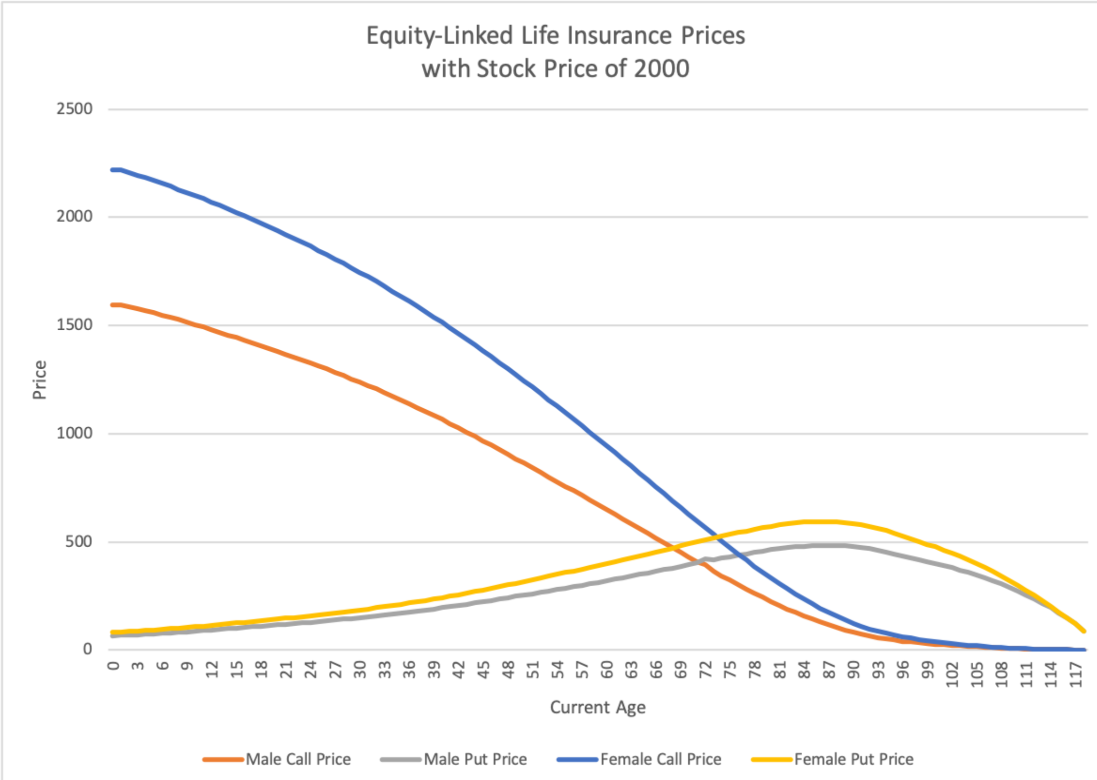

Figure 6

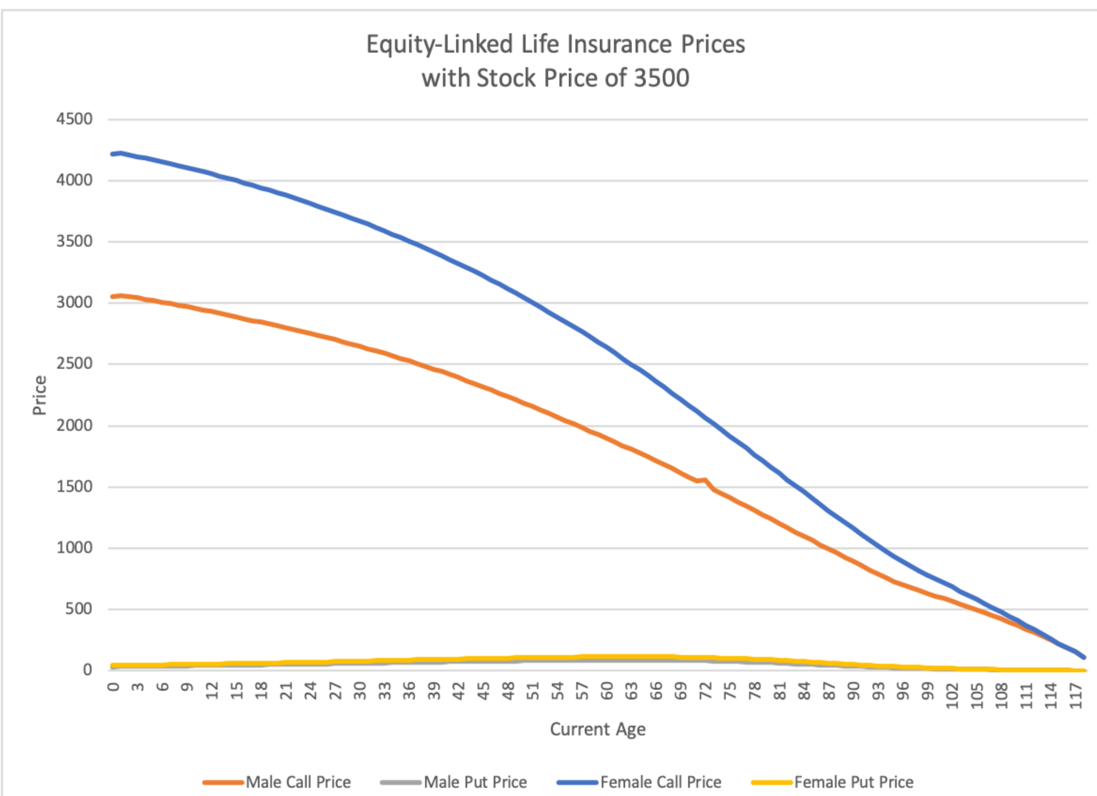

Figure 7

Lastly, I increased and decreased the volatility assumption from its original value of 0.14 to determine its effect on the overall prices of the equity-linked life insurance products. Again, I will use Figure 2 as a reference point as it uses my original assumption. Figure 8 shows the effect of decreasing the volatility to 0.10. This causes both the call and put-based prices to drop slightly, implying that the prices move in the same direction when volatility is changed. Figure 9 demonstrates this, as well, by illustrating that when the volatility assumption is increased to 0.20, both the call and put-based prices increase slightly. In these scenarios, the put prices never increase enough to be more than the call prices. All four prices simply converge as current age increases.

