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Rational decision-making in the face of uncertainty: An experiment in Jeopardy!

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RATIONAL DECISION-MAKING IN THE FACE OF UNCERTAINTY:

AN EXPERIMENT IN JEOPARDY!

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Abstract: This paper examines the quiz show "Jeopardy!." In the game, players accumulate points then wager them in a final round on a single question. My study finds an empirically-best wagering system for these players, then calculates the benefit of using this system through additional games that would be won with the correct bet. My regression model then finds the determinants of empirically-best (EB) betters. The results show players tend to act too aggressively. Half of non-first place players wager incorrectly and nearly 90% of those non-EB wagers are too large. The regression shows players are most likely to make the EB bet when the correct wager is predictable, simple, and large.

Introduction

This paper examines the betting strategies of contestants on the game show "*Jeopardy!*" Game shows have been popular among economists because of their well-defined rules and the significant amount of money on the line (Metrick 1995). Metrick started the trend of examining Jeopardy in 1995 with his ground-breaking research. His paper was the first to study Jeopardy, and it provided a statistical overview of how players wagered in two-person games.

Jeopardy is a quiz show in the United States, and each game has three contestants who vie for the most points by the end of the game. In the first round, or the "Jeopardy! round," six categories are revealed with five questions each. The first clue in the category is worth 100 points, the second 200 points, and so forth. A player selects one of the categories and values (such as U.S. Presidents for 300), and the question is read. All three players have an opportunity to buzz in and give an answer[1]. If the player guesses correctly, the points are added to her score. If the player guesses incorrectly, the points are subtracted from her score, and the others players are given an opportunity to buzz in. After a player answers correctly, she is given control of the board and chooses the next clue.

After all the clues are given or time runs out, the players move on to the "Double Jeopardy!" (DJ) round. In this second round, six new categories with five questions each are

[1] As Metrick notes, Jeopardy fans know that each clue actually reveals an "answer" and contestants are trying to provide a "question." I use the conventional wording for the sake of simplicity.

revealed and the points are doubled (the first clue is worth 200, the second 400, and so forth). The same rules apply.

At the end of the DJ round, all players with a positive point total move onto the "Final Jeopardy!" (FJ) round. A category is given, then each player writes down the number of points they wish to wager. After the wager is locked in, the clue is given, and each player writes down their answer. Then, the answers and wagers are revealed. Each player answering correctly gains the number of points wagered, and each player answering incorrectly loses the number of points wagered. No player can wager more points that he has accumulated throughout the Jeopardy and DJ rounds. Following FJ, the player with the most points is the champion and returns to the next show. The third place player wins $1000, the second place player wins $2000, and the first place player wins $1 for each point accrued. If two players are tied for the highest score, both return as champions.

The FJ round thus provides an interesting look at strategic decisions under uncertainty for players. It has been the subject of research for both mathematicians (Gilbert and Hatcher 1994) and economists (Metrick 1995), and has been studied both empirically and theoretically. This paper looks to synthesize that research to suggest an empirically-best (EB) bet and find the determinants of players making that bet.

Let us examine a hypothetical game to model these decisions. Going into FJ, player A has 10,000 points, player B has 9,000, and player C has 3,000. In most cases, player A will make a wager of 2*B – A + 1 where "B" represents player B's score and "A" represents player A's score. A correct answer will give player A a score of 2B + 1, effectively locking player B out of an opportunity to win the game. Player B, realizing this, must put herself in a situation to win if player A gives an incorrect answer. If player A answers incorrectly, his score will become A -

(2*B – A + 1), or 2*A – 2*B – 1. Player B thus, to beat player A, should bet such that B – X > 2*A – 2*B – 1, where X represents player B's wager. In this situation, 9,000 – 7,000 > 2*10,000 – 2*9,000 – 1. The wager to beat player A should be 0 ≥ Wager ≥ 7,000[2]. Of course, player B still needs to contend with player C. If she wagers such that B – X, where X is her wager, is greater than 2*C, she guarantees herself a score greater than player C. In this case, wagering 2,999 or less would satisfy that condition. Thus, player B's bet should be 0 ≥ Wager ≥ 2,999 to assure a higher score than player C and to allow her to win if player A is incorrect.

