What do young children know about numerosity? : implications for teachers

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What do young children know about numerosity? : implications for teachers

Abstract
"Numerosity" is the ability to count a set of objects using the correct number names in the proper order. Fuson and Hall (1983) assert that the use of cardinal words is one measure of the development of number knowledge. Using words in a cardinal context indicates that children are beginning to understand the "manyiness" of objects. The use of cardinal words is one of the best indicators that a child is beginning to understand counting and the underlying principles involved.

Numerosity is a concept which children often, but not always, demonstrate with overt behaviors. Understanding cardinality and numerical order, develops with an understanding of numerosity. Assessing numerosity skills is one of the most reliable gauges of overall mathematical development in the preschool child.
What Do Young Children Know About Numerosity?:

Implications for Teachers

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I. Introduction

Ask a group of people in the United States how many read for pleasure and many hands will be raised. Ask a group of people in the United States how many people do math for pleasure and fewer hands (if any) will be raised. Unless you are at a convention of engineers or scientists, the number of Americans that frequent mathematics clubs are few. Why do we as a culture treat mathematics as the academic pariah? Resnick (1989) offered an explanation for the general population's aversion to mathematics. She proposed that educators do not build on the informal knowledge that children possess when entering school, making learning basic mathematics skills a chore of memorization instead of building new concepts on known concepts. Similar to "Garcia's Syndrome" with food, many children get a "bad taste in their mouths" with mathematics at an early age and will never try it again.

The development of mathematical skills has been the focus of psychologist and educators for many years. In 1929, a report was released comparing the mathematical standing of the United States to twelve other industrial countries (Reeve, 1929). The report indicated that the United States was ranked tenth of the twelve countries rated. This report stunned and alarmed educators at the time and there was immediate interest in reforming the mathematics curricula used by the nation's schools (Reeve, 1929). However, after many years and many waves of reform, the United States continues to lag behind other industrialized nations in mathematics performance, as
demonstrated by the IEA’s Third International Mathematics and Science Study (Beaton, 1996). These test scores and others which obtained similar results, have led to an "explosion of research about the conceptualization of mathematical knowledge" (Resnick, 1989, 1992).

In seeking answers to the question of why American students seem to dislike mathematics and perform so poorly in this domain, researchers have investigated children’s intuitive understanding of mathematical concepts and how it relates to the mathematical concepts taught in school. There have been two major strands of research that focus on this interplay: first, studies of the mathematical processes used by individuals in “out-of-school” contexts, such as candy selling by Brazilian street children (e.g., Saxe, 1991) and second, studies of the mathematical knowledge young children possess prior to entering school (e.g., Resnick, 1992). In general, the studies that focus on mathematical knowledge in out-of-school contexts have compared the informal procedures individuals invent with their competence at using formal algorithms. In the second line of research, investigators have characterized the nature of young children’s informal mathematical understanding and then have tried to assess its “fit” to the to-be-learned formal knowledge learned in school (e.g. Gallistel & Gelman, 1992; Starkey, Spelke, & Gelman, 1991; Gelman & Greeno, 1989). In some cases, these researchers argue, the fit is poor; for example Gelman (1991) suggests that fractions do not map well to children’s intuitive number line; consequently fractions are difficult for children to learn. In other cases, as
Resnick and Greeno (1992) suggest, the "fit" is potentially good; for example, an understanding of fractions can be built on children's intuitive understanding of the "part/whole schema." The problem here, is that teachers do not build upon the rich base of intuitive prior knowledge children bring to school (Resnick, 1989).

Research supports the idea that when teachers are more sensitive to aspects of children's informal mathematical knowledge, and attempt to build on it rather than ignore it, children develop more powerful mathematical skills. For example, Fenema, Franke, Carpenter, and Carey (1993) investigated how teachers utilize their knowledge of children's cognitive development. Fenema et al. (1993) found that children in classrooms with teachers who had knowledge of children's thinking and beliefs congruent with the cognitive perspective also learned more than children in other classrooms (Fenema, et al., 1993). Fenema et al. (1993) concluded that teacher knowledge of student mathematical thinking has a direct effect on instruction. Therefore, it is reasonable to hypothesize that when teachers understand the role informal learning and prior knowledge in students' present learning, their instructional practices may improve.

There are many domains in which children develop informal knowledge. Like Piaget (1952), Ginsburg and Allardice (1984) stated that children develop elementary notions of mathematics, physics, causality, and the like by observing and interacting with the world around them. Within the domain of mathematics, many children learn basic concepts about weights and measures, approximation,
addition, subtraction and counting prior to beginning their formal education (Resnick, 1989). This paper reviews the literature related to informal knowledge and the impact this knowledge has on knowledge of number and counting. Resnick (1989) proposed that knowledge of number and counting “is where the most research has been done, because numbers and counting form the core of the elementary and middle-school curriculum.” Resnick (1989) further explained that, “there seems to be a general consensus that number concepts form the basis upon which higher mathematical competencies can develop.”