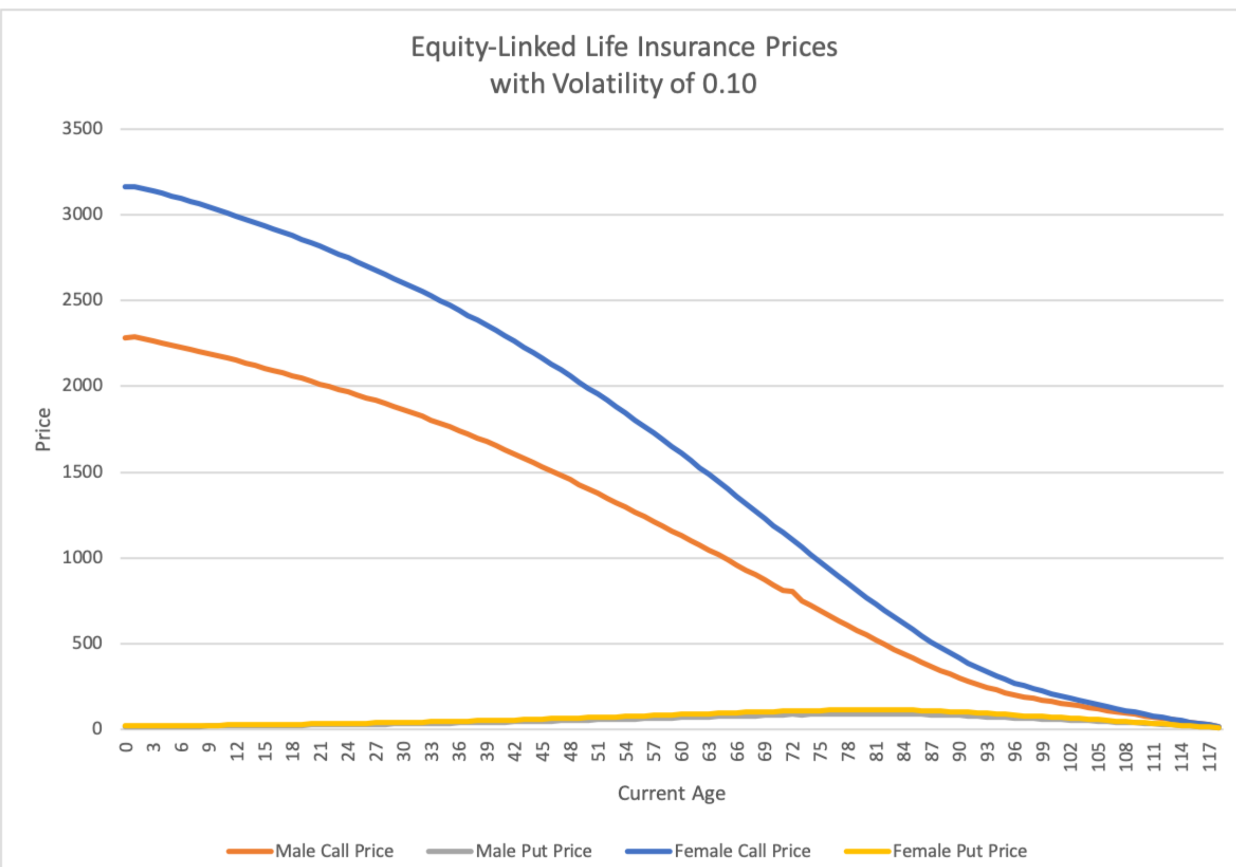

Figure 8

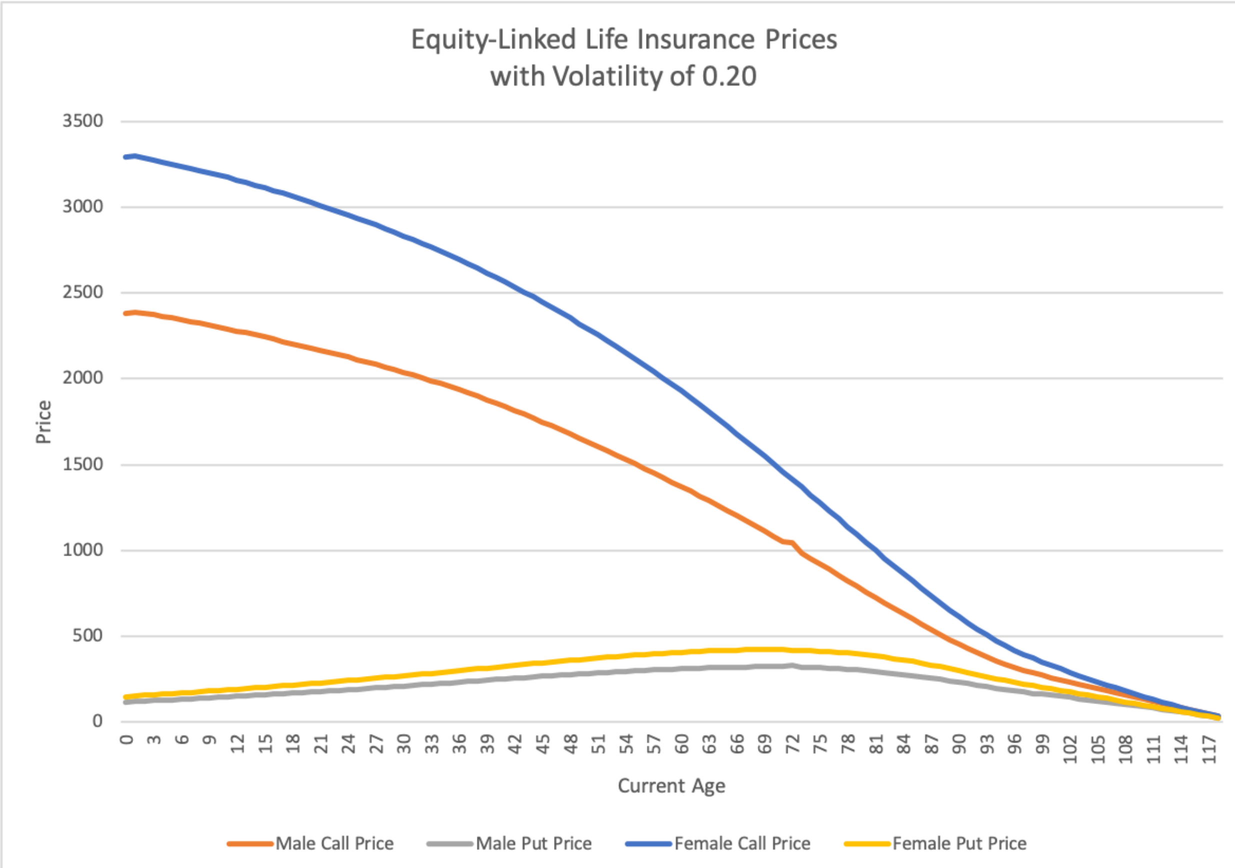
Equity-Linked Life Insurance Prices
with Volatility of 0.20
3500
3000
2500
2000
1500
1000
500
0
Price
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60 63 66 69 72 75 78 81 84 87 90 93 96 99 102 105 108 111 114 117
Current Age
Male Call Price
Male Put Price
Female Call Price
Female Put Price

