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Two Simple Exponential Decay Experiments

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When trying to teach a wide variety of experimental analysis techniques on a tight budget, teachers often have difficulty finding hands-on experiments which illustrate exponential decay and provide for the teaching of semi-log plotting skills. The authors have developed the following inexpensive and easy-to-implement experiments to meet that need.

Experiment 1 -- Discharging RC Circuits

The law is clear: If a capacitor is charged to a potential V_0 and then discharged through a resistance R, the potential drop across the resistance varies as:

$$V = V_0 \ e^{(-t/RC)} \text{ (Smithson and Pinkston, 1960)}$$

The usual implementation of this lab employs an RC circuit with a time constant on the order of milliseconds, driven by a square wave. An oscilloscope monitors the voltage. In this version of the discharging RC circuit lab, one of the new 1.0 F capacitors is used (the authors used an "NEC Super-Cap," currently available from Arbor Scientific, ph. 1-800-367-6695). Since the time constant is now much longer than was obtainable with the more common micro-farad capacitors, the square-wave generator and oscilloscope can be replaced by a DC power supply, switch, voltmeter and stopwatch. The circuit is shown in Figure 1. The dashed box represents the actual capacitor, which is treated as an ideal capacitor "C" with an internal resistance "R_{int}."

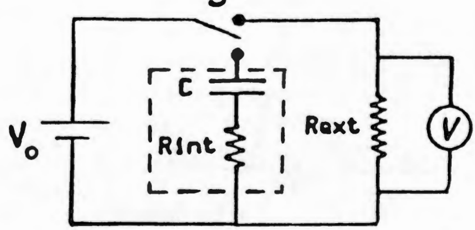

Figure 1. RC circuit used in the first experiment. The dashed box indicates the actual capacitor, which is modeled as an ideal capacitance and series internal resistance.