**Definitions of Terms Used in this Paper**

“Numerosity” is the ability to count a set of objects using the correct number names in the proper order. Counting and the accompanying knowledge of numbers are also referred to as “numerosity” (Fuson and Hall, 1983). Fuson and Hall (1983) assert that the use of cardinal words is one measure of the development of number knowledge. Using words in a cardinal context indicates that children are beginning to understand the “manyiness” of objects. The use of cardinal words is one of the best indicators that a child is beginning to understand counting and the underlying principles involved (Fuson and Hall, 1983). Numerosity is a concept which children often, but not always, demonstrate with overt behaviors. Understanding cardinality and numerical order, develops with an understanding of numerosity (Fuson and Hall, 1983). Assessing numerosity skills is one of the most reliable gauges of overall
II. The Numerosity Capabilities of Five-Year Olds

An understanding of how mathematical knowledge develops is important for two reasons. First, kindergarten teachers need to know what is developmentally appropriate so they can develop and utilize supportive curricula to support the growth of mathematical knowledge. This idea is now commonly known among constructivist educators as “developmentally appropriate instruction” (Case, 1985, 1992). Second, parents, caregivers, and preschool teachers need to know how mathematical knowledge develops so that children can be taught appropriate foundational concepts prior to entering school. Foundational concepts include prior knowledge that kindergarten teachers expect children entering school to possess and upon which new knowledge will be built. One category of these foundational concepts in mathematics is numerosity skills. By having basic numerosity skills children can enter kindergarten ready to build upon their prior knowledge instead of “playing catch-up” during that first formative year of school (Case, 1985, 1992). In this chapter, a historical overview is given of research on children’s early understanding of numerosity.

Summary of Previous Research on Children’s Numerosity Capabilities

Piaget’s theory of number concept development. Since the turn of the century, an entire body of literature has been amassed which describes how mathematical concepts develop in young children. Past and present experts in the field of child development have addressed mathematical development as
part of the development of the whole child (Resnick, 1992). Piaget was one such expert; he was intensely interested in how children develop knowledge in all areas, including mathematics. Piaget focused, in the period of early childhood, on children’s ability to learn one-to-one correspondence and conservation of number. To demonstrate conservation of number, a child is shown two parallel arrangements of counted objects. The objects are counted and then one row of objects is moved, so that it covers a greater overall distance than the other row and the child is then asked to identify which row has the greatest number of objects.

Figure 1

Piaget proposed that a child who has mastered number conservation will say that regardless of the extra spaces, the quantities in the two rows are the same. The conceptual understanding underlying success on this task was considered by Piaget to be the foundation of the mathematical knowledge necessary for children to be successful in the early grades of school. Piaget found that children at the age of five were able to understand one-to-one correspondence, but that their sense of number conservation was still developing. Piaget’s research and theories provided the basis for many major
studies concerned with the development of mathematical knowledge in this century (Kitchener, 1986). Specifically, his research and theory of mathematical development has been validated by the quality and quantity of researchers that followed in his footsteps (Ginsburg & Opper, 1988).

Since Piaget, research concerning mathematical development has flourished. However, the paradigm from which this research was approached shifted. Researchers began focusing on mathematical skills in the young child in terms their capabilities rather than their deficiencies. The previous focus, which had been examining the young child's lack of conservation and other skills, shifted to investigating the capabilities preschoolers possess. In other words, instead of trying to identify and define preschoolers' mathematical deficits from an adult perspective, the focus shifted to identifying and defining preschoolers' mathematical capabilities (Gelman, 1971).

Challenges to Piaget's theory. Researchers in the post-Piagetian era have followed two distinct paths. NeoPiagetians have focused their investigations on the definable prerequisites of mathematical capabilities instead of making attributional judgments concerning cognitive processes (Resnick, 1983). A second group of researchers have questioned some of Piaget's assertions. Gelman (1971) was one of the first researchers of mathematical cognition to challenge Piaget's theory of mathematical development. Gelman hypothesized that Piaget's methodology had masked children's true mathematical competence. Gelman (1971) replicated and refined many of
Piaget's original tasks in her research. She found that although children do not succeed on Piaget's version of the number conservation task until the age of 6 or 7, they succeed at variations of the task much earlier (Gelman, 1969). Based on these studies and others, Gelman and Gallistel (1978) identified five features, or principles of young children's counting: one-to-one correspondence (where each number is used for one object only), adjacent objects are counted consecutively, counting begins at an end (as opposed to the middle), each object is counted only once, and the last number counted in a given set represents the quantity of that set. The last feature was labeled by Gelman and Gallistel (1978) as the "cardinality principle." Gelman and Gallistel (1978) identified four behaviors which indicated a young child was using the cardinality rule: the ability to immediately respond when asked, "how much?," emphasis on the last word counted, repetition of the last word counted, and stating without recounting the last word counted. Pointing at objects, either manually or visually, and knowing the standard direction for counting are rules built upon these five foundational rules (Fuson and Hall, 1983).