Figure 9

Discussion and Conclusion

At the beginning of this paper, I stated my research question to be answered as, "what are reasonable prices for equity-linked life insurance products and how should they be priced?" I believe I was able to answer this question through my methodology and results. In response to the first part of this question, I found reasonable prices for a variety of equity-linked life insurance products. By changing my assumptions, I was able to see the impact each one had on the overall prices of the products. In doing this, I determined a range of different reasonable prices for both call-based and put-based equity-linked life insurance products and for both male and female policyholders. These prices also apply to all ages from 0 to 118, but in reality, prices would only be needed for older ages.

For the second part of my research question, I determined how equity-linked life insurance products should be priced. I found that the most reasonable method for my purposes is to use a combination of the present value formula, the Black-Scholes model, and the put-call parity equation. I used a present value method to find the death probabilities. I then used the Black-Scholes formulas to find the call prices for each age and multiplied those by the death probabilities. To find the put prices, I used prior assumptions with the put-call parity formula and then multiplied those by the death probabilities. Finally, I summed these prices across each age to determine the aggregate equity-linked life insurance prices. While this method does include a lot of different calculations and variables, it worked well for the purposes of my analysis.

In the summary of my results, I determined the effects of varying each of my four assumptions. At the beginning of my analysis, I established assumptions for stock price, strike price, risk-free rate, and volatility. For stock price, I used the S&P 500 price of 2740.37 from November 1, 2018. For strike price, I simply used an arbitrary value of 1520. For risk-free rate, I looked at recent Treasury Bill rates and decided to use 2.5%. Finally, I calculated the volatility parameter by calculating the standard deviation of historic S&P 500 prices.

By varying each of these assumptions separately, I was able to determine each of their effects on the overall equity-linked life insurance products, as illustrated in my results. I found that the call and put-based prices have inverse relationships in terms of strike price, risk-free rate, and current stock price. This means that as each of these assumptions are increased or decreased, the put and call-based prices move in opposite directions. As strike price increases, call prices decrease and put prices increase. On the other hand, when rate or current stock price are increased, the call prices also increase and the put prices decrease. Lastly, as volatility changes, the call and put-based prices move in the same direction. For instance, if volatility increases, each of the prices will also increase.

While I feel confident that I effectively answered my original research question, I do feel that some improvements could be made on this study in the future. For example, I would like to add some sensitivity measures to my analysis. Currently, I am only able to determine which direction the prices move when each parameter is changed, but sensitivity measures would help to understand the extent and impact of these price shifts. I would also like to add adjustments for additional features that these equity-linked life insurance products could have. These could be guaranteed benefit riders, surrender charges, or other fees. This would make the prices more reflective of real products on the market today and allow the prices to be more personalized to the policyholder’s needs. Adding calculations for fixed term products could also improve this study. I could determine prices for various fixed term lengths such as, 15, 20, or 30 years. It would be interesting to see the differences in prices among these fixed term prices and between fixed term and universal prices. I think it would also be beneficial to add some spreadsheet enhancements in order to improve user accessibility. This could be simply organizing the spreadsheets better or writing a macro to more easily input different parameters. There are always improvements that could be made, and since equity-linked life insurance is a growing field, I hope to see more studies examining the pricing of these products in the future.