Player C now examines his situation. He knows that to win, player A and player B both must answer incorrectly. He knows player A will wager 8,001. If he is incorrect, player A will fall to 1,999 points. He knows player B can wager 2,999 or less and lock him out of the game, so he must hope player B over-bets and is incorrect. Thus, a wager between 0 and 1,000 provides the greatest opportunity to win.

In looking at the EB bet, the key is to maximize the number of answer types that will provide a win. In our hypothetical situation, player A knows he will win in a RRR game (that is, player A is "right" in his answer, as is player B, and player C), a RRW game (again, player A and B are both "right" and player C is "wrong"), a RWW game, and a RWR game. He also stands to win in a WWW game if players B and C over-bet. Player B wins in WRR, WRW, WWR, and WWW games. Player C wins in WWR and WWW games if player B over-bets. [3]

When one player has negative points going into FJ, they are removed from the game, and only the remaining two players are allowed to wager. In these situations, the strategy becomes

[2] Occasionally, player A will give player B an opportunity to create a tie game by wagering 2B – A. Player B may then be prudent, in this example, to wager 0 ≥ Wager ≥ 6,999 to assure a sole win in the event of an incorrect answer by player A.

[3] This assumes players are acting as my empirical evidence suggests them to act. In this hypothetical game, if player A expects player B to make the empirically-best bet of 2,999, he may be better off wagering 2,000 to win in WWW, WWR, RWR, RRR, RRW, and RWW games. This paper does not examine the implications of players starting to implement this strategy.

much simpler. Let us look at another hypothetical situation in which player A has 15,000 points, player B has 8,000 points, and player C has -500 and will not continue on to FJ. In this case, player A again makes the lock-out bet (2*B – A + 1). She wagers 1001. Player B, expecting this bet, knows he can only win if player A is incorrect. He wagers between 6,000 and 8,000 to have a score large enough to win if player A is incorrect and he is correct. Looking at the answer types, player A wins in RR, RW, and WW games. Player B wins in a WR game.

In some games, player A's score will be greater than 2*B are the start of FJ. These are known as runaway games (J! Archive 2010) and were excluded from the data set because they provide no interesting strategic choices. Player A should always bet such that A – X > 2*B, where X is her wager, and she will always win.

The other interesting game type is ties. In 2000, a round of DJ ended with all three players tied at 5,200 points. Players A and B wagered all 5,200 in FJ and player C only wagered 5,000. All three players answered correctly and players A and B returned as co-champions while player C was eliminated (J! Archive 2010). Player C was probably hoping to hedge his bets by having some points left over if all three players were wrong. A wager of 0 would have accomplished the same goal and not left him susceptible to both a bet of 5,200 and a bet of 0. In the event of a tie, the EB wagers are everything or nothing.

This paper will gather data from a list of Jeopardy games, apply a formula for the EB bet, and then test for the determinants of EB betters.

Literature Review

Game Theory has a long and storied history in economics beginning with von Neumann and Morgenstern's *Theory of Games and Economic Behavior* (1944). The field evolved with

Nash's work in non-cooperative games and the development of what was later coined "Nash's Equilibrium" (1951). Game Theory explains the decision-making of people as rational beings. While this is not a novel concept, its innovative approach considered the actions of other players as rational decision makers before coming to a rational choice (Osborne and Rubinstein 1994).

Economists quickly latched onto Game Theory as a mathematical approach to solving utility problems. While mathematical models made economics problems easier to solve, the approach soon came under attack. Simon (1955) finds six criteria needed for a model of rational decision-making, including full knowledge of the odds of specific outcomes. Kahneman and Tversky (1979) found that people value certainty over risk more than a rational model would expect and proposed a new theory to accommodate this discrepancy. Loomes and Sugden (1982) suggest regret and rejoicing need to be factored into rational choice models.

More recently, economists have looked to empirical evidence to question the assumptions made by neoclassical theory. Game shows have become a popular way to do this because of their well-defined rules and the significant amount of money on the line (Metrick 1995). Metrick was the first to examine Jeopardy with his 1995 paper. While only studying the first and second place players, he found that players were making non-EB choices in a predictable pattern.