Research on counting: Rules and constraints. Briars and Siegler (1984) also investigated the numerosity capabilities of young children. However, they focused on how and when young children are able to identify counting mistakes. Briars and Siegler (1984) identified eight kinds of mistakes: skipping an object, double counting, omitting a word, putting in extra words, counting a nonadjacent object, beginning in the middle, double pointing, and reversing direction. Briars
and Siegler (1984) tested children's ability to identify these eight kinds of errors by counting objects in front of the child and making mistakes, then asking the child to tell them what had been done wrong. Briars and Siegler found that 5-year-olds performed significantly better than younger children at identifying errors, with the exception of reversing the direction of counting. When a 5-year-old child watched a researcher count the objects in correct order, but the researcher counted right to left instead of left to right, the 5-year-old rarely identified reversed counting order as a mistake. Briars and Siegler (1984) concluded that with increasing age children become more aware that correct counting operates under certain constraints. Their research demonstrated that learning what components are required and which are optional for accurate counting is a gradual process typically not yet complete by the age of five (Briars & Siegler, 1984).

Research conducted by Fuson and Hall (1983) supports the idea that 5-year-old children know many, though not all of the necessary components for accurate counting. After observations of 3-, 4-, and 5-year-old children counting, Fuson and Hall (1983) identified several distinctive features of 5-year-old's counting strategies. One feature was that 5-year-olds spontaneously point when they count, which implies that they still need to concretely pair an item with a word. In addition, Fuson and Hall (1983) found that most 5-year-old children correctly implement the cardinality rule as identified by Gelman and Gallistel
Fuson and Hall (1983) asserted that observance of the cardinality rule by a child indicates that the child understands cardinality and numerosity.

Siegler (1982) elaborated on the theme of numerosity development through research on how children count. He developed a construct for how children cognitively formulate counting. Siegler formulated a set hierarchical rules to illustrate how numerosity skills develop. The five rules and their subsequent diagrams gave the researchers guidelines for assessing the counting performance of children ages three to seven years. Of the five rules, 5-year-olds were most likely to use Rule II (see Figure 2). Rule II is a construct for describing how 5-year-olds tackled a counting task. A 5-year-old would first make a judgment, considering the quantity of the items to be counted, and then he or she would consider the kind of counting task: a numerosity task or a transformation task (involving numerosity). A transformation task involving numerosity is a problem where the quantity involved changes or is perceived to change. Finally, the child would arrive at an answer.

Figure 2

![Diagram of Rule II](image)
The diagram of Rule II illustrates an overlap between the constructs of Siegler and Piaget. Piaget's concept of number conservation indicated that 5-year-old children do not identify equal quantities if one set of objects is spread out farther than its equal partner (this task was considered unresolved until around six years of age). Siegler (1982) supported this finding on a limited basis; if a 5-year-old child is confronted with a large collection of objects to count they will make judgments concerning quantity according to the length of the row of objects. However, he found that around the 5-year-old children could successfully solve the number conservation task with smaller quantities.

Siegler (1983) further explored the progression of rules children use when counting. Young children form their rules for solving numerosity tasks by finding a rule that leads, at least initially, to success. Once a rule has been a success it functions as the rule of choice when the child is presented with a new task and will not be altered or replaced until it proves erroneous over multiple attempts (Siegler, 1983).

Siegler (1983) found that 5-year-olds approach tasks involving multiple properties by focusing on one attribute. When they were compared to a group of 8-year olds, the group of 8-year-olds could accurately resolve tasks which included multiple attributes (Siegler, 1983). Case also identified stages where children initially focused on one dimension of a task and subsequently built upon this prior knowledge (Case, 1992). Other researchers have also observed that
5-year-olds have difficulty focusing on more than one dimension (e.g. weight, measure, distance) of objects when performing a task (Gelman, 1978; Resnick, 1989; Case 1992; Siegler 1998).

The mental number line. Fuson and Briars (1980) studied the counting applications of 2- through 5-year-olds and identified several principles used by 5-year-olds for numerosity success. The most enduring of these concepts was that 5-year-old children who were successful at counting had developed mental picture of the number line. This mental number line gave young children a means of visualizing their counting (Fuson & Briars, 1980). Fuson and Briars asked children to verbalize their procedure as they counted and found that unsuccessful children did not have a mental representation of the number line. Fuson and Briars based their study on the work of Greeno, Riley and Gelman (1978), which indicated that children need to make successor relationships between numbers to count successfully. They found that young children could count reliably as long as they could make "next" connections and that having a mental representation of the number line was an integral part of success on numerosity tasks (Greeno, Riley, & Gelman, 1978).

Resnick (1983) expanded and defined this concept, which was labeled the "mental number line" (see Figure 3).
Figure 3 illustrates Resnick's conception of the mental number line and how it flows. Children count using "next" connections and are successful as long as they have prior knowledge on which to build next connections. Based on her research, Resnick (1983) expanded the concept of the mental number line. Resnick (1983) proposed that children use the mental line for "counting on" procedures involved in basic addition and subtraction problems. In order to achieve this level of numerosity understanding, Resnick (1983) asserted that children needed to grasp that the flow of the number line becomes reversible, meaning that one can count backwards as well as forwards.