Glossary

Term	Definition
Call Option	Agreement giving the call buyer the right to buy an asset at a set price during a specific time period. This does not obligate the call buyer to purchase the asset.
Certificate of Deposit (CD)	Security sold by banks that behaves similarly to a savings account in that interest accrues on the principal deposit at a fixed rate.
Equity-Indexed Annuity (EIA)	Deferred annuity with return linked to the stock market and no money value risk to investors
Equity-Linked Certificate of Deposit (ELCD)	Similar to a CD except interest rate depends on a stock market index with no money value risk to investors.
Equity-Linked Life Insurance	Form of variable annuity where payoff depends on the market value at the time of expiration; can be term or universal.
Expiration Date	The last day that an option can be exercised.
Geometric Brownian Motion (GBM)	Continuous time stochastic process widely used for modeling stock prices.
Put Option	Agreement giving the put buyer the right to sell an asset at a set price during a specific time period. This does not obligate the put buyer to sell the asset.
Risk-Free Rate of Return	The interest rate expected on a risk-free investment.

Stochastic Modeling	Mathematical tool that estimates probability distributions and allows for random variation of inputs.
Strike Price	The price at which a call or put is exercised.
Term Life Insurance Policy	Policy that provides life insurance coverage for a limited period of time.
Universal Life Insurance Policy	Policy that provides life insurance coverage until the policyholder dies.
Volatility (Stock)	A measure of the risk related to changes in stock price. Also called standard deviation.

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Table 1
Actuarial Life Table

Exact Age	Death Probability	Male Number of Lives	Life Expectancy	Death Probability	Female Number of Lives	Life Expectancy
0	0.006383	100,000	76.15	0.005374	100,000	80.97
1	0.000453	99,362	75.63	0.000353	99,463	80.41
2	0.000282	99,317	74.67	0.000231	99,427	79.44
3	0.00023	99,289	73.69	0.000165	99,405	78.45
4	0.000169	99,266	72.71	0.000129	99,388	77.47
5	0.000155	99,249	71.72	0.000116	99,375	76.48
6	0.000145	99,234	70.73	0.000107	99,364	75.48
7	0.000135	99,219	69.74	0.000101	99,353	74.49
8	0.00012	99,206	68.75	0.000096	99,343	73.5
9	0.000105	99,194	67.76	0.000092	99,334	72.51
10	0.000094	99,184	66.76	0.000091	99,325	71.51
11	0.000099	99,174	65.77	0.000096	99,316	70.52
12	0.000134	99,164	64.78	0.000111	99,306	69.53
13	0.000207	99,151	63.79	0.000138	99,295	68.53
14	0.000309	99,131	62.8	0.000174	99,281	67.54
15	0.000419	99,100	61.82	0.000214	99,264	66.56
16	0.00053	99,059	60.84	0.000254	99,243	65.57
17	0.000655	99,006	59.88	0.000294	99,217	64.59
18	0.000791	98,941	58.91	0.00033	99,188	63.61
19	0.000934	98,863	57.96	0.000364	99,156	62.63
20	0.001085	98,771	57.01	0.000399	99,120	61.65
21	0.001228	98,663	56.08	0.000436	99,080	60.67
22	0.001339	98,542	55.14	0.000469	99,037	59.7
23	0.001403	98,410	54.22	0.000497	98,990	58.73
24	0.001433	98,272	53.29	0.000522	98,941	57.76
25	0.001451	98,131	52.37	0.000546	98,890	56.79
26	0.001475	97,989	51.44	0.000572	98,836	55.82
27	0.001502	97,844	50.52	0.000604	98,779	54.85
28	0.001538	97,698	49.59	0.000644	98,719	53.88
29	0.001581	97,547	48.67	0.00069	98,656	52.92
30	0.001626	97,393	47.75	0.00074	98,588	51.95
31	0.001669	97,235	46.82	0.000792	98,515	50.99
32	0.001712	97,072	45.9	0.000841	98,437	50.03
33	0.001755	96,906	44.98	0.000886	98,354	49.07
34	0.0018	96,736	44.06	0.000929	98,267	48.11
35	0.001855	96,562	43.14	0.000977	98,175	47.16