Since Metrick, no major, overarching study on Jeopardy has been completed. Meanwhile, a simple three-player wagering strategy has been established (Gilbert and Hatcher 1994) and a fan website carefully compiles each day's games (J! Archive 2010). The conditions are perfect for new research.

Data

Data were collected from the fan created J! Archive. The site was created by 74 game Jeopardy champion Ken Jennings. It is updated daily by a team of former Jeopardy players and fans and includes every clue, answer, and wager as well as limited demographics of each player (J! Archive 2010). Game types were categorized using Gilbert and Hatcher's model (1994). Data were collected from games aired between September 2009 and December 2010. Excluded from the data set were runaway games, players who had a negative score at the end of DJ, and games where a special relationship existed between the scores.

Data on 239 games were collected and 62 were excluded, leaving 177 games and 523 FJ wagers. Because players with negative scores are eliminated before FJ, the data set includes eight more observations from player A and B than player C. The answer type distribution is as follows:

TABLE 1 - FREQUENCY OF ANSWERS TYPES

Type	Sample	Frequency
RRR	34	0.192
RRW	16	0.090
RWR	18	0.102
RWW	21	0.119
WRR	12	0.068
WRW	28	0.158
WWR	16	0.090
WWW	24	0.136
RR	2	0.011
RW	2	0.011
WR	2	0.011
WW	2	0.011
Total	**177**	**1.0**

TABLE 2 - FREQUENCY OF CORRECT ANSWERS BY PLAYER

Player	Correct Responses	Frequency
Player A	93	0.525
Player B	94	0.531
Player C	80	0.473
Total	**267**	**0.510**

Notes: "R" represents a "right" or correct answer in FJ. "W" represents a "wrong" or incorrect answer. The first letter represents the answer of the player in first place after DJ, the second letter, second place, and so forth.

Each game was categorized by a game type, corresponding to the game types proposed by Gilbert and Hatcher (1994). The condition for each type and the EB bet for players B and C are shown below. Player A's EB bet is always assumed to be the lock-out bet.

TABLE 3 - GAME TYPE CONDITIONS

Abbreviation	Condition	Wager for Player B	Wager for Player C
1	(3/2)B < A < 2B	B	C
2	B+(C/2) < A < (3/2)B, A+C < 2B	$min\begin{cases}3B-2A-1\\ \lvert B-2C-1\rvert\end{cases}$	C
3	B+(C/2) < A < (3/2)B, 2B < A+C	3B - 2A - 1	C
4	A < B+(C/2), (3C/2) < B < 2C	2C - B + 1	2B + C - 2A - 1
5	A < B+(C/2), 2C < B	B - 2C - 1	0
6	A < B+(C/2) < 2C, A + C < 2B	2C - B + 1	0
7	A < B+(C/2) < 2C, 2B < A+C	B - C - 1	0
2P	Two Player Games	Conditional	None

In two-player games, there are two different game types to consider. When 2A – 2B – 1 > B, player B should wager B ≥ Wager ≥ 2A – 3B. When B > 2A – 2B – 1, player B should wager 3B – 2A ≥ Wager ≥ 0.

Table 4 shows the frequency with which each game type occurs and the frequency in which players make the EB wager. The EB wager represents a range of values. A player is considered to have made the EB bet if she does not put herself in a position to lose to an answer type where she would have won with the EB wager. For example, look at our earlier hypothetical game where player A has 10,000 points, player B 9,000, and player C 3,000. This game would fall into game type 5. Player A's EB bet is 8,001. Again, I identify the answer types where he wins: RRR, RRW, RWR, and RWW. Player B's EB bet is B – 2C – 1, which is 2,999. She wins in WRR, WRW, WWR and WWW games. A wager of 2,000 would show player B to be risk adverse, but would still allow her to win with the same game types, so it is best bet empirically.

A wager of 4,000 opens up player B to a loss in WWR games; it is not EB. Assuming player B does not make too large of a bet, player C cannot win.