Figure 4

This extension of a child's mental number line concept allows a child to make backward "next" connections (see Figure 4). Resnick (1983) asserted that
many children have the capability to make "next" connections, backward and forwards, prior to entering kindergarten. Resnick (1991) felt that the mental number line was one of cognitive constructs she referred to as "central conceptual prerequisites." Resnick (1991) asserted that the mental number line was one of the central conceptual prerequisite necessary for success in future mathematics endeavors, such as addition and subtraction.

The concept of the mental line becomes increasingly important as children expand their mathematical knowledge. Siegler and Robinson (1982) found that children as young as 3 years of age understand that when an item is added to or taken away from a set the quantity changes. However, until a child has an understanding of the mental number line, around the age of 5, they do not have a concrete, quantified basis for understanding of addition and subtraction (Resnick, 1991).

Siegler and Robinson (1982) empirically investigated preschoolers' understanding of the mental number line and "next" connections. In the simple study design, the researchers asked children to count as high as they could. Siegler and Robinson (1982) found that counting capabilities were highly correlated with age, the 5-year-olds being the able to count the highest. Siegler and Robinson found that one of the prerequisite skills needed to be a proficient counter was a sense of the "next number." In definition and practice the concept of "next number" is identical to the concept of the mental number line. A related phenomenon serendipitously discovered through Siegler and Robinson's study
was that the 5-year-olds were significantly more likely to know the re-occurring
pattern which appears in the positive number line. Therefore, when a 5-year-old
came to a difficult transition such as 29, all they had to do was to remember the
next prefix and not the entire number. For example, if a 5-year-old came to 29
and remembered 30, the repeating pattern would follow: 31, 32, 33, etc. This
pattern was also evident by the fact that, when 5-year-olds discontinued
counting, the majority of the time it was at a "9" juncture. In effect, 5-year-old
children who are proficient counters integrate the concepts of a mental number
line and the pattern which numbers follow.
Ill. The Impact of Prior Knowledge on New Learning

Recently, a great deal of attention has been directed to the relatively low level of mathematics achievement of American students, when compared to those in other countries (Beaton, 1996). One possible contributing factor to this profile may be deficits or gaps in the mathematical knowledge children possess when they enter kindergarten. Resnick (1983), and Griffin, Case, and Siegler (1994) addressed the issue of children’s “knowledge gaps” and indicated that knowledge gaps could set in motion a downward spiral of mathematics achievement throughout the formal schooling years. Several programs have attempted to rectify this potential problem by providing math enrichment to preschoolers; among them are Headstart, educational programming on television, and other special enrichment programs. Still, much remains to be learned about the typical student’s mathematics readiness skills and the effects of weak entrance skills on later mathematical achievement. Researchers have suggested that a child’s numerosity capabilities are critical prerequisites for other mathematical skills such as addition and subtraction (Gelman & Gallistel, 1978, Resnick, 1989, 1991, 1993).

The entrenched nature of prior knowledge. Ginsburg (1983) investigated the role of prior knowledge in children’s mathematical thinking. Ginsburg found that children’s attachment to prior concepts was so powerful that often they would rely on previously learned concepts even after being instructed in new procedures. Ginsburg (1983) also found that children integrate known and
unknown procedures into self-invented methods. These self-invented methods are dependent on the child having some kind of prior knowledge to integrate. Siegler (1983) also provided some insights into the role of prior knowledge in learning. The rules that Siegler identified built upon one another and each were dependent upon prior knowledge (Siegler, 1983). Siegler found that children form their rules for solving tasks by finding a rule that leads, at least initially, to success. As discussed above prior knowledge is so entrenched that once a rule has been a success it functions as the rule of choice when presented with a task and will not be altered or replaced until it proves erroneous over multiple attempts (Siegler, 1983).

Siegler (1983) further asserted that when children do not learn the prerequisite numerosity skills, subsequent mathematical learning may be jeopardized. Siegler (1983) indicated that one of the most serious effects of inadequate prior knowledge may be learning difficulties. When foundational principles, such as those identified by Gelman and Gallistel (1978), are not learned, children may experience knowledge gaps. Siegler (1995) compared the strategies of 5-year-olds and 8-year-olds who were able to count to 100 with those children who were able to count to 50 and 20. The research indicated that a child's cardinal counting proficiency was dependent upon the ability to make "next connections." For example, children that understood the pattern of numbers counted much higher. In addition, children that understood the pattern of numbers and the transitions (e.g. 19 to 20, 29 to 30, 39 to 40, etc.) counted
even higher than did children without this prior knowledge. Siegler (1995) noted that not all counting is done by knowing the pattern of number names; eleven and twelve as well as the prefixes had to be memorized for a child to be successful.

The Rightstart Program. Griffin, Case, and Siegler (1994) examined the effect of "knowledge gaps" on future learning and the feasibility of implicitly teaching basic knowledge principles through their "Rightstart" program.