36	0.00192	96,383	42.22	0.001034	98,080	46.2
37	0.001988	96,198	41.3	0.001098	97,978	45.25
38	0.00206	96,006	40.38	0.001171	97,870	44.3
39	0.002141	95,809	39.46	0.001253	97,756	43.35
40	0.00224	95,603	38.54	0.001347	97,633	42.41
41	0.002362	95,389	37.63	0.001452	97,502	41.46
42	0.002509	95,164	36.72	0.001571	97,360	40.52
43	0.002684	94,925	35.81	0.001706	97,207	39.59
44	0.00289	94,671	34.9	0.001857	97,041	38.65
45	0.003121	94,397	34	0.002022	96,861	37.72
46	0.003386	94,102	33.11	0.002204	96,665	36.8
47	0.003707	93,784	32.22	0.002411	96,452	35.88
48	0.004091	93,436	31.34	0.002648	96,220	34.96
49	0.004531	93,054	30.46	0.00291	95,965	34.06
50	0.005013	92,632	29.6	0.003193	95,686	33.15
51	0.005524	92,168	28.75	0.003491	95,380	32.26
52	0.006059	91,659	27.9	0.003801	95,047	31.37
53	0.006611	91,103	27.07	0.004119	94,686	30.49
54	0.007187	90,501	26.25	0.004449	94,296	29.61
55	0.0078	89,851	25.43	0.004813	93,877	28.74
56	0.008456	89,150	24.63	0.005201	93,425	27.88
57	0.009144	88,396	23.83	0.005583	92,939	27.02
58	0.009865	87,588	23.05	0.005952	92,420	26.17
59	0.010622	86,724	22.27	0.006325	91,870	25.32
60	0.011458	85,802	21.51	0.006749	91,289	24.48
61	0.01235	84,819	20.75	0.007238	90,673	23.64
62	0.013235	83,772	20	0.007776	90,017	22.81
63	0.014097	82,663	19.27	0.008368	89,317	21.99
64	0.014979	81,498	18.53	0.009032	88,569	21.17
65	0.015967	80,277	17.81	0.009794	87,769	20.36
66	0.017109	78,995	17.09	0.010673	86,910	19.55
67	0.018392	77,644	16.38	0.011676	85,982	18.76
68	0.019836	76,216	15.68	0.012815	84,978	17.98
69	0.021465	74,704	14.98	0.014105	83,889	17.2
70	0.023351	73,100	14.3	0.015616	82,706	16.44
71	0.025482	71,393	13.63	0.017318	81,414	15.69
72	0.027794	69,574	12.97	0.019118	80,004	14.96
73	0.030282	67,640	12.33	0.020996	78,475	14.24
74	0.033022	65,592	11.7	0.023033	76,827	13.54
75	0.036201	63,426	11.08	0.025413	75,058	12.85
76	0.039858	61,130	10.48	0.028197	73,150	12.17
77	0.043891	58,693	9.89	0.031313	71,088	11.51

78	0.048311	56,117	9.33	0.034782	68,862	10.86
79	0.053228	53,406	8.77	0.038689	66,466	10.24
80	0.058897	50,564	8.24	0.043258	63,895	9.63
81	0.065365	47,585	7.72	0.04849	61,131	9.04
82	0.072491	44,475	7.23	0.054223	58,167	8.48
83	0.080288	41,251	6.75	0.060446	55,013	7.93
84	0.088916	37,939	6.3	0.067338	51,688	7.41
85	0.098576	34,566	5.87	0.075133	48,207	6.91
86	0.109438	31,158	5.45	0.084033	44,585	6.43
87	0.121619	27,748	5.06	0.094177	40,838	5.98
88	0.135176	24,374	4.69	0.105633	36,992	5.54
89	0.150109	21,079	4.35	0.118407	33,085	5.14
90	0.166397	17,915	4.03	0.132476	29,167	4.76
91	0.183997	14,934	3.73	0.147801	25,303	4.41
92	0.202855	12,186	3.46	0.164331	21,563	4.09
93	0.222911	9,714	3.21	0.182012	18,020	3.8
94	0.244094	7,549	2.99	0.200783	14,740	3.54
95	0.265091	5,706	2.8	0.219758	11,781	3.3
96	0.285508	4,193	2.63	0.23863	9,192	3.09
97	0.304926	2,996	2.48	0.257065	6,998	2.9
98	0.322919	2,083	2.34	0.274706	5,199	2.73
99	0.339065	1,410	2.22	0.291189	3,771	2.57
100	0.356018	932	2.11	0.30866	2,673	2.42
101	0.373819	600	2	0.32718	1,848	2.27
102	0.39251	376	1.89	0.34681	1,243	2.14
103	0.412135	228	1.79	0.367619	812	2
104	0.432742	134	1.69	0.389676	514	1.88
105	0.454379	76	1.59	0.413057	313	1.76
106	0.477098	42	1.5	0.43784	184	1.64
107	0.500953	22	1.41	0.464111	103	1.53
108	0.526	11	1.33	0.491957	55	1.43
109	0.5523	5	1.25	0.521475	28	1.33
110	0.579915	2	1.17	0.552763	13	1.24
111	0.608911	1	1.1	0.585929	6	1.15
112	0.639357	0	1.03	0.621085	2	1.06
113	0.671325	0	0.96	0.65835	1	0.98
114	0.704891	0	0.89	0.697851	0	0.9
115	0.740135	0	0.83	0.739722	0	0.83
116	0.777142	0	0.77	0.777142	0	0.77
117	0.815999	0	0.71	0.815999	0	0.71
118	0.856799	0	0.66	0.856799	0	0.66
119	0.899639	0	0.61	0.899639	0	0.61