TABLE 4 - FREQUENCY OF GAME TYPES

Game Type	Number of Games	Frequency	Number EB	Number of Wager	Freq EB
1	41	0.23	111	123	0.90
2	39	0.22	83	117	0.71
3	8	0.045	15	24	0.63
4	21	0.12	34	63	0.54
5	18	0.10	31	54	0.57
6	19	0.11	37	57	0.65
7	23	0.13	28	69	0.41
2P	8	0.045	10	16	0.63
TOTAL	**177**	**1.0**	**349**	**523**	**0.63**

For each game, I then applied this new strategy for each player to see if the outcome of the games changed. Table 5 shows the results by player.

TABLE 5 – EMPRICALLY-BEST WAGERS BY PLAYER

Player A		**Player B**		**Player C**	
Situation	**#**	**Situation**	**#**	**Situation**	**#**
Total Games	177	Total Games	177	Total Games	169
Wins Originally	115	Wins Originally	49	Wins Originally	13
Wins EB	115	Wins EB	64	Wins EB	16
# of Wagers Not EB	17	# of Wagers Not EB	88	# of Wagers Not EB	69
Frequency Not EB	0.096	Frequency Not EB	0.497	Frequency Not EB	0.408
Number of Overbets	3	Number of Overbets	80	Number of Overbets	56
Frequency Overbet	0.176	Frequency Overbet	0.909	Frequency Overbet	0.812

Using the EB strategy, players B and C were able to pick up a combined 18 additional wins. Note that when players do not make the EB bet, players B and C make a wager larger than the EB wager a vast majority of the time. This explains why they make the correct bet in game type 1 90% of the time; the EB wager is to bet everything.

Models

My model for the tendency to make the EB wager is as follows:

$$EB = \beta_1 X + \varepsilon$$

Where:

EB = Dummy variable for making the EB wager (1 for EB bet, 0 for non-EB bet);

X = A vector of explanatory variables;

ε = An error term.

The vector of explanatory variables includes:

DJPLC = A player's standing at the end of DJ;

PERCBET = The EB bet divided by the player's score;

TEACH = A dummy variable for a profession involving teaching (1 for teacher, 0 for non-teacher);

GT4 = A dummy variable for game type four (1 for game type four, 0 for any other game type).

DJPLC is a variable to account for the difficulty of wagers. First place always has the same wager to make. The game is in the player's hands and she does not have to consider the actions of other players. Conversely, the third place player is often in a dire situation. Many games require both players A and B to miss the question and for player B to overbet. In situations that appear this desperate, player C will often bet everything and hope for a miracle. PERCBET accounts for a player's propensity to overbet. When the EB wager is to bet everything, players may luck into the correct wager. I attempted to include a variable to measure the natural selection of Jeopardy's rules. Players making EB bets should be more likely to return as champions and returning as a champion provides more opportunities to learn the EB wagers.

These variables were highly correlated with DJPLC. Unsurprisingly, the more successful the player has been in previous games of Jeopardy, the more likely they were to be in first place at the end of DJ. GT4 was selected because of its relative difficulty in calculating the correct wager. Other game types with unusually high or low EB wager rates were correlated with PERCBET. TEACH was chosen as a measure of overestimating one's odds of correctly answering the FJ question. Another measure attempted was players with an advanced degree, inferred from profession. With the underemployed rate at over 20% (Marlar 2010) and players likely being better educated than the general population (Metrick 1995), it is not an accurate measure.

TABLE 6 - SUMMARY STATISTICS

NAME	N	MEAN	ST. DEV	VARIANCE	MINIMUM	MAXIMUM
EB	523	0.67	0.47	0.22	0	1
PERCBET	523	0.52	0.38	0.15	0	1
DJPLC	523	1.98	0.81	0.66	1	3
TEACH	523	0.13	0.33	0.11	0	1
GT4	523	0.12	0.33	0.11	0	1

Empirically-Best Wager Regression Results

I use a logit regression because of the dummy dependent variable. Table 7 shows the results.