Rightstart extended Resnick's (1983) work which investigated the role of particular prior knowledge constructs children needed to be successful in mathematics in the early grades. Resnick proposed that the "mental number line" was a necessary for successful completion of the mathematical expectations of the early elementary years. It is easy to imagine that, without a representation of a mental number line, understanding basic mathematics would be difficult or impossible. Rightstart was based on the assumption that the concept of a mental number and its functions could be explicitly taught to children in the preschool years. According to Griffin et al., if low-SES children could learn this knowledge through instruction, they would begin school with a knowledge foundation to build upon instead of a knowledge deficit to overcome.

Griffin, Case, and Siegler (1994) compared number line knowledge possessed by mid- and upper socio-economic status (SES) children entering kindergarten to the number line knowledge of low-SES children. Griffin et al. found that "knowledge gaps" were more likely to exist in groups of socio-
economically disadvantaged groups, especially those from inner-city areas, than in groups of mid- and high SES children living in urban or suburban areas.

The Rightstart program used thirty interactive games to teach basic mathematics concepts. The games were designed to be "hands-on" and concrete. The games were intended to aid the children in constructing, consolidating, and understanding basic mathematical concepts. Griffin, Case, and Siegler (1994) identified three main foci of the instruction included in the Rightstart program: teach relative magnitude of any concrete set of objects, teach the "increment rule" (the addition or subtraction result in a change of cardinal value), and teach relative position on the number line. Participating children were divided into groups of no more than five per instructor and the duration of the program was four months. Three different groups of first grade children participated in the program over a period of three years. Each group was followed-up with once a year for two years after the original program administration to ascertain whether there were long-term benefits to the program.

The results of the Rightstart program are promising. As compared to the control group, the Rightstart group showed greater gains in mathematics in kindergarten and in first grade (Griffin et al., 1992). The children in the Rightstart program were also compared to children in mid- to upper- SES groups. The researchers found that children who had participated in Rightstart were closer to approximating the gains made by those groups as compared to
children in the low-SES group that had not participated in the Rightstart program. The authors concluded that the Rightstart program is effective in teaching the targeted central conceptual prerequisites of primary level mathematics (Griffin et al., 1992).

The Impact of Knowledge Gaps on Minority Students

Many of the studies which address the issue of knowledge gaps were conducted in the context of investigating why minority children continue to do relatively poorly on mathematics achievement tests than do majority children. There are specifically defined social groups in the United States that traditionally do poorly in mathematics (Resnick, 1989). Usually these groups are defined as the cultural minority and/or of low socio-economic status (Resnick, 1989).

Griffin, Case, and Siegler (1994) found that children from low socio-economic status (SES) groups did not possess the same prior mathematical knowledge that children from the middle- and upper-socio-economic status groups possessed. Specifically, they found that more children from low SES groups did not possess a mental representation of the number line, hence the Rightstart Program designed to fill in these knowledge gaps (Griffin, Case, and Siegler, 1994).

Ginsburg and Allardice (1984) researched the impact of prior knowledge on the current learning of African-American students by conducting a cross-cultural study. Ginsburg and Allardice (1984) compared the counting and basic addition abilities of five and six-year-olds in the Ivory Coast and in the United
States. All of the participants were African or African-American (people of the Ivory Coast are also descendants of American slaves). Ginsburg and Allardice (1984) found that children in the Ivory Coast were better at basic mathematics than were their U.S. counterparts. This countered the claim that differences in mathematics performance were due to cognitive deficits (Ginsburg & Allardice, 1984). Ginsburg and Allardice (1984) proposed that the difference between the two groups was that the children in the Ivory Coast were taught mathematics in a culturally meaningful way. In the Ivory Coast children had access to enrichment programs which presented basic mathematical knowledge in ways which were integrative with their culture and therefore, their informal learning (Ginsburg and Allardice, 1984). Ginsburg and Allardice (1984) hypothesized that minorities in the United States would make better gains in mathematics if there were a better fit between formal instruction and culturally driven informal learning (1984).

Resnick (1989) explained this discrepancy between socio-economic groups differently than Ginsburg and Allardice. Resnick proposed that it was not socio-economic status that determined whether a child spent their preschool years in an academically enriched environment, but the level of connectedness the family felt toward the educational system (Resnick, 1989). Resnick faulted educators for not building on the informal learning all cultures generate; she stated, "current school practice however, seems not to build on this informal knowledge, and in some cases, it even suppresses it deliberately (1984)."

Resnick (1989) also discussed the long-range effects of this cultural bias in
mathematics. She cited the statistically low percentages of minorities in careers that emphasized mathematics as an indication that many minorities begin formal instruction at a deficit (Resnick, 1989). In addition, Resnick (1989) voiced many of same concerns about girls and mathematics as she did for minorities and mathematics.