Table 2

Equity-Linked Life Insurance Prices for Males and Females

Current Age	Male Call Price	Male Put Price	Female Call Price	Female Put Price
0	2482.50884	12.4372986	3412.73669	16.0057247
1	2491.76682	12.7474101	3423.50132	16.3973287
2	2486.27227	12.979175	3416.49238	16.7050859
3	2480.26098	13.2103809	3408.93462	17.0136549
4	2474.03536	13.4422545	3401.02513	17.3237709
5	2467.5102	13.6743664	3392.80223	17.6356338
6	2460.79251	13.9070683	3384.3337	17.9493636
7	2453.88745	14.1401463	3375.62732	18.2647586
8	2446.79628	14.3733415	3366.69334	18.5816103
9	2439.50168	14.6063408	3357.52875	18.8996929
10	2431.9937	14.8389324	3348.12553	19.218811
11	2424.25749	15.071001	3338.46768	19.5388427
12	2416.29052	15.302653	3328.53246	19.8597612
13	2408.11477	15.5342553	3318.3014	20.1816537
14	2399.78279	15.7663522	3307.77457	20.5046808
15	2391.35096	15.9993943	3296.96823	20.8289682
16	2382.83341	16.2335196	3285.89057	21.1545452
17	2374.21138	16.4687454	3274.52051	21.4813435
18	2365.50089	16.7052842	3262.84102	21.8092679
19	2356.71554	16.9432691	3250.82717	22.1380881
20	2347.85802	17.1827124	3238.45956	22.4675461
21	2338.95272	17.4235807	3225.74823	22.7973747
22	2330.01524	17.6654944	3212.73518	23.1272549
23	2321.02253	17.9075695	3199.46004	23.4566577
24	2311.90005	18.1485568	3185.94161	23.7849587
25	2302.57658	18.3873503	3172.17695	24.1115188
26	2293.01	18.6231786	3158.14694	24.4356946
27	2283.20347	18.8555604	3143.84557	24.7568709
28	2273.14567	19.0839173	3129.27175	25.0744819
29	2262.84127	19.3077414	3114.43281	25.3879648
30	2252.2962	19.5264424	3099.33544	25.6966548
31	2241.50954	19.739283	3083.98445	25.9997795
32	2230.46858	19.9453925	3068.37543	26.296453
33	2219.16483	20.1438852	3052.48818	26.5855944