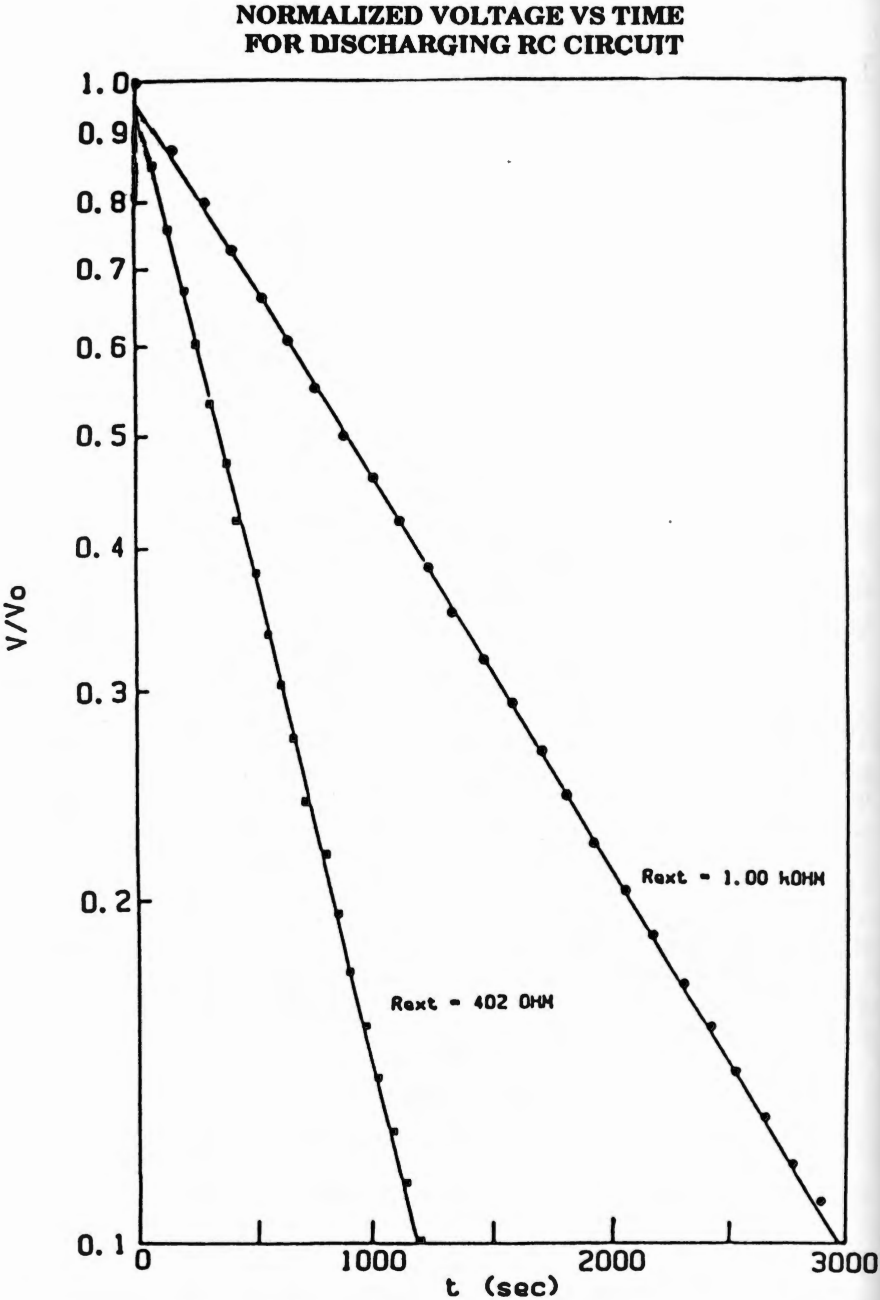

Figure 2. A semi-log plot of the discharging capacitor results for two different external resistances.

Concern arose that the high capacitance capacitors would not behave ideally. Because the internal resistance of the capacitor was suspected to be quite high, the experiment was performed twice (using two different values for the discharging/external resistor). Figure 2 shows the results of these trials. Since the slopes of the lines are given by $-1/RC$ on this semi-log plot, and $R = R_{ext} + R_{int}$, it is clear that $R_{int} = 20 \pm 20$ Ohms and $C = 1.26 \pm 0.06$ F. The capacitor's specifications sheet lists C as 1.0 F with an uncertainty of -20 percent to +80 percent, so the result of the trials is well within tolerances. Furthermore, the internal resistance is small enough to neglect for reasonable values for the external resistance. One nice feature of this experiment is that teachers may choose values for the external resistance to fit their lab periods. Not all brands of the high capacitance capacitors behave so ideally, however. The authors have found that performing this experiment with another brand of capacitor yielded a very non-linear semi-log plot.

Experiment 2 -- Leaking Water Columns

This is the authors' version of the leaking water column experiment described by Smithson and Pinkston (1960) and Skinner (1971). If a cylindrical column of water has a small leak at the bottom, the height of the column decreases exponentially (provided that the leak rate is low enough). This theory is deduced from the following relationships:

1) In the hydrostatic case, the total pressure at the bottom of a column of water of depth h is figured:

$$p(h) = p_o + \rho g h$$

where ρ is the density of the water and p_o is the air pressure outside of the tube. If there is a leak at the bottom, this equation will still be approximately valid if the leak rate is reasonably low.

2) Provided that the leak results in laminar flow, the flow rate is given by Poiseuille's equation:

$$Q = (\Delta P)\pi R^4 / (8L\eta)$$

where Q is the flow rate through a cylindrical pipe of radius R and length L, ΔP is the difference in pressure at each end of the pipe and η is the coefficient of viscosity of the water.

Also, for a cylindrical water column, the flow rate is given by:

$$Q = -dV/dt = -A\, dh/dt$$

where V is the volume of the water in the column, A is the cross-sectional area of the column and h is again the height of the column. The minus sign is due to the fact that the volume of water is decreasing in time.

The first equation shows that the pressure difference ($p(h) - p_0 = \Delta\rho$) is proportional to the height of the water column. The other two equations show that this pressure difference is also proportional to the rate at which the height of the water column changes:

$$dh/dt = -k h$$

(where k is a constant of proportionality). Finally, the solution of this differential equation clearly is:

$$h = h_0 e^{-kt}$$

where h_0 is the water column height at time $t = 0$.

So, if conditions are such that the leak rate is low enough and that the flow is laminar, the column height should decrease exponentially in time. Smithson and Pinkston (1960) and Skinner (1971) met these conditions by connecting a capillary tube to the base of the water column. A simpler method is to use a rubber hose and hose clamp to adjust the leak rate (shown in Figure 3). When performing this experiment, the authors use a "Resonance of Air Columns" apparatus, which consists of a tube connected to an adjustable height water reservoir by a rubber hose in the base. Since the resonance tubes are marked off in millimeters, the height measurements are easily made. If a "Resonance of Air Columns" apparatus is not available, teachers may use a glass tube with a single hole rubber stopper at the bottom. Simply attach a rubber hose and clamp to the stopper, and use a meter stick to measure the water height.