One point of convergence in the literature discussed above is that formal instruction has to meaningfully bridge to a child's informal knowledge. Although this concept was developed and elaborated through work with minority students, it holds true for all students. Some children will be able to memorize basic mathematics facts, but when they need to make connections in order to perform higher mathematical functions they may experience difficulties or failure. The formation of mathematical knowledge should be viewed much like knitting a sweater. You can knit all the rows of a sweater in isolation, but unless they are interconnected you will not have anything to wear.
IV. Teachers' expectations of Incoming Five-Year Olds' Numerosity Skills

Anyone observing a kindergarten class during the first few weeks of school will realize that teachers expect children to enter their classroom equipped with certain types of knowledge. In some school districts, children are asked questions during “Kindergarten Roundup” to ascertain whether they are entering with “appropriate” prior knowledge. Further evidence of this emphasis on preschool knowledge is the focus of educational programming such as Sesame Street and Barney, whose goals are to provide an accessible forum for all children to acquire certain knowledge prior to entering school. Further evidence is provided by the proliferation of preschools and preschool-based day-care centers for young children that have an academic focus. The success of such television and preschool programs suggests that American society, as a whole, expect children to have prior knowledge of certain domains before entering school.

Background

There is a long history of research on teachers' expectations (e.g., Rosenthal 1973, 1995); however, it is not within the scope of this paper to review it. Nonetheless, it is noteworthy that this literature suggests teachers' expectations may exert a powerful influence on students' academic achievement. Because kindergarten is a student's first introduction to formal
schooling, it would make sense that teachers' expectations may influence the direction a student takes throughout school.

Despite the American emphasis on preschool knowledge, little research has been conducted on the subject of teachers' expectations of the numerosity capabilities of children entering school. Exceptions are the few related relevant studies on elementary and secondary teachers' expectations of students' mathematical abilities, which provide insights into teachers' expectations of mathematical knowledge at other levels. These studies, reviewed below, generate questions concerning teachers' expectations and provide a foundation from which to research the question of prior numerosity knowledge of kindergartners and examine its compatibility with teachers' expectations.

Ginsburg (1977) demonstrated that teachers' expectations may not always be in line with a student's actual capabilities. Ginsburg (1977) interviewed a teacher to ascertain what her perceptions were of a particular student's mathematical ability. Ginsburg found that the teacher's expectations were low due to the fact that the child had poor verbalization skills and could not explain his computations. In actuality, there were no differences between the mathematical capabilities of this student and the students in the class deemed as average by the teacher (Ginsburg, 1977).

Teachers' impact of expectations on instructional practices. When children experience mathematical difficulties in the early elementary years, the child's teacher is often the first person who is aware of a problem. When
academic difficulties arise, parents and students depend on teacher expertise for assistance. Fenema, Franke, Carpenter, and Carey (1993) investigated how teachers utilize their knowledge of children's cognitive development. Teachers' expectations of children's mathematical skills important because they may dictate the type of instruction a child receives (Ginsburg, 1980). Fenema, et al. (1993) carried out a study that demonstrates the impact of teachers' expectations on instruction. This study investigated the impact of first grade teachers' understanding of children's cognitive processes on mathematical instruction. The study compared a teacher who had been instructed on the cognitive processes of learning mathematical concepts and a teacher who had not received any additional instruction. This study indicated that the teacher, who had knowledge of how children learned and knowledge of what constitutes developmentally appropriate curricula, experienced more success than her peer. Success in this study was measured by students' knowledge on pre- and post-tests, indicating that the children in the treatment group learned more than the children in the control group during the study. Fenema et al. (1993), concluded that teachers' expectations play a significant role in children's learning.

A study similar to that conducted by Fenema et al. (1993) was carried out by Funkhouser (1994), who investigated the role of first grade teachers' expectations in mathematics. Funkhouser (1994) hypothesized that teacher awareness of their own expectations in mathematics would allow for more mathematical success in the early grades. In addition, he predicted that this
information would impact the curricula used to teach mathematics in the early grades. Funkhouser surveyed 64 first grade teachers at the beginning and again at the end of the school year. The survey included questions about eleven basic areas: numeration, addition, subtraction, multiplication, fractions, measurement, money, time, calendar, symbols, and geometry. Funkhouser found that teachers' expectations were highest in numeration (counting) than in any other area. Specifically, Funkhouser found that participating teachers had expectations that students would be able to identify, write, and count numbers from "0" to "10". The teachers who responded indicated that their main expectations for first graders' numerosity capabilities were in the concrete rather than abstract sphere. These expectations are consistent with predictions generated from Piagetian theory: 100% of the teachers expected that if 0-10 objects were laid in front of a child, that child should be able to accurately count the objects. It remains to be seen whether kindergarten teachers will share these expectations of their incoming students' counting abilities.

Funkhouser (1994) also found that other mathematical skills that require competent numerosity skills such as adding, sum less than zero, were the next highest in terms of the teachers' expectations. This is consistent with Resnick's (1982) findings that simple addition is an extension of numerosity skills. Resnick saw this issue not only as a question of skill, but as a question of development. Resnick asserted that children could memorize simple counting sequences; however, the more complex simple addition skills indicated that the child had and
was able to use a mental representation of the number line. We can infer that Funkhouser's study indicated that a majority of teachers (55%) expect that children will have and be able to use their mental number line in simple addition tasks.