34	2207.58709	20.3338228	3036.29689	26.8660278
35	2195.71858	20.514255	3019.77194	27.1365614
36	2183.55999	20.6843509	3002.90495	27.396132
37	2171.12016	20.8432448	2985.7077	27.6437365
38	2158.39086	20.9898936	2968.18395	27.8782731
39	2145.35962	21.1232483	2950.33655	28.0986579
40	2132.01344	21.2423488	2932.1567	28.3037771
41	2118.35224	21.3464221	2913.63784	28.4925804
42	2104.38359	21.4348083	2894.76766	28.6639752
43	2090.11492	21.5069042	2875.53847	28.8169469
44	2075.55556	21.5621932	2855.94422	28.9505405
45	2060.72751	21.6002554	2835.98565	29.0638126
46	2045.62822	21.6205695	2815.64604	29.1557704
47	2030.246	21.6228294	2794.88988	29.2255095
48	2014.60469	21.6072176	2773.69549	29.2723542
49	1998.74924	21.5740602	2752.06323	29.2957547
50	1982.72544	21.5235016	2729.99575	29.2949759
51	1966.56481	21.4553345	2707.49943	29.2691101
52	1950.27734	21.369013	2684.56906	29.2170092
53	1933.8702	21.2638244	2661.19877	29.1373712
54	1917.32693	21.1388413	2637.36223	29.0287082
55	1900.62759	20.9932184	2613.02703	28.8895782
56	1883.76866	20.8263115	2588.21016	28.7191043
57	1866.76968	20.6375187	2562.91362	28.5160376
58	1849.62591	20.4259099	2537.05922	28.2781859
59	1832.32892	20.1904657	2510.54773	28.0030887
60	1814.82132	19.9300946	2483.27129	27.6886883
61	1797.15148	19.6444447	2455.26784	27.3341672
62	1779.39882	19.3325787	2426.68237	26.939008
63	1761.54445	18.9922662	2397.63288	26.5021943
64	1743.48674	18.6210266	2368.19036	26.0228387
65	1725.09518	18.2173204	2338.35775	25.5005549
66	1706.3309	17.7813217	2308.1352	24.9356721
67	1687.25022	17.3141183	2277.57623	24.3290329
68	1667.85571	16.8165598	2246.71706	23.6816539
69	1648.14858	16.289839	2215.5826	22.9948247
70	1628.07572	15.7354976	2184.13358	22.2701299
71	1607.71184	15.1561456	2152.49068	21.5106442
72	1632.30129	14.9678012	2120.77092	20.7186088

73	1566.48974	13.9304157	2088.88143	19.8943092
74	1545.54633	13.2865086	2056.58588	19.0379285
75	1524.1375	12.6246865	2023.58445	18.1515779
76	1502.29242	11.9496703	1989.90216	17.2408961
77	1480.20978	11.2662539	1955.78895	16.3122505
78	1457.89714	10.5776365	1921.30447	15.3703717
79	1435.23004	9.88698194	1886.39119	14.4201174
80	1411.92647	9.19836902	1850.82277	13.4673046
81	1388.01694	8.51784605	1814.72377	12.5202604
82	1363.75154	7.85094032	1778.35	11.5861959
83	1339.1309	7.20112977	1741.58878	10.6694686
84	1313.96005	6.57153534	1704.07261	9.77404141
85	1288.01339	5.96598791	1665.4794	8.90549402
86	1261.12779	5.38892021	1625.65696	8.07085285
87	1233.21878	4.84474645	1584.61521	7.27748801
88	1204.27703	4.33729052	1542.47806	6.53203136
89	1174.35492	3.86940704	1499.42576	5.83963212
90	1143.51651	3.44282253	1455.61611	5.20363515
91	1111.8416	3.05826268	1411.16704	4.62569218
92	1079.41623	2.71557105	1366.14805	4.10597258
93	1046.33419	2.41381477	1320.56878	3.64332668
94	1012.71448	2.15141311	1274.3834	3.23549339
95	980.220223	1.92647112	1228.98252	2.87949708
96	949.255093	1.73360315	1184.79032	2.56877208
97	920.236191	1.56721157	1142.262	2.29636265
98	893.55895	1.42158813	1101.87201	2.0552089
99	869.560369	1.29100448	1064.09519	1.83837485
100	844.72738	1.1692822	1025.66496	1.63874282
101	819.006541	1.05611393	986.524475	1.45559887
102	792.343191	0.95116986	946.617344	1.28818355
103	764.680127	0.8540984	905.881519	1.13569151
104	735.955151	0.76452872	864.254745	0.99728777
105	706.106155	0.68207674	821.670036	0.8721086
106	675.066017	0.60634429	778.058873	0.75927459
107	642.765192	0.53692444	733.347165	0.65789554
108	609.131665	0.47340388	687.462041	0.56708433
109	574.087178	0.41536451	640.323099	0.48596084
110	537.552222	0.36238889	591.853752	0.41366973
111	499.441967	0.31405957	541.976796	0.34938952

112	459.666722	0.26995978	490.635042	0.29235789
113	418.129139	0.22966749	437.834394	0.24190025
114	374.712598	0.19273445	383.814247	0.19748372
115	329.238509	0.15862527	329.761763	0.15887738
116	281.265005	0.12654145	281.265005	0.12654145
117	228.997832	0.09487042	228.997832	0.09487042
118	162.054178	0.05877513	162.054178	0.05877513