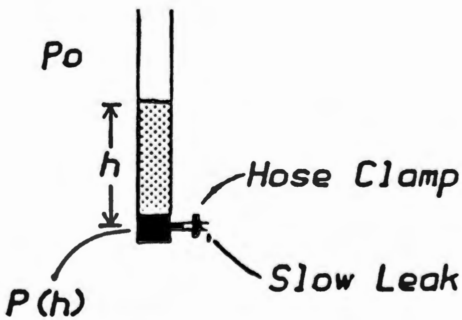

Figure 3. Rubber hose and hose clamp apparatus used in the second experiment. Po is the external air pressure and P(h) is the water pressure at the depth h.

COLUMN HEIGHT VS TIME
FOR LEAKING WATER COLUMN
(RESONANCE TUBE)

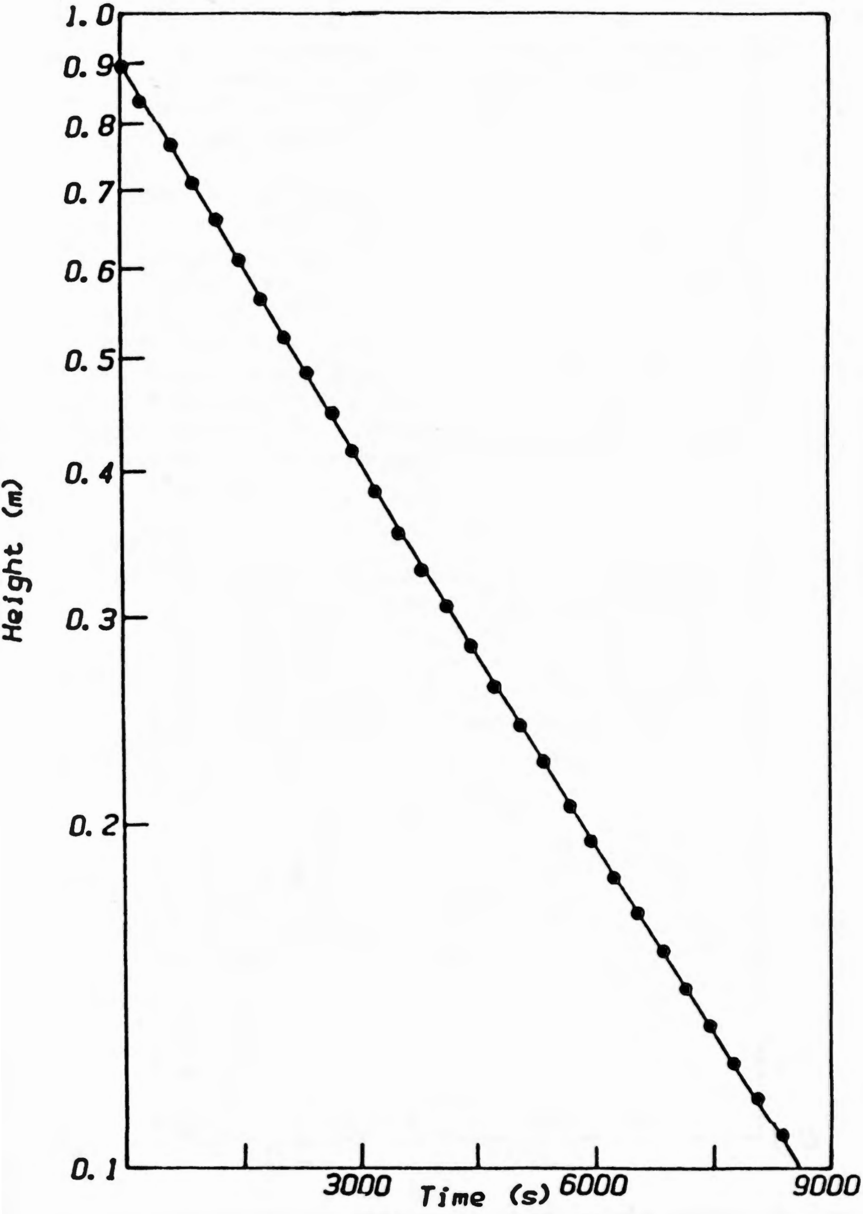

Figure 4. A semi-log plot of the dripping tube results.