**Influences on teachers' expectations in mathematics.** In addition to knowledge of child development and personal experience, teachers' expectations of children's prior knowledge of mathematics may also be influenced by professional groups. The National Council of Teachers of Mathematics periodically publish standards of mathematics competence. The *NCTM Standards* have been adopted by thirty-eight states (including Iowa) as the state standards of mathematical competence (National Council of Teachers of Mathematics, 1989). The standards are intended to give teachers a "yardstick" with which to measure mathematics performance and progression (Burrill, 1997). The standards identified two major challenges for math educators: first, to make classroom practices more meaningful and, second, to rethink the definition of the where, what, and when of basic mathematical knowledge (NCTM Standards, 1989). Given the widespread publicity and implementation of the NCTM Standards (1989), it is likely they have impacted teacher beliefs about what constitutes acceptable math progression.

The National Council of Teachers of Mathematics (NCTM) has authored guidelines for the mathematical knowledge preschoolers should have before entering kindergarten. It is important to note however, that these guidelines
have a theoretical foundation but not an empirical one (NCTM Standards, 1989). The most recent NCTM Standards were written in 1989 by a committee composed of mathematics educators. The standards were written in four grade bands, preK-2, 3-5, 6-8, and 9-12+. No preschool mathematics experts were involved in the preparation of this document. Due to the emphasis on grades other than preschool in the pre-k-2 band, the recommendations for what children should learn prior to kindergarten are not very elaborate (Burrill, 1997). These recommendations are that children should enter kindergarten knowing how to identify 10 objects and count to 10, and that they should have an understanding of counting principles. Citing Gelman and Gallistel (1978), the guidelines outlined that in this process of counting to 10, children should demonstrate knowledge of one-to-one correspondence, the correct order of counting, only to count objects once, and that the last number counted indicated the cardinal value of the set (NCTM Standards, 1989, 1991). The standards acknowledged the fact that children have an understanding of many basic mathematics principles prior to entering school and that this knowledge should be built upon in subsequent grades (Pepper, 1998). A more elaborate set of guidelines for preschoolers could serve as a guide for kindergarten teachers in their expectations of incoming kindergartners mathematical readiness. In addition, more comprehensive guidelines for preschoolers could guide the assessment practices of school psychologists. School psychologists would be able to use in
depth guidelines to test mathematical readiness and recommend early interventions.

More elaborate guidelines for pre-kindergarten mathematical readiness are on the horizon. The NCTM is currently writing mathematics standards for the year 2000. The NCTM Standards (1989) are being used as the foundation for the new standards, however, a pledge was made in the initial announcement of the new standards for more comprehensive preschool guidelines (Burrill, 1997). The basic structure and bands of grades will remain the same in the new standards as in the original NCTM Standards. The authors of Standards 2000, have announced that mathematical development will play a larger role in the formulation of the new standards. To meet this goal, developmental experts will work with mathematics educators to develop Standards 2000 (Burrill, 1997). At the meeting launching the new guidelines the role and importance of preschool mathematical knowledge was acknowledged as a vital component of mathematical development and success in future mathematical development (Burrill, 1997). The Standards 2000 document was launched in September 1997 with an initial draft to be released in October 1998 (Burrill, 1997).

**Implications for Educational Professionals**

Although little is known about how educators use research about informal learning to alter instruction (Fenema et al., 1993), there are certain implied conclusions. Educators and other educational professionals need to have a basic understanding of the role of informal learning and prior knowledge in order
to improve instruction and assessment procedures. Resnick (1989) challenged educators to develop approaches of mathematical instruction "specifically tuned" to children's needs. Specifically tuned or individualized instruction would address the informally learned knowledge base which each child enters school with and build upon it, bridging informal and formal learning.

Similar to instructional practices, assessment practices of early mathematical capabilities could be improved by first understanding the role informal learning and prior knowledge play in the acquisition of new knowledge. Ginsburg (1977), Greeno, et al. (1984), and Resnick (1989) proposed that one way of assessing children's informally learned knowledge was to analyze mistakes made in developmentally appropriate mathematical tasks. Often through analyzing children's mistakes a teacher can identify knowledge gaps. Ginsburg (1977) asserted that documenting children's invented methods for solving mathematical tasks was a valid assessment tool. More precise assessment of informal knowledge, especially early mathematical capability, would assist educators in developing more germane mathematics enrichment and remediation programs.
V. Conclusions

Summary

As the literature reviewed above indicates, there is a consensus among researchers that informal learning plays an important role in new learning. Researchers have proposed that the informal learning of basic mathematical concepts begins in the first months of an infants' life (e.g., Spelke, 1990, Gelman, 1972). By the time a child begins formal schooling, many children have foundational knowledge in mathematics gained simply by interacting with the world around them (e.g., Siegler, 1997).