TABLE 7 - REGRESSION RESULTS

NAME	MARGINAL EFFECT	T-RATIO	SIGNIFIGANCE
DJPLC	-0.189	-6.397	**
PERCBET	0.586	9.570	**
TEACH	-0.142	-2.129	*
GT4	-0.115	-1.785	

**-significant at 1% level
*-significant at 5% level

The regression generated a McFadden R-square of 0.25. This pseudo R-square takes one minus the sum of the log-likelihood function of the model divided by the log-likelihood function of the intercept. If there is no difference between the log-likelihoods, the R-square will equal zero (Wooldridge 2009, 581). Other R-squares varied between 0.27 and 0.38. The low R-square is due to a variety of reasons. Each FJ has a unique category that is given to players before they submit their wagers. Players may believe themselves more or less likely to answer a question correctly based on that category. Without interviewing contestants, it is impossible to derive confidence. I also have no way of gauging familiarity with the Jeopardy community. Sites such as J! Archive extensively discuss wagering strategies, giving an advantage to contestants who frequent them.

The model reveals contestants are far more likely to correctly wager when the EB bet is large, simple, and predicable. This result is predicted by Gilbert and Hatcher (1994) who found players make overly-aggressive wagers.

Behavioral Economics

Behavioral economics is a new approach to economics that challenges the traditional neoclassical model. It seeks to dispel the idea of rational consumers in favor of modeling how they actually act. DellaVigna (2009) argues behavioral economics differs by "asserting (1) nonstandard preference, (2) nonstandard beliefs, and (3) nonstandard decision making" (315). It focuses on real world and experimental data of individual choice making rather than modeled behavior.

The emergence of behavioral economics has the potential to alter our understanding of human behavior, and has already been adopted by entrepreneurs and government entities. Thaler and Benartzi (2004) recognize our lack of self-control and our focus on nominal, rather than real wages. Their Save More Tomorrow plan allows employees to commit their future wage increases to a retirement plan, and has drastically increased savings rates. Seventy-eight percent of sampled employees joined the plan, and 80 percent stayed with the plan through their fourth pay raise (S165). If employees had wanted to invest these raises, traditional economic theory suggests they would have done it without the plan. Yet, a vast majority of individuals joined the plan and continued with it.

Behavioral economics can even have an impact in lunchrooms. Moving fruit next to the cash register, salads to the middle of the room, and allowing students a choice in their vegetable side reduces waste and leads to more nutritious meals for K-12 students (Just and Wansink 2009). Again, traditional economic models would not have predicted these behaviors. Rational consumers are not impacted by the location of goods.

My paper adds to this growing field. Gilbert and Hatcher's wagering strategy is not new. It gives players the ability to increase their winning percentage, by a considerable amount, without any additional costs or risks. Yet, in the nearly twenty years since it has been published, little has changed in players' wagering strategies. Neoclassical theory predicts arbitrage will eliminate this advantage in the market, but in reality players are exhibiting nonstandard decision making.

The implications of this finding are not obvious. I find that players struggle to make complicated, unpredictable, and small wagers. While this may not lend itself to any new product or service, it is a starting place to finding arbitrage in other markets.

Future Research

A common wager in Jeopardy for players B and C is to bet everything but a point or two. The logic is to still have a couple of points left if everyone answers incorrectly. In some circumstances, this can boost a player from 3rd place into 2nd place. The model I used, however, makes no accommodations for this strategy; it focuses solely on winning. While the EB bet can allow players to beat players they otherwise would not, any measure of mean outcome is tainted by the inability to compete with the aforementioned strategy. I hope a future model can find the ideal number of points to leave, which would enable us to see the real advantage of EB wagers.

Another topic for future research is professions. The model makes it clear players wager empirically-best when the bet is simple and large. Contestants educated in statistics or economics may be better able to wager EB with complicated and/or small bets. My sample size was too small to test these variables, but future research could find a large sample of these contestants and test their results against the general population of Jeopardy players.

Conclusion

This paper was an attempt to show the advantages of making the EB wager in Final Jeopardy and to find the determinants of EB bettors. Seventeen years after Gilbert and Hatcher's paper (1994), players still have not adapted to the empirical best bet. Contestants still wager too aggressively and leave room for third place players to walk away with wins.

I find that players will have a far greater chance of winning a game of Jeopardy by following Gilbert and Hatcher's (1994) wagering outline. This assumes that other players are not knowledgeable of the EB wagers, but given the amount of time this information has been available with no empirical changes, the assumption is far from bold. I also find large, simple, and predicable bets as the key determinants of EB wagering players and that overconfidence in the ability to correctly answer a question may lead to sub-EB bets.

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