COLUMN HEIGHT VS TIME
FOR LEAKING WATER COLUMN
(BURET)

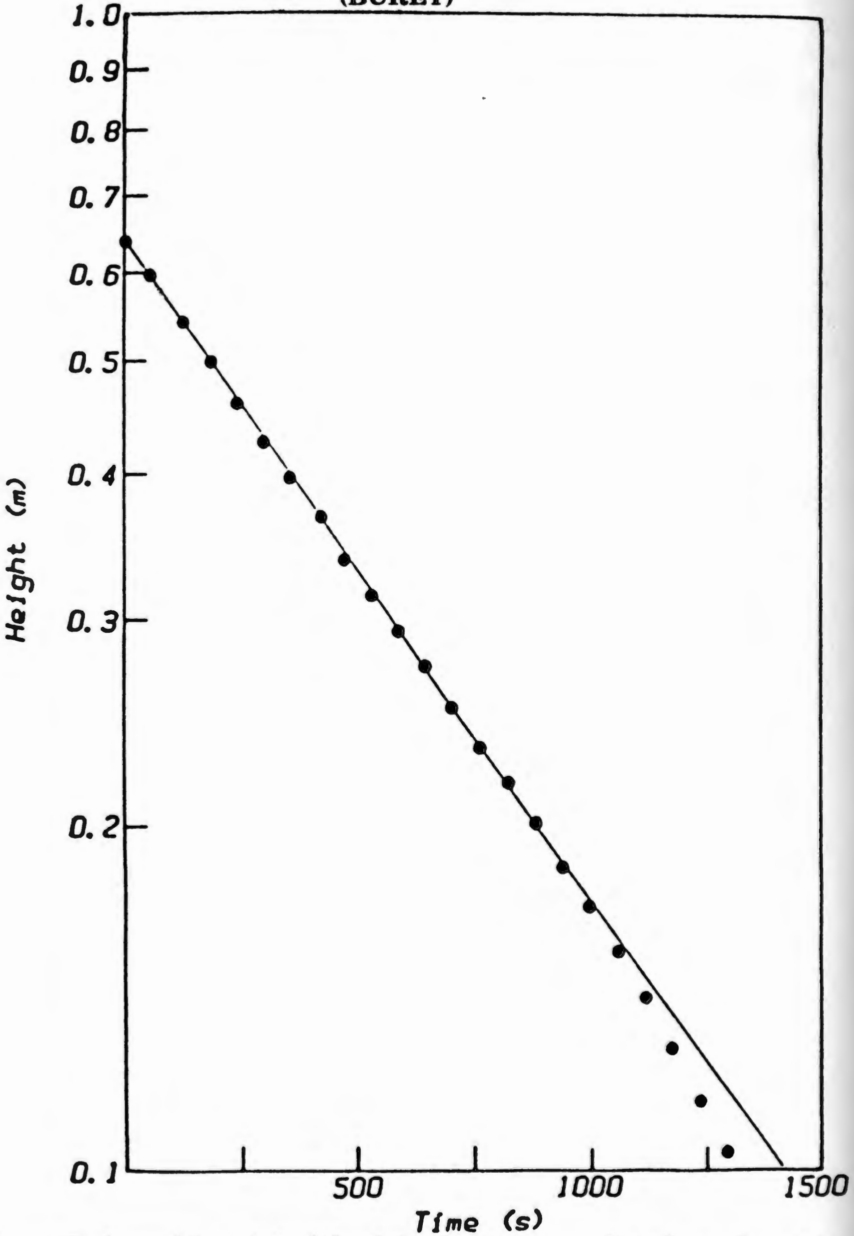

Figure 5. A semi-log plot of the dripping tube results when a buret is used. The deviation from a straight line at the bottom of the graph is due to equation (6) not being valid at the flow rate used.

In this experiment, the leak must take place at h=0 since a zero shift in the height will cause a non-linear plot on semi-log paper. Make certain, too, that no air bubbles are present in the tube and that room temperature water is used. Some trial and error practice may be needed to choose an initial leak rate which satisfies the conditions under which the $h = h_o e^{-kt}$ equation are valid and still be able to finish taking data in a reasonable amount of time. Figure 4 shows data for one trial.

The authors did attempt to perform this experiment using a buret with the valve barely open. With the narrow diameter buret used, however, getting a reasonable leak rate at the same time as meeting the conditions for the $h = h_o e^{-kt}$ equation was not possible. Figure 5 shows the buret testing results.

After using these experiments in the lab, the authors have found that the dripping water experiment does not hold student interest very well. It does, however, make a very nice lecture demonstration to show exponential decay, with the instructor pausing in his or her lecture every five minutes or so to make a height measurement and place a point on the plot.

Conclusion

Both of these experiments are easily implemented with a minimum of equipment and yield data which clearly show the exponential decay curve when plotted on linear paper and a straight line when plotted on semi-log paper. Furthermore, students get a "gut-level" understanding of the exponential decay, since they readily notice that the voltage or water height decreases much more quickly at the earlier times than the later times.

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