An area of focus in the study of the role of prior knowledge in mathematics is how children develop a sense of number and quantity. The literature reviewed above characterizes children's' developing understanding of numerosity. Over the past 90 years, research in developmental psychology has defined many of the numerosity capabilities of preschool children. Preschoolers are capable of using the basic principles of counting as described by Gelman and Gallistel (1978), and Fuson and Briars (1983). In addition, there are definable rules or procedures which preschoolers will follow when asked to complete a counting task (Siegler, 1982). The research of Resnick (1991) among others, has also identified and refined the role of informal learning in mathematical understanding of concepts such as the mental number line.
Limitations of the Reviewed Literature

One problem in the research on numerosity development is the lack of an agreed upon definition of what constitutes mastery of numerosity skills. Because each researcher formulated their own definition of mastery, the lack of an overall definition makes studies difficult to interpret or replicate. Piaget asserted that skill mastery was indicated by stable usage, that is, the implementation of a concept across time and situations. However, other researchers, including Gelman and Gallistel (1978) and Resnick (1982) have argued that initial competency, the implementation of a concept in a given situation, is an indication of mastery. Piaget’s number conservation task provides a classic forum for this debate. Most children are not successful with the number conservation task until the age of 6 or 7 (Kitchener, 1986), yet, almost all children are successful on variants of the task by the time they are 2 or 3 years of age (Gelman 1972).

What has not been established through research is how informal knowledge is used by teachers, in the early grades, to build new knowledge. Resnick (1989) claims that this bridging of informal and formal knowledge is the biggest challenge faced by teachers of mathematics today. The teacher’s role in bridging informal and formal knowledge is paramount. Teachers’ expectations drive instructional planning (Fenema et al., 1993). Research has indicated that instruction planned by teachers with an understanding of children’s cognitive processes is more effective than instruction planned by teachers without
understanding of children's cognitive processes (Fenema et al., 1993). Fenema et al. (1993) proposed that when teachers understand the role of prior knowledge in their children's classroom learning, instruction is significantly improved. The interaction between informal and formal learning is one of the cognitive processes identified by Fenema et al. (1993) which is important for teachers to understand for improved instruction.

**Educational Implications of Studies on Numerosity**

Understanding the mathematical capabilities of preschool children is of great benefit to caregivers, parents, and educators. New knowledge is built on prior knowledge (Bruer, 1993). Understanding preschool children's' mathematical capabilities allows caregivers and parents of preschool-aged children to provide mathematically enriching activities during the preschool years. This same understanding allows educators to write, choose, and implement curricula for students that builds on their existing knowledge. It also allows educators to write, choose, and implement remedial programs for children who do not enter school with the type of prior knowledge necessary for new learning. Through better understanding of the mathematical capabilities of preschoolers, children in the future may begin school with a firm foundation of prior knowledge instead of the deficits so many children have when they enter the public school system.

When there are gaps in a child's prior knowledge base the child is left with a shaky foundation on which to build future constructs (Siegler, 1983). Focusing
on the mathematics skills of the preschool-age child, researchers have been able to identify certain central conceptual prerequisites necessary for mathematics success in the early grades. Given the importance of these central conceptual prerequisites in mathematical success, it is urgent that educational professionals understand the key role prior knowledge plays learning. One of the first steps in bringing the United States up in world standards in mathematics will be to ensure that American children begin their academic careers with the necessary central conceptual prerequisites in mathematics. With a solid foundation in mathematics at the preschool and kindergarten levels, children would be better able to build strong mathematical constructs ensuring greater future success and less frustration in mathematics.

Directions for Future Research

One direction for future research is to investigate the interaction between the informal knowledge children possess when they enter school and what teachers expect children to know prior to entering school. Due to the lack of research on this subject at the preschool and kindergarten levels, it is possible that there is a mismatch between children's informal knowledge and teachers' expectations. If teachers had a better understanding of the informal mathematical knowledge with which children enter school they could tailor their instruction to bridge informally and formally learned knowledge in meaningful ways. This assessment of informal mathematical knowledge would have to consider aspects such as the ethnicity and cultural background of the children as
well as their access to preschool enrichment programs. Knowing what level of informal learning children enter school with could also assist the authors of published curricula to strive for a better fit between their curricula and children's level of mathematical knowledge when entering school.

Suggestions for Future Research

To gain information about the "fit" of teachers' expectations of prior knowledge and the actual prior knowledge children enter school researchers could interview teachers, prior to the beginning of the school year, about the extent of prior knowledge of numerosity with which they expect children to enter their kindergarten class with. Teachers' responses could be coded according to how well they match the preschool informal learning identified in the literature. The teachers could be shown a number of tasks and then asked about their expectations concerning their incoming children's' performance. In addition, a subset of their class could be given the same tasks used with the teachers and their performance measured. The teachers' predictions could then be compared to the children's performances, to assess the "fit" between the two.

The role of informal knowledge and its interaction with teachers' expectations could provide valuable information and influence instruction, which in turn, could help raise overall mathematics scores of children in the United States. If teachers could help children bridge informal and formal learning in meaningful ways, it is possible that instead of a downward spiral of mathematics failure, they could generate an upwards spiral of mathematics success.
References


